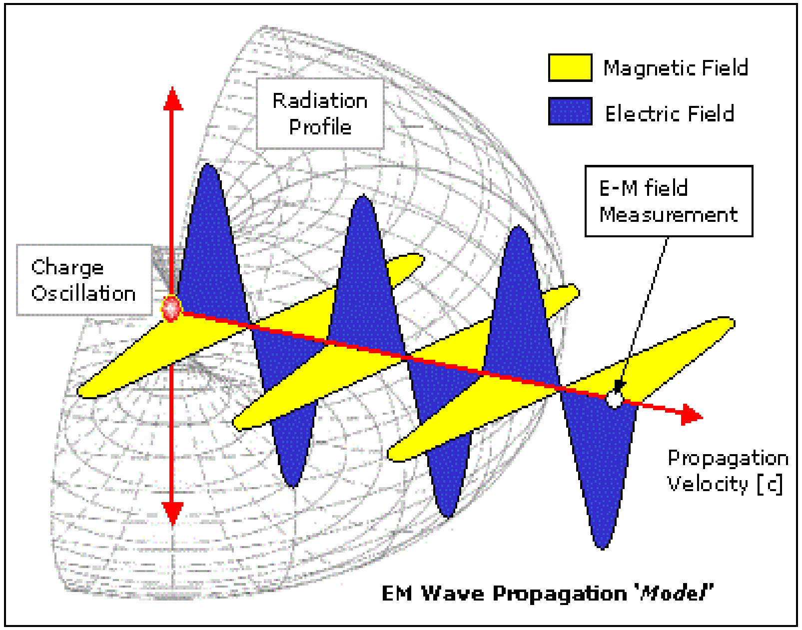
Unit-1
Electromagnetic Theory

Electromagnetic Waves and useful Mathematics

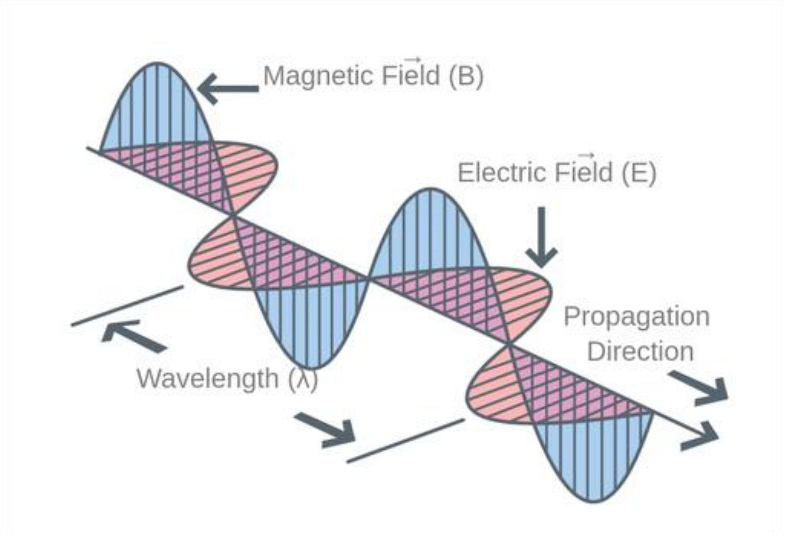
Electromagnetic (EM) wave



It is one of the waves that are propagated by simultaneous periodic variations of electric and magnetic field intensity. The electromagnetic waves are created by oscillating electric and magnetic fields.

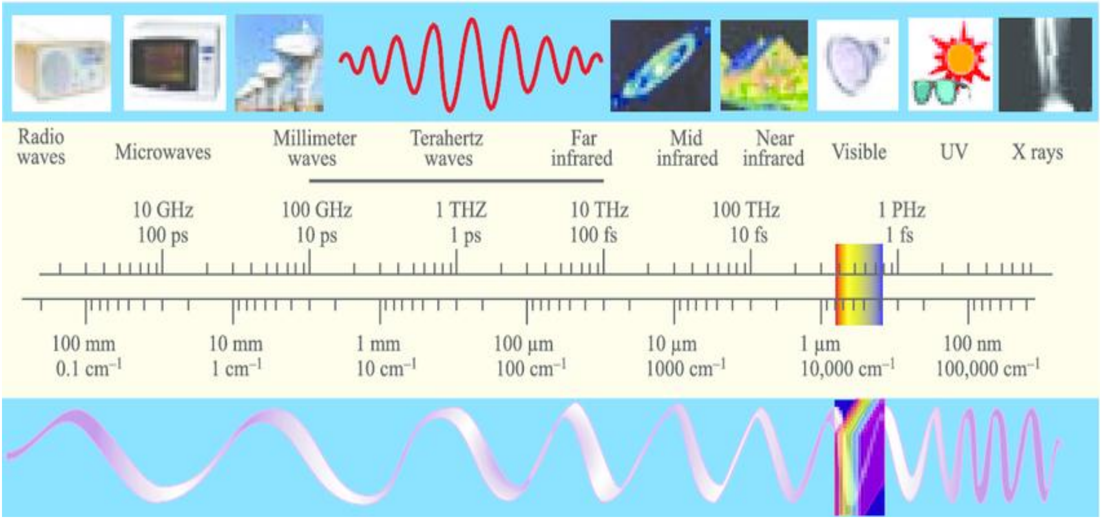
Properties:

1) Transverse in nature
2) Can travel through vacuum
3) Can be reflected and refracted
4) Transfer energy from one place to another



