Unit-1

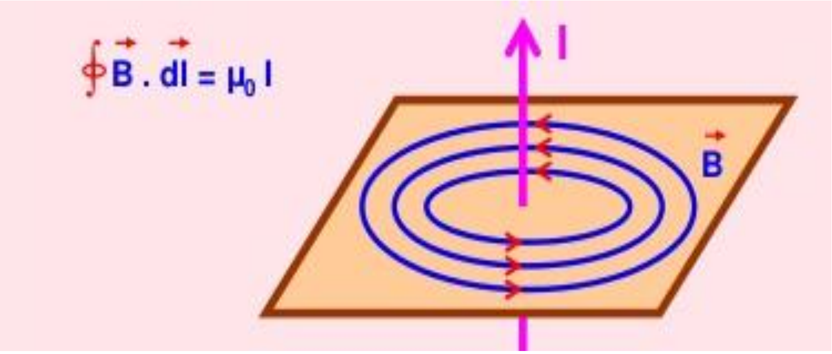
Electromagnetic Theory

- 4^{th} Maxwell's Equations (Differential form)
- Maxwell Displacement current and Correction in Ampere's Circuital Law
- Maxwell's Equations (Integral form and significance)

Maxwell's 4th Equation

This is also called as the modified Ampere's Circuital Law of Electromagnetism.

According to Ampere's law

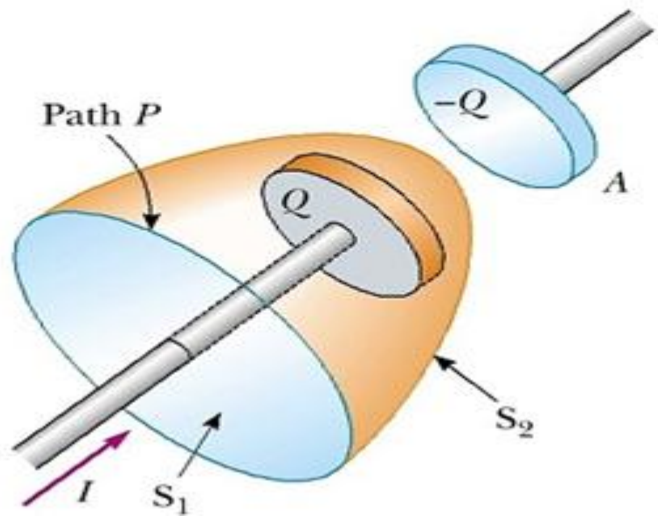


Maxwell addressed the inconsistencies of Ampere's circuital law when applied to electric circuits with capacitors.

Maxwell's 4^{th} Equation

We know electric field produces magnetic field and Maxwell predicted that a time varying electric field produces a magnetic field. This means that a changing electric field gives rise to a current which flow through a region so long as electric field is changing there.

The inconsistencies were modified by James Clerk Maxwell by adding the displacement current term.



When the path P is considered as bounding S_1, $\oint \mathbf{B}.ds = \mu_0 I$ because the conduction current passes through S_1. When the path is considered as bounding S_2, however, $\oint \mathbf{B}.ds = 0$ because no conduction current passes through S_2.

Thus, we arrive at a contradictory situation that arises from the discontinuity of the current! Maxwell solved this problem by postulating an additional term, which includes a factor called the **displacement current** I_d:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 I_{d,enc}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

The changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current and named as displacement current.

Maxwell told that current generates the same magnetic field as the conduction current can generate. The displacement current term is now seen as a crucial addition that completed Maxwell's equations and is necessary to explain many phenomena, most particularly the existence of electromagnetic waves.

Maxwell's 4th Equation

The work done in carrying a unit magnetic pole once around closed arbitrary path linked with the current is expressed by

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I$$

I = current enclosed by the path and $I = \int J\,\hat{n}\,ds$

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \int J\,\hat{n}\,ds$$

Maxwell's 4th Equation

On applying Stoke's Theorem

$$\oint \vec{B}.\overrightarrow{dl} = \int Curl\ B\ \hat{n}\ ds$$

$$\int Curl\ B\ \hat{n}\ ds = \mu_0 \int J\ \hat{n}\ ds$$

$$\int Curl\ B\ \hat{n}\ ds - \mu_0 \int J\ \hat{n}\ ds = 0$$

$$\int (\,Curl\ B\ \ - \mu_0\, J)\hat{n}\ ds = 0$$

Maxwell's 4th Equation

$$Curl\ B - \mu_0 J = 0$$

$$Curl\ B = \mu_0 J$$

It is known as the unmodified fourth equation of Maxwell.

It was found that this is not a valid equation as explained following.

Maxwell's 4th Equation

Taking divergence of both sides

$$Div.(Curl\ B) = \mathrm{Div.}(\mu_0 J)$$

$By\ using$ $$Div.(Curl\ B) = 0$$

$$\mu_0 \mathrm{Div.}(J) = 0$$

Hence $$\mathrm{Div.\,J} = 0$$

$$\mathrm{Div.}(\mu_0 J) = 0$$

Maxwell's 4th Equation

This means that the current is always closed and there are no source and sink. It also show conflict with the equation of continuity which is

$$\mathrm{Div.}(J) = -\frac{d\rho}{dt}$$

So this equation fails and it need of little modification. So Maxwell include the displacement current which makes this equation correct.

