

Problem 2 Boxes and Coins [RunC]

Two friends, Ehrenfeucht and Fraïssé, decide to play a game one afternoon, since their internet connection is dead.

They have two rows of boxes, one with N_0 boxes and one with N_1 boxes. In row i , $i \in \{0, 1\}$, the boxes are arranged in a line, numbered 0 to $N_i - 1$ from left to right. In addition, each of Ehrenfeucht and Fraïssé has a set of K coins, numbered 0 to $K - 1$. Ehrenfeucht's i th coin is indistinguishable from Fraïssé's i th coin.

The game is played in K successive rounds, numbered 0 to $K - 1$, as follows:

- In the i th round, Ehrenfeucht chooses one of the two rows of boxes, and places his i th coin in some box in the chosen row.
- Fraïssé responds by placing his i th coin in some box in the *other* row.

There are no further restrictions on which boxes are chosen; in particular it is legal to place a coin in a box that already has coins.

Note that at the end of the game, after K rounds have been played, each row contains K coins, numbered 0 to $K - 1$. The winner is determined as follows: Fraïssé wins if the relative positions of the coins are the same in both the rows, and Ehrenfeucht wins otherwise. More precisely, Fraïssé wins if and only if, for all $0 \leq i \leq j < K$, all the following hold:

- Coin i is in a lower numbered box than coin j in row 0 if and only if coin i is in a lower numbered box than coin j in row 1.
- Coin i and coin j are in the same box in row 0 if and only if coin i and coin j are in the same box in row 1.
- Coin i is in a higher numbered box than coin j in row 0 if and only if coin i is in a higher numbered box than coin j in row 1. (This is implied by the previous two conditions)

Fraïssé manages to convince Ehrenfeucht to choose the numbers N_0 , N_1 , and K such that $N_0, N_1 \geq 2^K$, since this will guarantee that Fraïssé has a winning strategy. Your task is to play this game as Fraïssé, and win.

You must implement functions `fraisse_start(N0,N1,K)` and `fraisse_turn(row,box)` in `fraisse.cpp`.

- In the beginning, `fraisse_start` will be called exactly once, with the values of N_0 , N_1 , and K .
- Then `fraisse_turn(row, box)` will be called exactly K times, corresponding to the K rounds. In the i th call, the details of Ehrenfeucht's i th move will be supplied: the row ($\text{row} \in \{0, 1\}$) and the box number ($0 \leq \text{box} < N_{\text{row}}$).

`fraisse_turn` must return the box in the other row where Fraïssé places his coin (this must be a number between 0 and $N_{1-\text{row}} - 1$ inclusive).

Test Data

In all subtasks, $1 \leq K \leq 60$, $1 \leq N_0, N_1 \leq 3 \times 10^{18}$, and $N_0, N_1 \geq 2^K$.

- *Subtask 1 (5 marks)* : $N_0 = N_1$.
- *Subtask 2 (25 marks)* : $N_1 = N_0 + 1$.
- *Subtask 3 (70 marks)* : No further constraints on N_0 and N_1 .

Limits

The time limit for this problem is 2 seconds. The memory limit is 64MB.