Types of EM Waves

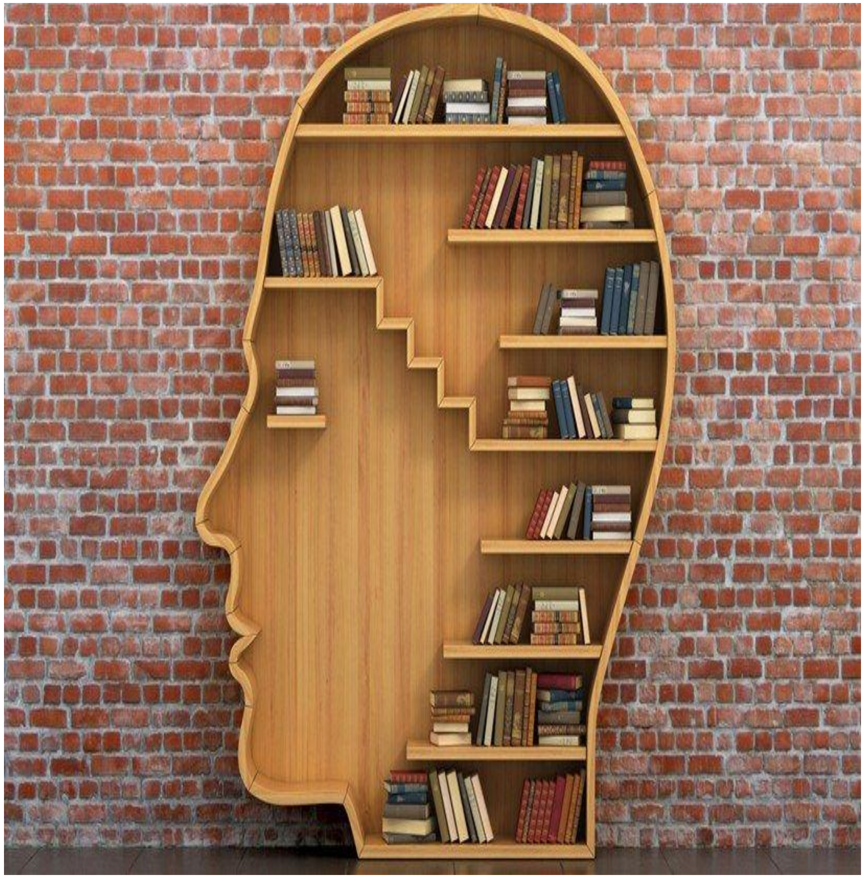
1) Radio waves
2) Microwaves
3) Infrared Waves
4) Visible Light
5) Ultraviolet Radiation
6) X-Rays
7) Gamma Rays
8) THz Radiation



Electromagnetic Spectrum

To understand EM Waves

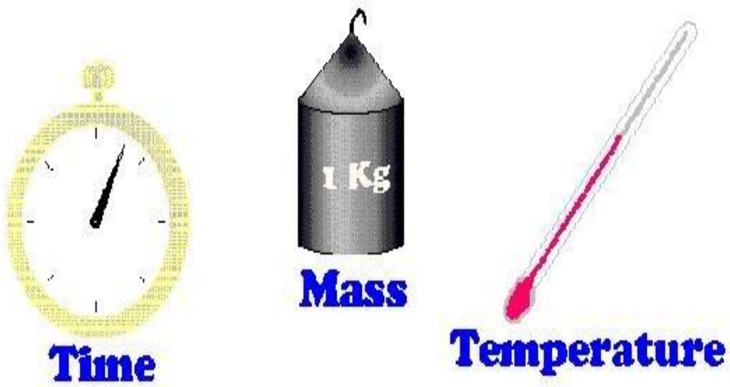
We should have strong basics :



Scalar field
and
Vector field

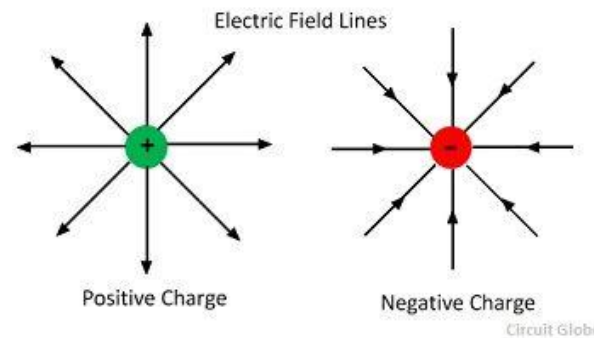
Scalar Quantity

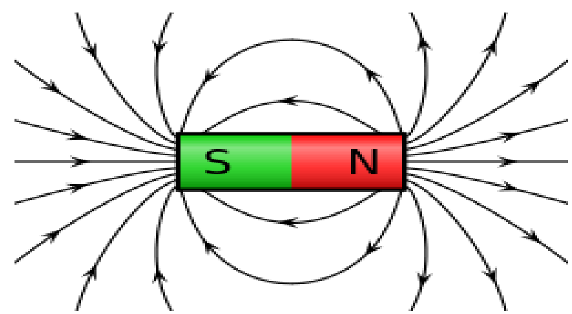
Which has magnitude only.



Vector Quantity

Which has magnitude and as well as definite direction





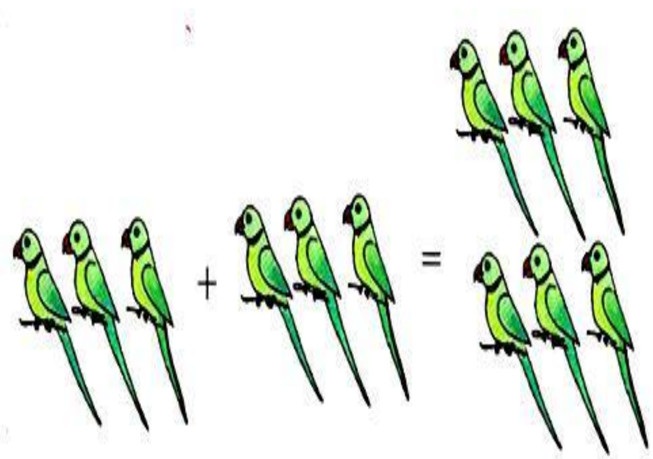
Magnetic field

Scalar field and Vector field

- A scalar field is a kind of function that gives a single value of some variable for every point in space. e.g. temperature
- A vector is a quantity which has both a magnitude and a direction in space. E.g velocity, momentum, acceleration