$$Curl\ B = \mu_0 J + \mu_0 J_d \qquad \ldots\ldots\ldots(6)$$

Maxwell's 4th Equation

Taking Divergence on both sides

$$Div.(Curl\ B) = \mu_0(Div.J) + \mu_0(Div.J_d)$$

$using \qquad Div.(Curl\ B) = 0$

$$0 = \mu_0(Div.J) + \mu_0(Div.J_d)$$

$$\mu_0(Div.J_d) = -\mu_0(Div.J)$$

This equation holds the equation of continuity. Therefore, by using equation of continuity

$$\mathrm{Div}.J = -\frac{d\rho}{dt}$$

Therefore,

$$\mathrm{Div.\,J_d} = \frac{d\rho}{dt}$$

Maxwell’s 4th Equation

$$and\ by\ using\ Maxwell's first\ equation\ \mathrm{Div}.D=\rho$$

$$\mathrm{Div.J_d}=Div.\frac{\partial D}{\partial t}$$

$$\mathrm{J_d}=\frac{\partial D}{\partial t}$$

Putting in equation (6), we get

$$Curl\ B=\mu_0\left(\ J+\frac{\partial D}{\partial t}\ \right)=\ \mu_0 J+\mu_0\varepsilon_0\frac{\partial E}{\partial t}\qquad \ldots\ldots\ldots\ldots.(7)$$

$$we\ know\ B=\mu_0 H$$

$$Curl\ H=(\ J+\frac{\partial D}{\partial t}\ )\ldots\ldots\ldots\ldots.(8)$$

Equation (7) and (8) both are the modified Maxwell’s fourth equation.

Maxwell's Equations in Medium

1. $\vec{\nabla}.\vec{D} = \rho = \overrightarrow{\nabla}.\varepsilon_0\vec{E}$ $\quad Gauss's\ Law\ of\ electrostatics$

2. $\vec{\nabla}.\vec{B} = 0\ Gauss's\ Law\ of\ Magnetostatics$

3. $\nabla \times E = -\frac{\partial B}{\partial t}\ Faraday's\ law$

4. $\nabla \times B = \mu_0\left(\ J + \frac{\partial D}{\partial t}\ \right) \ = \ \mu_0 J + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

$modified\ Ampere's\ Circutal\ Law$

Maxwell's Equations in Vacuum

To convert these Maxwell's Equations in medium to Maxwell's equations in vacuum, Let's consider light travelling in a vacuum, i.e. a region in which there are no electric charges, that is ρ = 0, J = 0 and $\boldsymbol{\sigma = 0}$.

1) $\vec{\nabla}.\vec{E} = 0$

2) $\vec{\nabla}.\vec{B} = 0$

3) $\nabla \times E = -\frac{\partial B}{\partial t}$

4) $\nabla \times B = \mu_0 \varepsilon_0 \frac{dE}{dt}$

Maxwell's Equations (Integral Form)

1) Maxwell's First Equation in Integral Form

We have

$$\vec{\nabla}.\vec{D} = \rho \qquad \text{Differential form of Maxwell's Equation}$$

On Integrating over a volume V, we can write as

$$\iiint(\vec{\nabla}.\vec{D})dV = \iiint \rho \, d\mathrm{V} \qquad (1)$$

By Gauss's Divergence theorem $\iiint(\vec{\nabla}.\vec{D})dV = \oint \vec{D}.\overrightarrow{dS}$

(1) becomes

$$\oint \vec{D}.\overrightarrow{dS} = \iiint \rho \, d\mathrm{V}$$

If $\iiint \rho \, dV = q$ is the total charge enclosed by V and S is the surface bounding the volume

$$\oint \vec{D}.\overrightarrow{dS} = q$$ This is the required equation

- It means total electric displacement through the surface S enclosing a volume V is equal to total charge contained within V.

2) Maxwell's 2nd Equation in Integral Form

Differential form is $\vec{\nabla}.\vec{B} = 0$

On Integrating over a volume V, we can write as

$$\iiint (\vec{\nabla}.\vec{B})dV = \iiint 0 \, dv = 0 \qquad (2)$$

By Gauss's Divergence theorem $\iiint (\vec{\nabla}.\vec{B})dV = \oint \overrightarrow{B}.\overrightarrow{dS}$

(2) becomes $\oint \vec{B}.\overrightarrow{dS}=0$ This is required equation

- It means total flux through any closed S is equal to zero.

3) Maxwell's 3rd Equation in Integral Form

We have $$\nabla \times E = -\frac{\partial B}{\partial t}$$

On Integrating over a surface S bounded by closed surface

$$\iint (\nabla \times E)\,.\,ds = -\iint \frac{\partial B}{\partial t}\,.\,ds \qquad (3)$$

By Stoke's theorem $\iint (\nabla \times E)\,.\,ds = \oint \overrightarrow{E}.\overrightarrow{dl}$

(3) becomes $\oint \vec{E}.\overrightarrow{dl} = -\iint \frac{\partial B}{\partial t}.\,ds = -\frac{\partial}{\partial t}\iint B\,.\,ds$ This is the required equation

- It means thta the emf around closed S is equal to time derivative of magnetic flux through the closed surface.

4) Maxwell's 4th Equation in Integral Form

We have $$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Step1) On Integrating over a surface S bounded by closed surface

$$\iint (\nabla \times H) \,.\, ds = \iint (J + \frac{\partial D}{\partial t}) \,.\, ds$$

Step 2) By Stoke's theorem $\iint (\nabla \times H) \,.\, ds = \oint \overrightarrow{H} . \overrightarrow{dl}$

This is the required equation is $$\oint \vec{H} . \vec{dl} = \iint (J + \frac{\partial D}{\partial t}) \,.\, ds$$

Thank You…….