Froe

September 3, 2017

In [12]:
```
from sklearn.preprocessing import PolynomialFeatures
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import mean_squared_error
from tqdm import tqdm

%pylab inline
pylab.rcParams['figure.figsize'] = (20, 10)
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
C:\Users\Matteo\Anaconda3\envs\py35\lib\site-packages\IPython\core\magics\pylab.py:160: UserWa
`%matplotlib` prevents importing * from pylab and numpy
  "\n`%matplotlib` prevents importing * from pylab and numpy"
```

In [37]:
```
# y -> outlet liquid temperature
# q(t) -> liquid flow rate
# Narx -> yhat(t+1|t) = f(y(t) ... y(t-3) u(t) ... u(t-9))

#read data
data = open('../exchanger/exchanger.dat')
lst = []
for line in data:
    lst += [line.split()]
time = [float(x[0]) for x in lst]
u = [float(x[1]) for x in lst]
y = [float(x[2]) for x in lst]
avg_y = np.mean(y)
avg_u = np.mean(u)
```

In [38]:
```
#divide identification set and validation set
ID_LENGHT = 3000
VAL_LENGHT = 1000
time_id = time[:ID_LENGHT]
u_id = u[:ID_LENGHT]
y_id = y[:ID_LENGHT]
```

```
avg_y_id = np.mean(y_id)
avg_u_id = np.mean(u_id)

time_val = time[ID_LENGHT:]
u_val = u[ID_LENGHT:]
y_val = y[ID_LENGHT:]
avg_y_val = np.mean(y_val)
avg_u_val = np.mean(u_val)

AR_deg = 4
X_deg = 10
poly_degree = 2
```

In [39]:
```
features = []
for i in reversed(range(X_deg)):
    if i==0:
        features.append("u(t)")
    else:
        features.append("u(t-{})".format(i))
for i in reversed(range(AR_deg)):
    if i==0:
        features.append("y(t)")
    else:
        features.append("y(t-{})".format(i))

print("Model: y(t+1|t) = f({}, ... , {}, {}, ... ,{})".format(features[-1],features[-
                                                              features[X_deg-1],featu
print("Where f() is a polynomial expansion of the {} degree".format(poly_degree))
```

```
Model: y(t+1|t) = f(y(t), ... , y(t-3), u(t), ... ,u(t-9))
Where f() is a polynomial expansion of the 2 degree
```

In [40]:
```
poly = PolynomialFeatures(poly_degree)
Y = np.array(y_id)
reg_u = np.full(X_deg,avg_u_id)
reg_y = np.full(AR_deg,avg_y_id)
PHI = []
for i in range(ID_LENGHT):
    if i!=0:
        reg_y = np.append(reg_y, Y[i])[1:]
        reg_u = np.append(reg_u, u_id[i])[1:]
    regressors = np.append(reg_u, reg_y)
    PHI.append(poly.fit_transform([regressors])[0])
PHI = np.array(PHI)
regressor_terms = poly.get_feature_names(features)
print("Nř of regressors: ", len(regressor_terms))
```