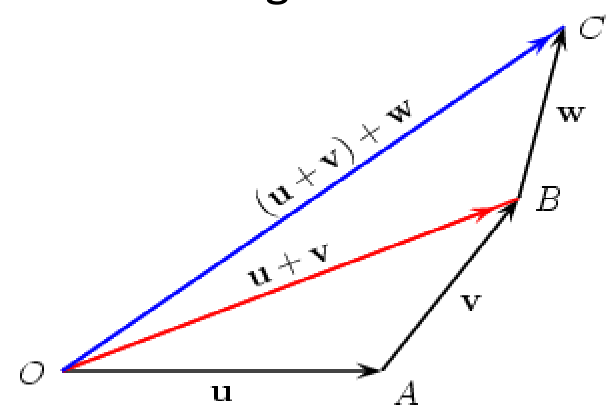
Simple/Scalar Algebra

Simply deal with number or magnitude only.



Vector algebra:

Deal with magnitude and direction



This is solved by using triangular law of vectors.

Del Operator

Del, or nabla, is an **operator used** in mathematics, in particular in vector calculus, as a vector differential **operator**, usually represented by the nabla symbol $\vec{\nabla}$.

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).$$

Gradient

- Gradient of a scalar function is a vector quantity. ∇f

Divergence

- Divergence of a vector is a scalar quantity. $\nabla . A$

Curl

- Curl of a vector is a vector quantity. $\nabla \times A$

Gradient of Scalar Function

1) It is always of scalar function.

$$\vec{\nabla}\Phi = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)\Phi .$$

2) Gradient is always a normal vector to the level surface $\nabla\phi$.

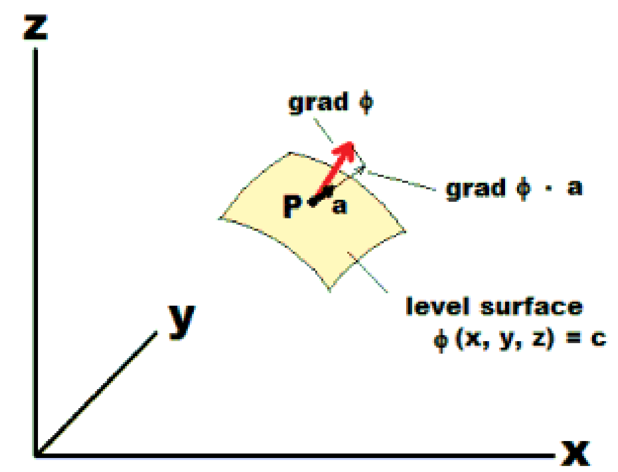


Fig. 1

3) Unit normal vector to the level surface $\hat{n} = \dfrac{\nabla\phi}{|\nabla\phi\,|}$

Find the gradient and direction derivative of the surface $f(x,y) = x^2 + xy$ in the direction of $\vec{a} = \vec{\imath} + 2\vec{\jmath}$ at the point(1,1).

We have the given surface $f(x,y) = x^2 + xy$

Gradient of $f(x,y)$ $\quad \vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right) f$

$$= \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)(x^2 + xy)$$

$$= \left(\frac{\partial(x^2+xy)}{\partial x}\hat{\imath} + \frac{\partial(x^2+xy)}{\partial y}\hat{\jmath} + \frac{\partial(x^2+xy)}{\partial z}\hat{k}\right)$$

$$\nabla f = (2x + y)\hat{\imath} + x\hat{\jmath} + 0\hat{k} \text{ which is gradient of } f.$$

Gradient at $(1,1)$ ***is***

$$\nabla f(1,1) = 3\hat{\imath} + 1\hat{\jmath}$$

Direction derivative of $f(x,y)$ *in direction of* $\vec{a}$ *is*

$$\nabla f.\hat{a} = \nabla f.\frac{\vec{a}}{|\vec{a}|}$$

here $$|\hat{a}| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$$

$$\nabla f.\hat{a} = \frac{\left((2x+y)\hat{\imath}+y\hat{\jmath}+0\hat{k}\right).(\vec{i}+2\vec{j})}{\sqrt{5}}$$

$$\nabla f.\hat{a} = \frac{5}{\sqrt{5}} = \sqrt{5}$$ *Which is required direction derivative?*

Divergence of Vector Field

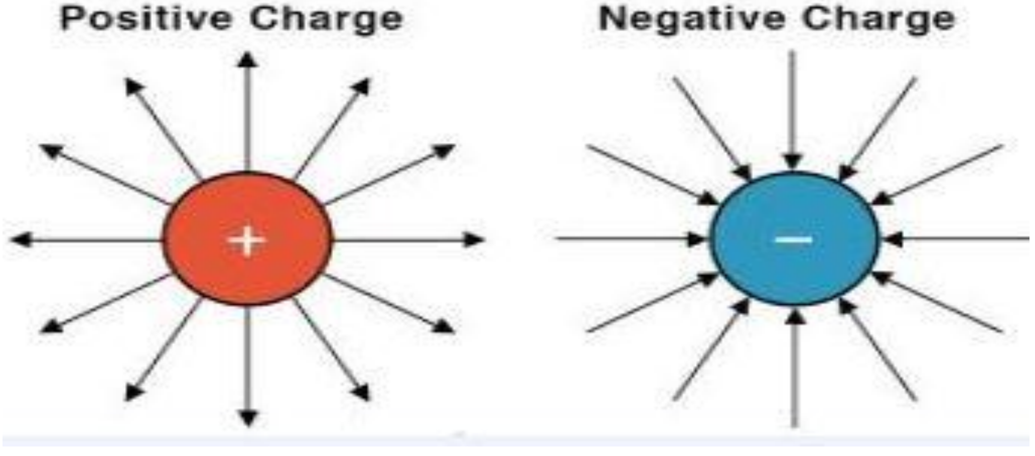
Divergence measures the net flow of liquid *out of* (i.e., *diverging* from) a given point. If fluid is instead flowing *into* that point, the divergence will be negative.

