

# Bayesian inference and data analysis

05/11/2021

Conditional probability:

$I =$  "normal 52-card deck"

$$p(\text{black-card} | I) = \frac{26}{52} = \frac{1}{2}$$

↑  
given

Ques. probability of drawing 2 black-suit card from a pack of 52 cards?

Trial 1:  $T_0 = \frac{26}{52} = \frac{1}{2}$

Trial 2:  $T_1 = \frac{26-1}{52-1} = \frac{25}{51}$

Probability of drawing 2 black-suit card is

$$T_1 \times T_0 = \frac{25}{102}$$

Mathematically,

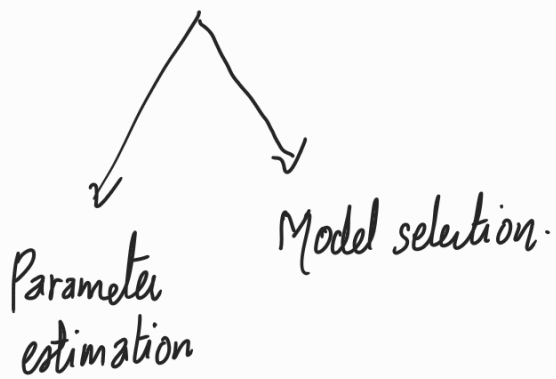
$$P(T_0, T_1 | I) = \underbrace{P(T_1 | T_0, I)}_{= 25/51} \underbrace{P(T_0 | I)}_{= 1/2} = \frac{25}{102}$$

$$P(T_1 | T_0, I) = \frac{\text{number of black cards}}{\text{number of cards}} = \frac{25}{51}$$

Conditional probability:  $P(A, B | I) = P(A | B, I) P(B | I)$   
 $= P(B | A, I) P(A | I)$

Bayes' theorem  $\Rightarrow P(A | B, I) = \frac{P(B | A, I) P(A | I)}{P(B | I)}$

Inference: figuring something from the data.



I. Parameter estimation: figuring out model parameters  $\theta$  given some model  $M_A$ .

Let's denote data by  $\bar{d}$ .

$$P(\theta | M_A, \bar{d}) = \frac{P(\bar{d} | M_A, \theta) P(\theta | M_A)}{P(\bar{d} | M_A)} \rightarrow \text{normalizes the distribution.}$$

$$\text{or, } P(\theta | M_A, \bar{d}) \propto P(\bar{d} | M_A, \theta) P(\theta | M_A)$$

In general,

$$\underbrace{P(\theta | M_A, \bar{d})}_{\text{posterior}} \propto \underbrace{\mathcal{L}(\bar{d} | M_A, \theta)}_{\text{Likelihood}} \underbrace{\pi(\theta | M_A)}_{\text{prior}}$$

Example: One-dimensional posterior distribution.

Consider specific data  $\bar{y} = [y_0, y_1, y_2, \dots, y_N]$  observed at times

$$\bar{t} = [t_0, t_1, \dots, t_N]$$

Model:  $M_A$ :

$$y_A(t; \omega) = \sin(\omega t)$$

One model parameter,  $\theta = [\omega]$

Jupyter notebook.

Data + Noise : Try to figure out frequency ( $\omega$ ).  
Gaussian

$$y_{obs} = y_{det} + \text{noise} \Rightarrow \text{noise} = y_{obs} - y_{det}$$

For a single data point  $y_i, t_i$ , then given a particular  $\omega$

$$\begin{aligned} \mathcal{L}(y_i, t_i | \omega, M_A, \sigma = 0.5) &= \text{Normal}(y - y_A(t; \omega); \sigma = 0.1) \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - y_A(t_i))^2}{2\sigma^2}\right) \end{aligned}$$

Then, log-likelihood

$$\ln \mathcal{L}(y_i, t_i | \omega, M_A, \sigma = 0.5) = -\frac{1}{2} \left( \frac{(y_i - y_A(t_i))^2}{\sigma^2} + \ln(2\pi\sigma^2) \right)$$

For all the data,

$$\mathcal{L}(\bar{y}, \bar{t} | \omega, M_A, \sigma = 0.5) = \prod_i \mathcal{L}(y_i, t_i | \omega, M_A, \sigma = 0.5)$$

product

$$\ln \mathcal{L}(\bar{y}, \bar{t} | \omega, M_A, \sigma = 0.5) = \sum_i \ln \mathcal{L}(y_i, t_i | \omega, M_A, \sigma = 0.5)$$

sum

Posteriors

↙ data.

$$P(w|\bar{d}) \propto \mathcal{L}(\bar{d}|w) \times \pi(w)$$

- If  $\pi(w) \propto \text{constant}$ , then  $P(w|\bar{d}) \propto \mathcal{L}(\bar{d}|w)$
- If not, then we need to multiply by the prior.

Consider the easier case.