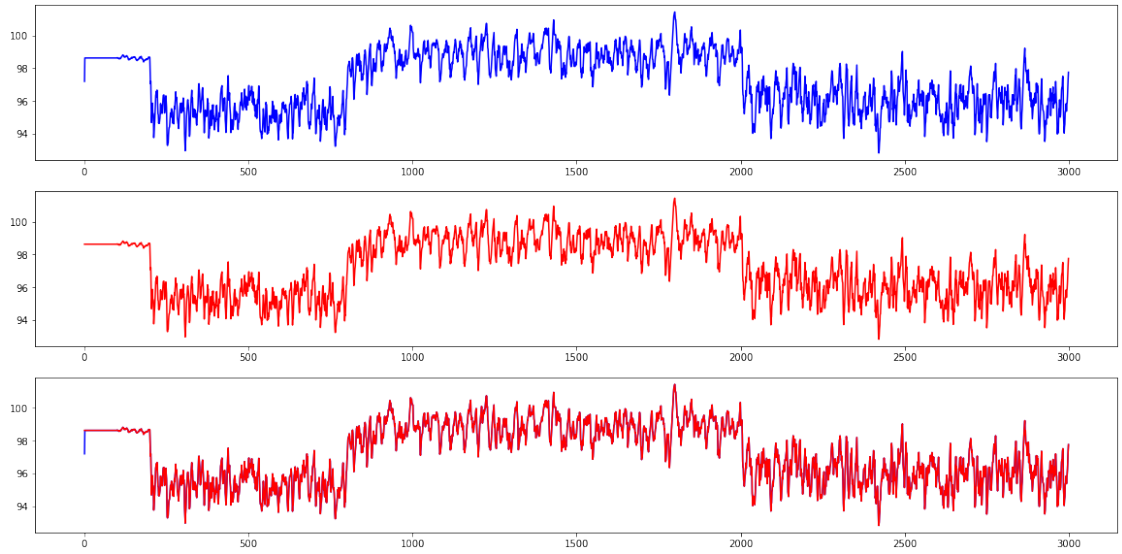
```
Nř of regressors:  120
```

In [41]:
```
#FROE Implementation
A = np.zeros((PHI.shape[1],PHI.shape[1]))
W = np.zeros(PHI.shape)
g_hat = np.array([])
np.fill_diagonal(A,1)
regressor_selected = np.array([], dtype=int)
err_sum = 0
threshold = 0.05

for k in range(PHI.shape[1]):
    err = np.array([])
    g = np.array([])
    if k == 0:
        for i in range(PHI.shape[1]):
            W[:,0] = PHI[:,i]
            g_i = np.dot(W[:,0],Y)/np.power(np.linalg.norm(W[:,0]),2)
            g = np.append(g, g_i)
            err_i = np.power(g_i,2) *  np.power(np.linalg.norm(W[:,0]),2) / np.power(n
            err = np.append(err, err_i)
        j = np.argmax(err)
        print(err[j])
        print(regressor_terms[j])
        W[:,0] = PHI[:,j]
        g_hat = np.append(g_hat, g[j])
        regressor_selected = np.append(regressor_selected, j)
        err_sum += err[j]
        if (1-err_sum < threshold):
            print(1-err_sum)
            print('Threshold exceeded!')
            break;
    else:
        for l in range(PHI.shape[1]):
            if l not in regressor_selected:
                temp = np.zeros(PHI.shape[0])
                for i in range(k):
                    A[i,k] = (np.dot(W[:,i],PHI[:,l]))/np.power(np.linalg.norm(W[:,i])
                    temp += A[i,k] * W[:,i]
                W[:,k] = PHI[:,l] - temp
                #g = np.append(g, (np.dot(W[:,k],Y))/np.power(np.linalg.norm(W[:,k]),2
                g_i = np.dot(W[:,k],Y)/np.power(np.linalg.norm(W[:,k]),2)
                g = np.append(g, g_i)
                err_i = np.power(g_i,2) *  np.power(np.linalg.norm(W[:,k]),2) / np.pow
                err = np.append(err, err_i)
            else:
                err = np.append(err, 0)
                g = np.append(g, 0)
        j = np.argmax(err)
        print(err[j])
```

```
                temp = np.zeros(PHI.shape[0])
                for i in range(k):
                    A[i,k] = (np.dot(W[:,i],PHI[:,j]))/np.power(np.linalg.norm(W[:,i]),2)
                    temp += A[i,k] * W[:,i]
                print(regressor_terms[j])
                W[:,k] = PHI[:,j] - temp
                g_hat = np.append(g_hat, g[j])
                regressor_selected = np.append(regressor_selected, j)
                err_sum += err[j]
                if (1-err_sum < threshold):
                    print('Threshold exceeded!')
                    print(1-err_sum)
                    break;

        theta = np.zeros(len(g_hat))
        for i in reversed(range(len(g_hat))):
            if i == len(g_hat):
                theta[i] = g_hat[i]
            else:
                temp = 0
                for k in range(i+1, len(g_hat)):
                    temp += A[i,k] * theta[k]
                theta[i] = g_hat[i] - temp
```

```
0.999999927661
y(t)
7.23391251345e-08
Threshold exceeded!
```