A point or region with positive divergence is often referred to as a "source" (of fluid, or whatever the field is describing), while a point or region with negative divergence is a "sink".

Divergence of Vector Field

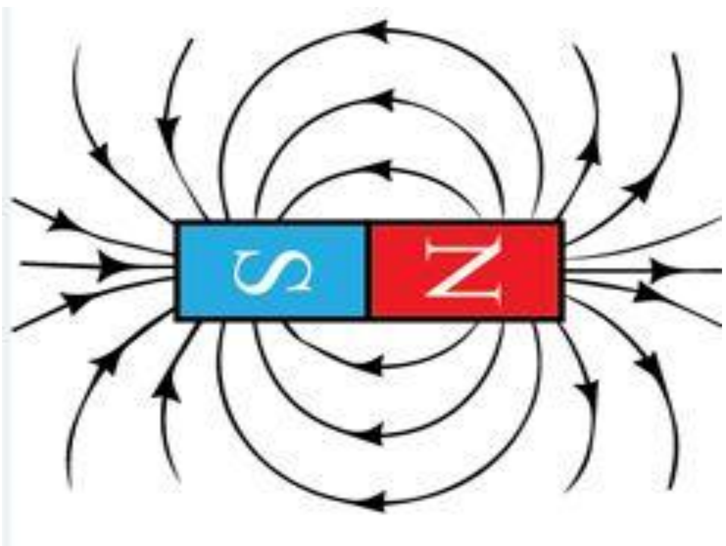
1) Vector field is called source and the lines of field are outward if the divergence is positive.

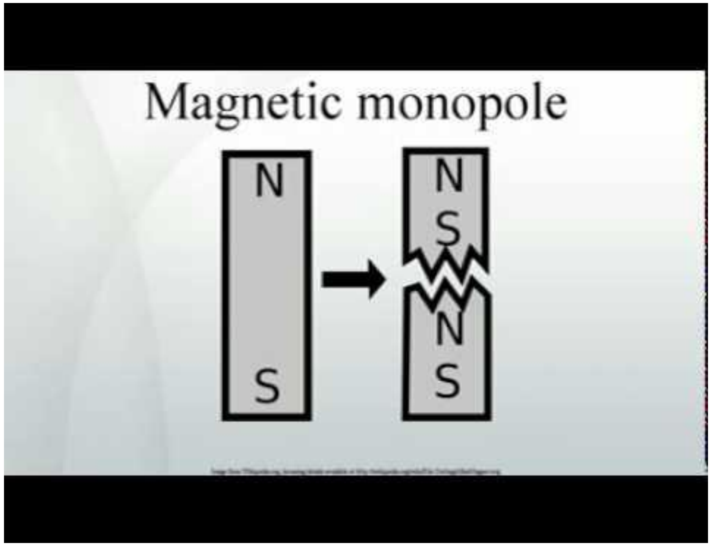
2) Vector field is called sink and the lines of field are inward if the divergence is negative.



3) Divergence of any vector field is always a scalar quantity.

4) Field is solenoid if the divergence is zero. For example the magnetic field divergence is always zero and that’s why natural magnetic monopole does not exist.





Calculate the divergence of vector field

$$\vec{F} = (x - y)\hat{\imath} + (x + y)\hat{\jmath} + z\,\hat{k}.$$

Divergence of $\vec{F}$ *is* $\vec{\nabla}.\vec{F}$

$$= \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).\left((x - y)\hat{\imath} + (x + y)\hat{\jmath} + z\,\hat{k}\right)$$

Note that because of orthonormalize condition

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

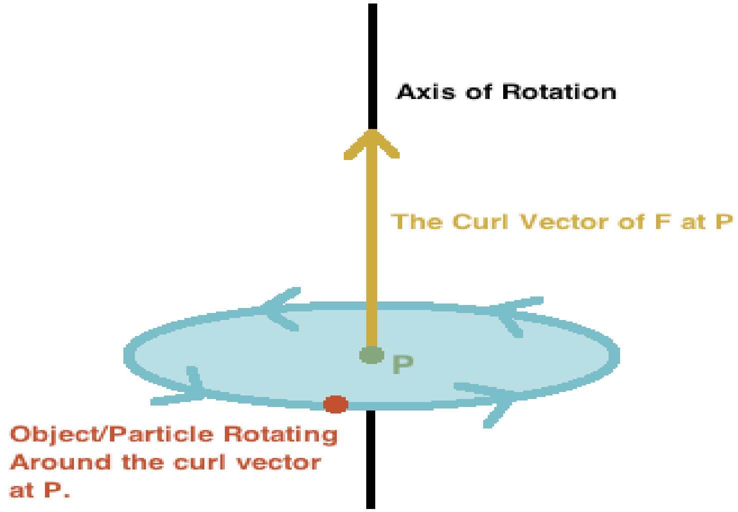
and all other combination equal to zero.

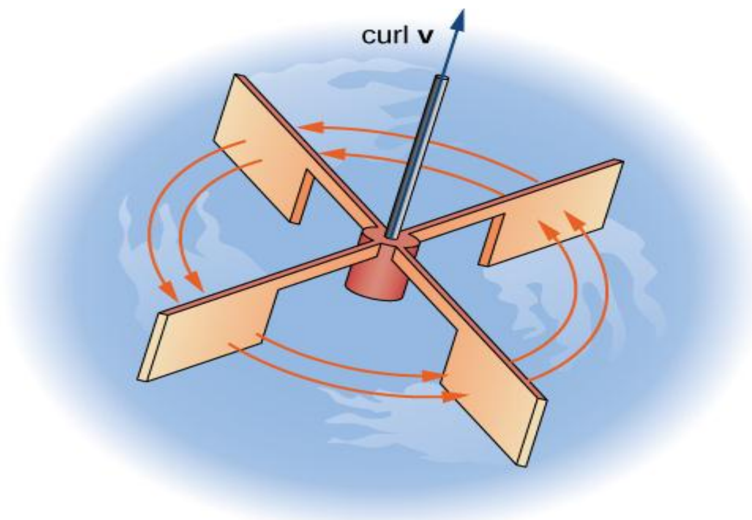
Therefore $\vec{\nabla}.\vec{F} = 1 + 1 + 1 \;= +3$

which is divergence and + ve sign indicate $\vec{F}$ *is source.*

Curl of Vector Field

Curl of any vector field is the measurement of rotation of vector field and the direction of curl of vector is along axis of rotation which is measured by right hand rule.





