IOI Training Camp 2010 – Final 1, 25 June, 2010

Problem 2 Colouring Brackets[1]

Given a matched parentheses expression, we form a pictorial representation of it as follows.

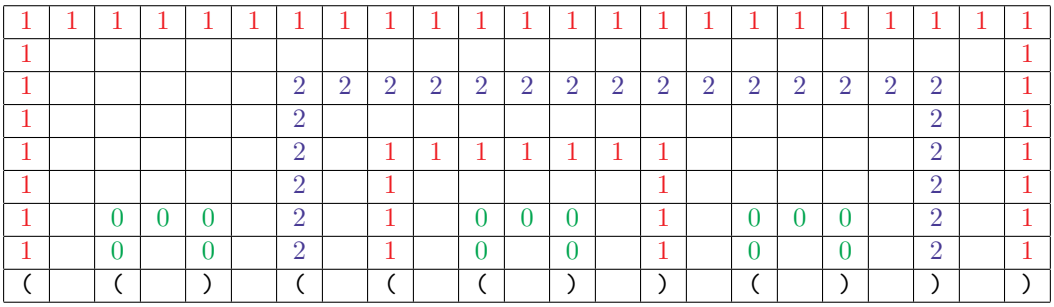
- Take a large grid of squares, and write the parenthesis expression in one row, leaving a gap of one square between two consecutive symbols.
- For a pair of matched parentheses, take the square lying above one of them, follow upwards upto a height, then travel horizontally, and then downwards upto the square lying above the matching parenthesis. Mark the squares along this path. The height is the minimum needed for the horizontal section to pass above any intermediate marked squares, with a clearance of 1.

For example, for `(()((())()))`, we have the following picture.

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
X																						X
X						X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X
X						X														X		X
X						X		X	X	X	X	X	X	X						X		X
X						X		X						X						X		X
X		X	X	X		X		X		X	X	X		X		X	X	X		X		X
X		X		X		X		X		X		X		X		X		X		X		X
(		(		)		(		(		(		)		)		(		)		)		)

The contiguous path of marked cells from a symbol to its matched symbol is called a segment. We will now colour the segments. Colours are nonnegative integers. For each segment, assign it the smallest colour that does not occur amongst the colours of the outermost segments inside it. In the above example, the colours assigned will be as follows.

[1]Problem formulated by Prateek



After colouring the segments, we want to partition the colours into two groups, *light* and *dark*, such that the absolute difference between the number of dark coloured squares and light coloured squares is minimized. In this example, if we designate the colours {0,2} as light and {1} as dark, we have 40 light squares and 50 dark squares so the difference is 10. It can be checked that this is the minimum difference that one can get, across all possible ways of partitioning the colors {0,1,2} into light and dark sets.

Input format

The first line contains an integer N, the number of symbols in the bracketed expression. The second line consist of a well-bracketed expression with N symbols, made up of the characters `'('` and `')'`.

Output format

The output should be a single number, the minimum absolute difference between the number of light and dark coloured squares.

Test Data

You may assume that $2 \leq N \leq 3 \times 10^6$.

Sample input	Sample output
12 (()((())()))	10

Time and memory limits

The time limit for this task is 1 second. The memory limit is 140 MB (actual limit 128 MB, plus 12 MB buffer for 64-bit compilation).