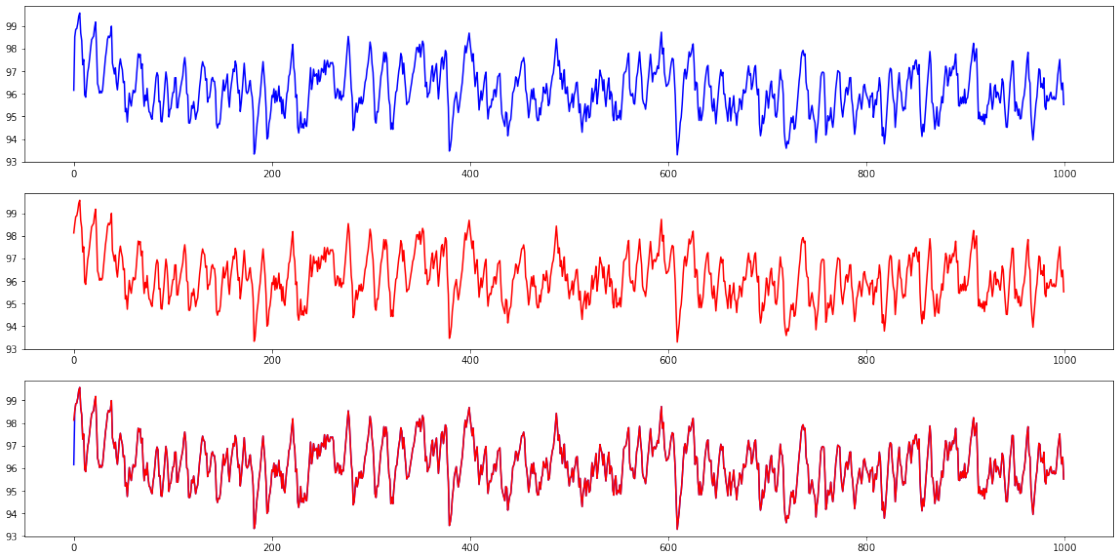
```
In [42]: PHI_final = np.zeros((PHI.shape[0], len(regressor_selected)))
         for i in range(len(regressor_selected)):
             PHI_final[:,i] = PHI[:, regressor_selected[i]]

In [43]: y_hat = np.dot(PHI_final, theta)

In [44]: plt.subplot(311)
         plt.plot(y_hat, color='blue')
         plt.subplot(312)
         plt.plot(y_id, color='red')
         plt.subplot(313)
         plt.plot(y_hat, color='blue')
         plt.plot(y_id, color='red')
         plt.show()

         MSE_id = mean_squared_error(y_id,y_hat)
         print("MSE on identification: ", MSE_id)
```




```
MSE on identification:  0.0006836127158 43
```

```
In [45]: #Validation
         poly = PolynomialFeatures(poly_degree)

         Y_val = np.array(y_val)
         reg_u = np.full(X_deg,avg_u_val)
         reg_y = np.full(AR_deg,avg_y_val)
         PHI_val = []
         for i in range(VAL_LENGHT):
             if i!=0:
                 reg_y = np.append(reg_y, Y_val[i])[1:]
                 reg_u = np.append(reg_u, u_val[i])[1:]
             regressors = np.append(reg_u, reg_y)
             PHI_val.append(poly.fit_transform([regressors])[0])
         PHI_val = np.array(PHI_val)
         regressor_terms = poly.get_feature_names(features)

In [46]: PHI_final_val = np.zeros((PHI_val.shape[0], len(regressor_selected)))
         for i in range(len(regressor_selected)):
             PHI_final_val[:,i] = PHI_val[:, regressor_selected[i]]

In [47]: y_hat_val = np.dot(PHI_final_val, theta)

In [48]: plt.subplot(311)
         plt.plot(y_hat_val, color='blue')
         plt.subplot(312)
         plt.plot(y_val, color='red')
```

```
plt.subplot(313)
plt.plot(y_hat_val, color='blue')
plt.plot(y_val, color='red')
plt.show()

MSE_val = mean_squared_error(y_val,y_hat_val)
print("MSE on validation: ", MSE_val)
```