Important points for Curl

1)Vector field $\overrightarrow{\mathrm{A}}$ is called irrotational vector if curl of vector is zero $\nabla \times \overrightarrow{\mathrm{A}} = 0$.

2)Vector field $\overrightarrow{\mathrm{A}}$ is called rotational vector if curl of vector is not zero.$\nabla \times \overrightarrow{\mathrm{A}} \neq 0$

Curl

DEFINITION. *The curl of a vector field is a vector function defined as the cross product of the vector operator* ∇ *and* $\vec{v}$*,*

$$\begin{aligned} \text{Curl}\,\vec{v} = \nabla \times \vec{v} &= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix} \\ &= (\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z})i - (\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z})j + (\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y})k \end{aligned}$$

EXAMPLE. Compute the curl of the vector function $(x-y)\vec{i} + (x+y)\vec{j} + z\vec{k}$.

SOLUTION:

$$\begin{aligned} \text{Curl}\,\vec{v} = \nabla \times \vec{v} &= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x-y) & (x+y) & z \end{vmatrix} \\ &= (\frac{\partial z}{\partial y} - \frac{\partial (x+y)}{\partial z})i - (\frac{\partial z}{\partial x} - \frac{\partial (x-y)}{\partial z})j + (\frac{\partial (x+y)}{\partial x} - \frac{\partial (x-y)}{\partial y})k \\ &= (0-0)\vec{i} - (0-0)\vec{j} + (1-(-1))\vec{k} \\ &= 2\vec{k} \end{aligned}$$

Calculate the curl of the given vector $\vec{v} = xy\hat{\imath} + yz\hat{\jmath} + zx\hat{k}$ at point (2,4,6).

$$\nabla \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix}$$

$$= \hat{\imath}\left(\frac{\partial(zx)}{\partial y} - \frac{\partial(yz)}{\partial z}\right) - \hat{\jmath}\left(\frac{\partial(zx)}{\partial x} - \frac{\partial(xy)}{\partial z}\right) + \hat{k}\left(\frac{\partial(yz)}{\partial x} - \frac{\partial(xy)}{\partial y}\right)$$

$$= \hat{\imath}(0 - y) - \hat{\jmath}(z - 0) + \hat{k}(0 - x)$$

$$= -y\hat{\imath} - z\hat{\jmath} - x\hat{k}$$

$$\nabla \times \vec{v} \text{ at } (2,4,6) = -4\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$$

Laplacian Operator (∇^2)

Laplaceian is a differential operator given by the divergence of the gradient of a scalar function

$$\vec{\nabla}.\overrightarrow{\nabla} = \nabla^2$$

$$= \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).\left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

Gauss's/Divergence Theorem

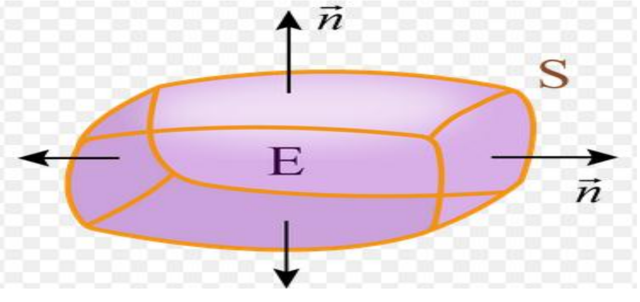
It states that the integral of a divergence over a volume is equal to the closed surface integral of the function or value of the function at the surface of the volume.

$$\int_v (\boldsymbol{\nabla}.\boldsymbol{F})dV = \oint_S \boldsymbol{F}.\boldsymbol{dS} = \oint_S F\,ndS$$

This theorem converts the volume integral into the closed surface integral. For example: Gauss's Law of electrostatics

Note: It also states that the surface integral of a vector field over a closed surface, which is called the "flux" through the surface, is equal to the volume integral of the divergence.

$$\phi_E = \oint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$



Stokes theorem

It states that the integral of the curl of a vector function over a patch of surface is equal to the closed line integral of the function or the value of the function at the perimeter of the patch.

$$\int_{s} (\nabla \times \boldsymbol{E})\boldsymbol{ds} = \oint_{l} \boldsymbol{F}.\boldsymbol{dl}$$

- This theorem converts the surface integral into the closed line integral.
- Right hand rule will be used to check direction of line integral.
- Foe example

Ampere's Law:

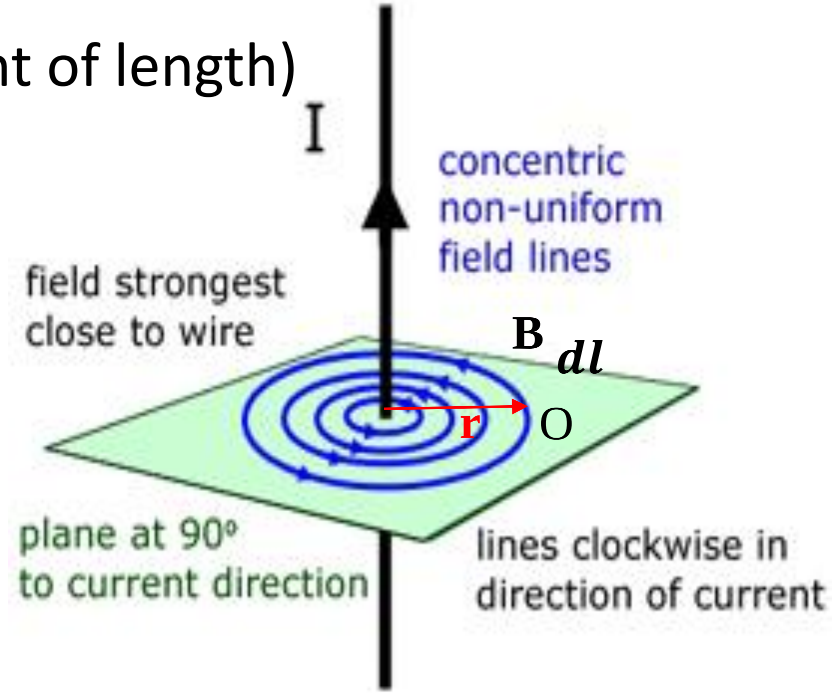
$$\oint_{C} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$$

Ampere's Circuital Law

It is analogous to the Gauss's Law of electrostatics. It states that the line integral of magnetic field **B** round any closed loop is equal to μ_0 times the net current flowing through the area enclosed by the closed loop.

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I \qquad (dl = \text{Element of length})$$

μ_0= permeability of the free space.



Derivation: Consider a straight conductor carrying current I.

By using Biot-Savart law, B at O is given by

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} \qquad \text{(a)}$$

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I \qquad \oint \vec{B}.\overrightarrow{dl} = \oint B dl \cos\theta$$

B and dl are in same direction i.e. $\theta=0$

$$\oint \vec{B}.\overrightarrow{dl} = B \oint dl$$

By using (a)

$$\oint \vec{B}.\overrightarrow{dl} = \frac{\mu_0}{4\pi}\frac{2I}{\mathrm{r}} \oint dl \quad = \frac{\mu_0}{4\pi}\frac{2I}{\mathrm{r}} (2\pi r) \qquad Because \oint dl = 2\pi r$$

$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I$ *This is Ampere's Circuital Law*

By applying Stoke's theorem $\oint \vec{B}.\overrightarrow{dl} = \iint (\nabla \times B).ds$

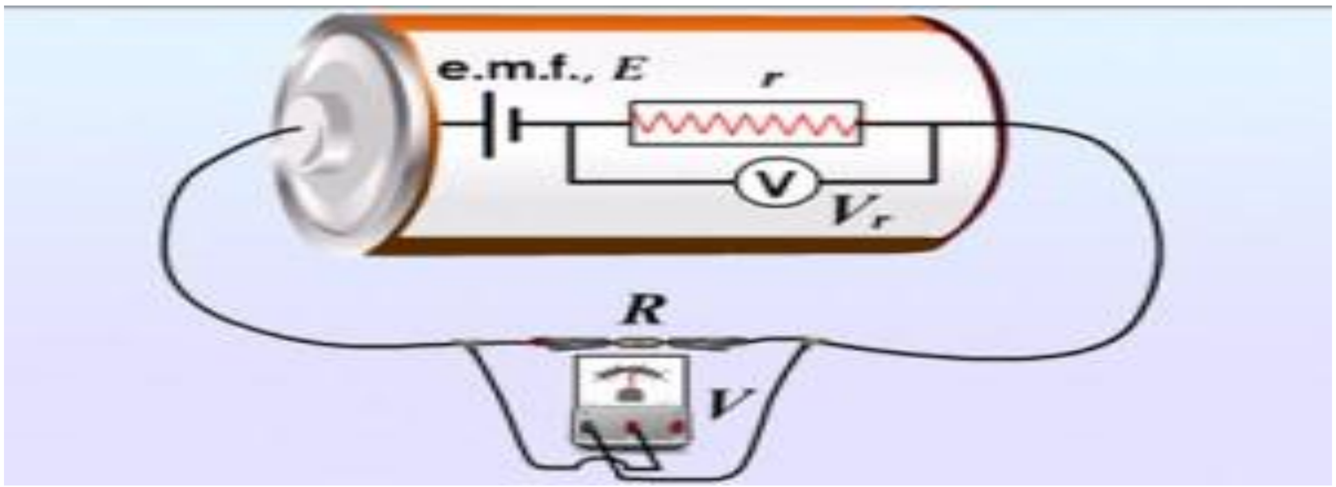
Since $\mathrm{I} = \iint J.ds \qquad \iint (\nabla \times B).ds = \mu_0 \iint J.ds$ $\qquad J$=Current density

$(\nabla \times B) = \mu_0 J$ *Another form of Ampere's Circuital Law*

Unit-1
Electromagnetic Theory

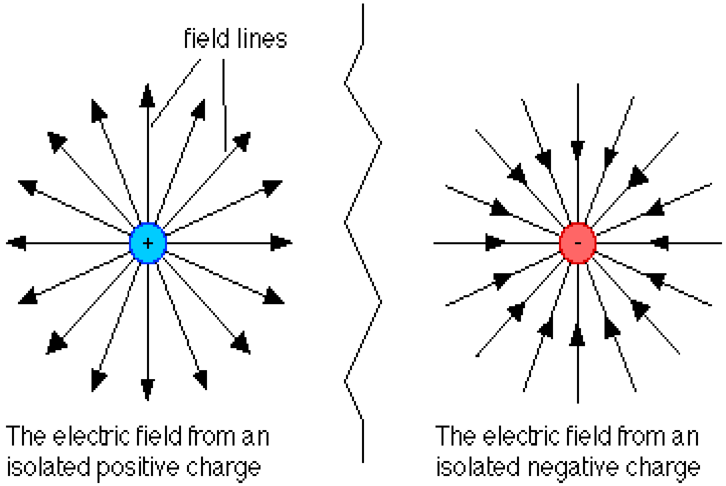
Dielectric Constant

Relation Between E and V



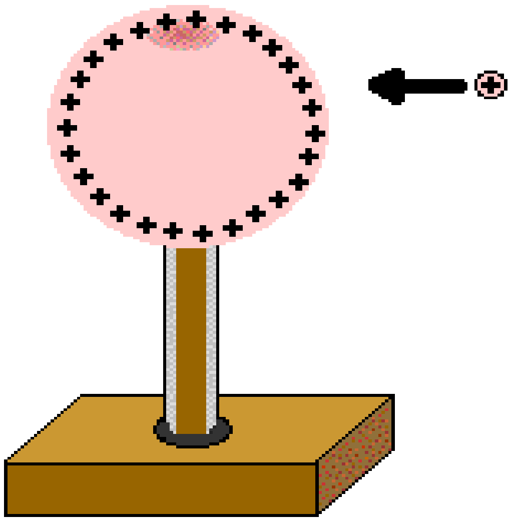
Electric Field (E)

Space around the electric charge where electric force can be experienced.