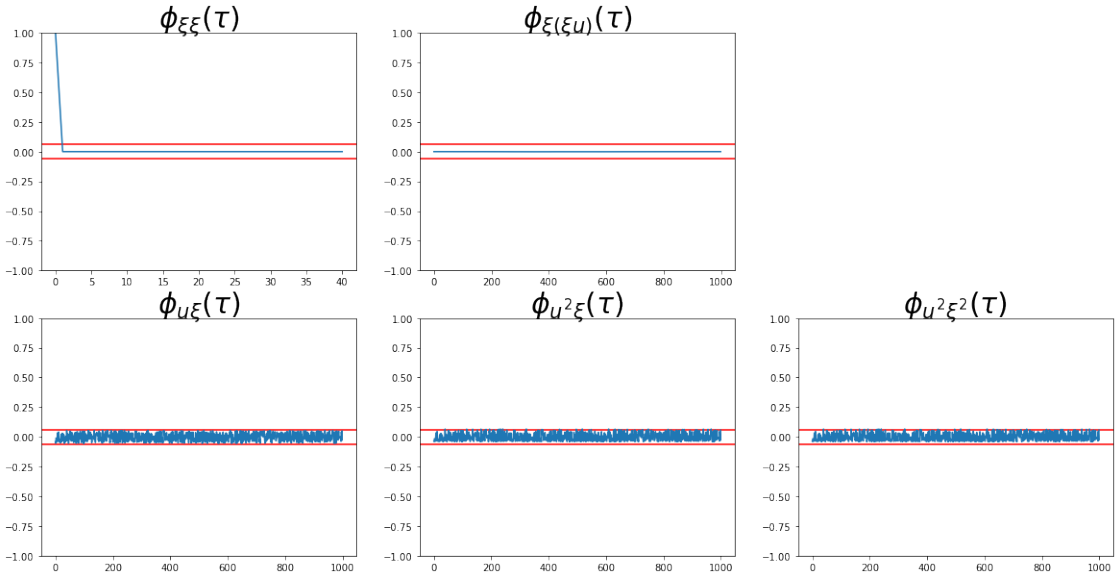



```
MSE on validation:  0.00388656705618
```

```
In [49]: #MODEL VALIDATION - CORRELATION FUNCTIONS
         from statsmodels.tsa.stattools import acf , ccf
         epsilon = np.array(y_val - y_hat_val)
         u_val = np.array(u_val)

         #Autocorrelation epsilon
         corr_ee = acf(epsilon)

         #Cross-correlation u-epsilon
         corr_ue = ccf(u_val, epsilon,unbiased=False)

         #Cross-correlation epsilon ( epsilon*u)
         corr_e_eu = ccf(epsilon,np.multiply(epsilon[1:],u_val[1:]),unbiased=False)

         #Cross-correlation delta(u^2)-epsilon
         corr_du2_e = ccf(np.power(u_val,2) - np.mean(np.power(u_val,2)),epsilon, unbiased=Fals

         #Cross-correlation delta(u^2)-epsilon
```