Electric Potential (V)

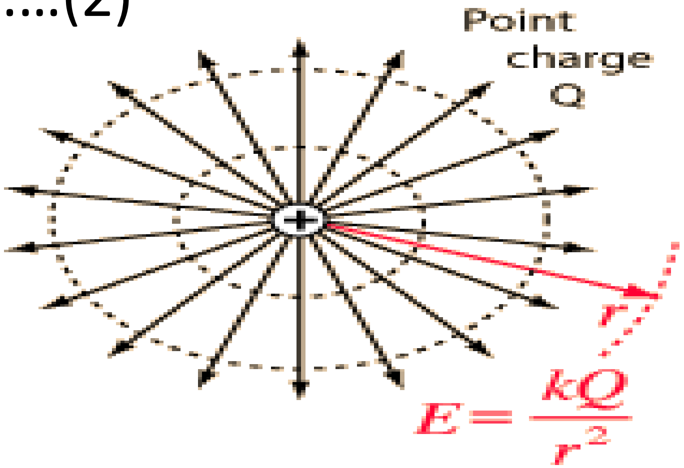
Required work to carry unit charge from reference point to point inside electric field.



E and V due to point charge

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$ (for free space) ....(1)

$$E = \frac{Q}{4\pi\varepsilon r^2}$$ (for medium) ....(2)

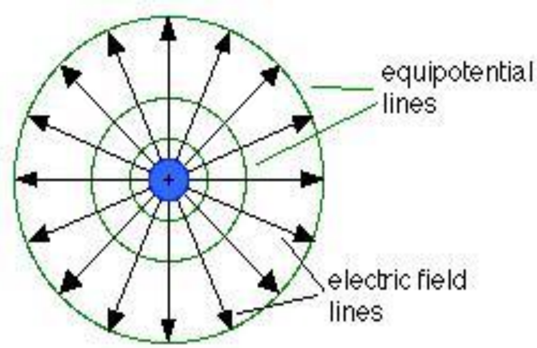


$$V = \frac{Q}{4\pi\varepsilon_0 r}$$ (for free space)

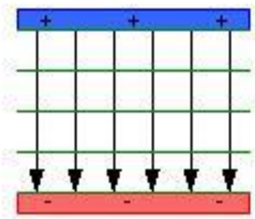
$$V = \frac{Q}{4\pi\varepsilon r}$$ (For a medium)

Equipotential Surfaces

The surfaces having same electric potential are called equipotential surfaces



Field and equipotential lines for a positive point charge

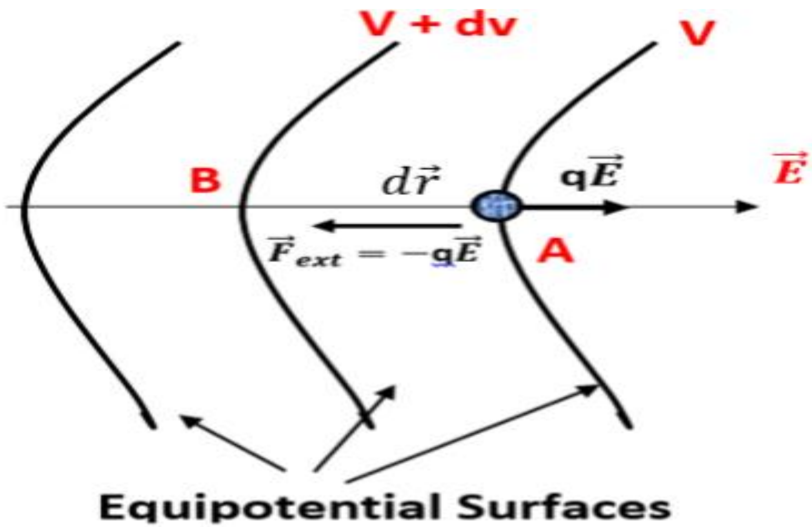


Field and equipotential lines for a set of parallel plates

Relation between E and V

$$F = qE$$

$$dW = -F.dr$$



$$dW = -F.\,dr = -q(E.\,dr) = qdV$$

$$or$$

$$-E.\,dr = dV$$

$$or$$

$$E = -\frac{dV}{dr}$$

Electric field is the negative gradient of the scalar potential.

Poisson's and Laplace's Equations

We have $\vec{\nabla}.\vec{D} = \rho$ Gauss′s Law in a medium

$\vec{D} = \varepsilon\vec{E}$ Which is electric displacement

Because $\vec{E} = -\vec{\nabla}V$

$$\nabla^2\mathrm{V} = -\rho\ /\varepsilon$$ This is the Poisson's Eq.

The solution to Poisson's equation is the potential field caused by a given electric charge distribution; with the potential field known, one can then calculate electrostatic field.

For charge free medium that is ρ=0, then

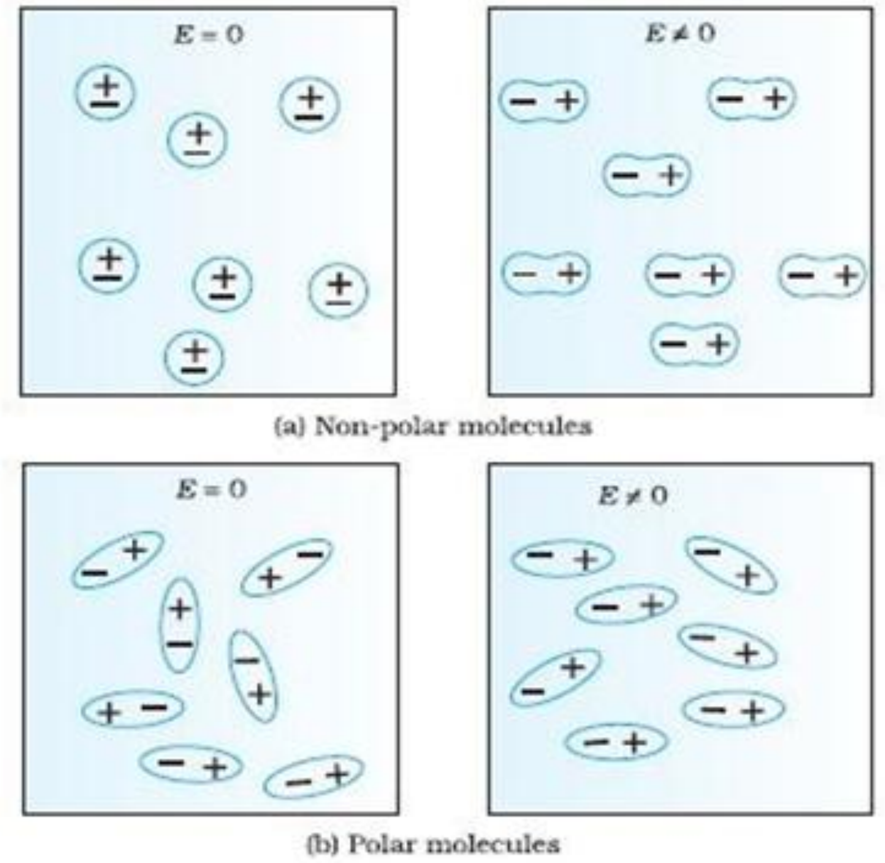
$$\nabla^2\mathrm{V} = 0$$ This is the Laplace′s Eq.

It is a useful approach to the determination of the electric potentials in free space or region.

Dielectric

A dielectric material is a type of insulator which becomes polarized when it comes in contact with an electrical field. It can easily support an electrostatic field even though it is not a conductor of electricity. E.g. Mica, Plastic, glass etc.

Mica is a dielectric material and mostly used in capacitor



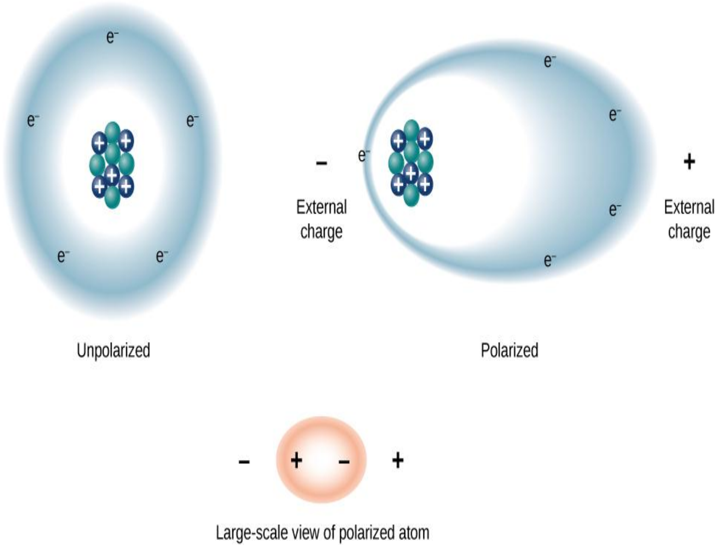
It is of Two types: Polar and Non-polar dielectric

Non-polar dielectric: Center of mass of positive charges coincides with the center of mass of the negative charges. E.g. H_2, O_2 etc.

Polar Dielectrics: center of mass of positive charges does not coincide with the center of mass of the negative charges. E.g. . HCL, CO_2, H_2O etc.

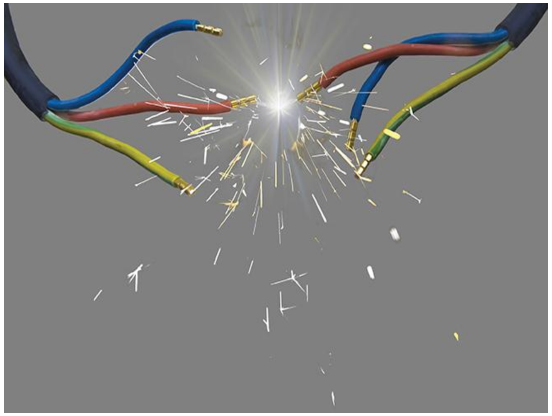
Dielectric Polarisation

When an external electric field is applied, the bound electrons of an atom are displaced such that the centroid of the electronic cloud is separated from the centroid of the nucleus. hence, an electric dipole is created and the atom is said to be polarized.



Examples: Electric sparking and lightning, air breakdown

For air breakdown: An electric field of about ***3 x 10⁶ V/m***



Dielectric constant

The dielectric constant measures the ability of the medium to store electrical energy. It can be calculated by

$$K = \varepsilon(r)/\varepsilon(0)$$

Units and dimensions of K

It is a unit-less, dimensionless quantity because it is the ratio of two like entities.

substance	relative permittivity, ε_r
vacuum	1.0 (definition)
dry air	1.000536
mica (mineral)	2.5 - 7
bakelite (plastic)	3.5 - 5.0
glass	3.7 - 10
laminated paper	4.5
phenolic resin (plastic)	2.8 - 4.5
nylon	4.0 - 5.0
porcelain	5.0 - 7.0
hemp fiber	13.5
calcium copper titanate ($CaCu_3Ti_4O_{12}$)	~12,000

THANK YOU……