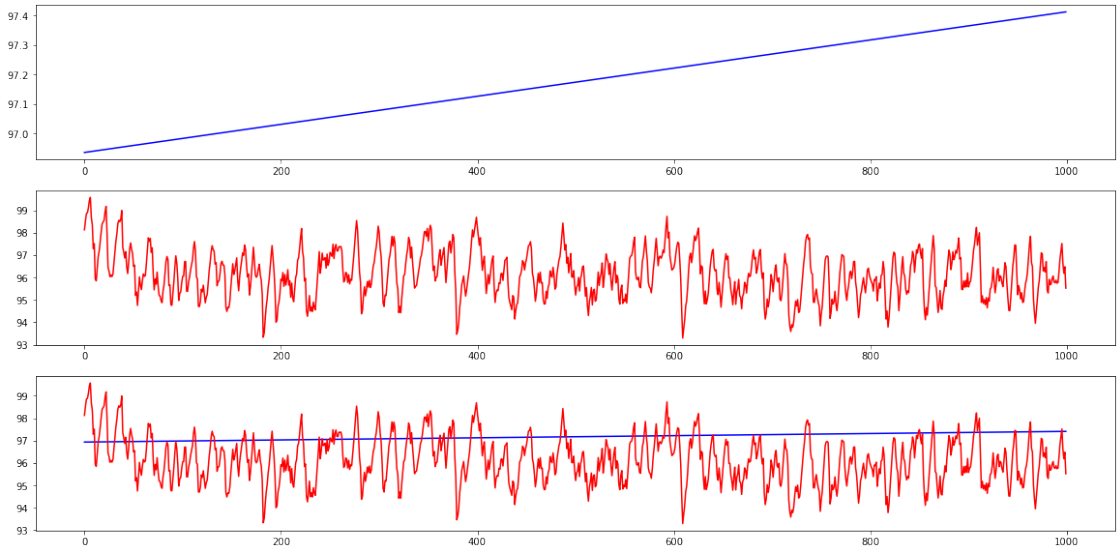
```
corr_du2_e2 = ccf(np.power(u_val,2) - np.mean(np.power(u_val,2)),np.power(epsilon,2),

#confidence interval   -95%
conf_interval_sup = 1.96 / np.sqrt(VAL_LENGHT)
conf_interval_inf = -1.96 / np.sqrt(VAL_LENGHT)
plt.subplot(231)
plt.title(r'$\phi_{\xi\xi}(\tau)$', fontsize=30)
plt.axhline(y=conf_interval_sup, color = "red")
plt.axhline(y=conf_interval_inf, color = "red")
plt.plot(corr_ee)
plt.ylim((-1,1))
plt.subplot(232)
plt.title(r'$\phi_{\xi(\xi u)}(\tau)$', fontsize=30)
plt.axhline(y=conf_interval_sup, color = "red")
plt.axhline(y=conf_interval_inf, color = "red")
plt.plot(corr_e_eu)
plt.ylim((-1,1))
plt.subplot(234)
plt.title(r'$\phi_{u \xi}(\tau)$', fontsize=30)
plt.axhline(y=conf_interval_sup, color = "red")
plt.axhline(y=conf_interval_inf, color = "red")
plt.plot(corr_ue)
plt.ylim((-1,1))
plt.subplot(235)
plt.title(r'$\phi_{u^2\xi}(\tau)$', fontsize=30)
plt.axhline(y=conf_interval_sup, color = "red")
plt.axhline(y=conf_interval_inf, color = "red")
plt.plot(corr_du2_e)
plt.ylim((-1,1))
plt.subplot(236)
plt.title(r'$\phi_{u^2\xi^2}(\tau)$', fontsize=30)
plt.axhline(y=conf_interval_sup, color = "red")
plt.axhline(y=conf_interval_inf, color = "red")
plt.plot(corr_du2_e2)
plt.ylim((-1,1))
```

```
Out[49]: (-1, 1)
```




```
In [50]: ## Simulation
         # start from initial phi, then build step by step each ne element
         poly = PolynomialFeatures(poly_degree)
         reg_y = np.full(AR_deg,avg_y)
         reg_u = np.full(X_deg,avg_u)
         reg = np.append(reg_u,reg_y)
         reg = poly.fit_transform([reg])[0]

In [51]: model_reg = reg[regressor_selected]  # initial values for the regression

In [52]: #simulate the process
         y_hat_sim  = []
         for i in range(VAL_LENGHT):
             y_i = np.dot(model_reg,theta) #simulated
             y_hat_sim.append(y_i)
             reg_y = np.append(reg_y, y_hat_sim[i])[1:]
             reg_u = np.append(reg_u, u_val[i])[1:]  #append at beggining, then remove last on
             reg = np.append(reg_u,reg_y)
             reg = poly.fit_transform([reg])[0]
             model_reg = reg[regressor_selected]

In [53]: plt.subplot(311)
         plt.plot(y_hat_sim, color='blue')
         plt.subplot(312)
         plt.plot(y_val, color='red')
         plt.subplot(313)
         plt.plot(y_hat_sim, color='blue')
         plt.plot(y_val, color='red')
```

```
plt.show()

MSE_sim = mean_squared_error(y_val,y_hat_sim)
print("MSE on simulation: ", MSE_sim)
```




```
MSE on simulation:  2.20168655113
```

In [ ]: