# Unit 5: Introduction to Morphological Image Processing

## Introduction

*Morphology* commonly denotes a branch of biology that deals with the form and structure (*shape)* of animals and plants.

The same word *morphology her we called* ***Mathematical morphology*** is used as a tool for extracting image components that are useful in the representation and description of region shape such as boundaries. It is also used for pre or post processing, such as filtering, thinning, and pruning.

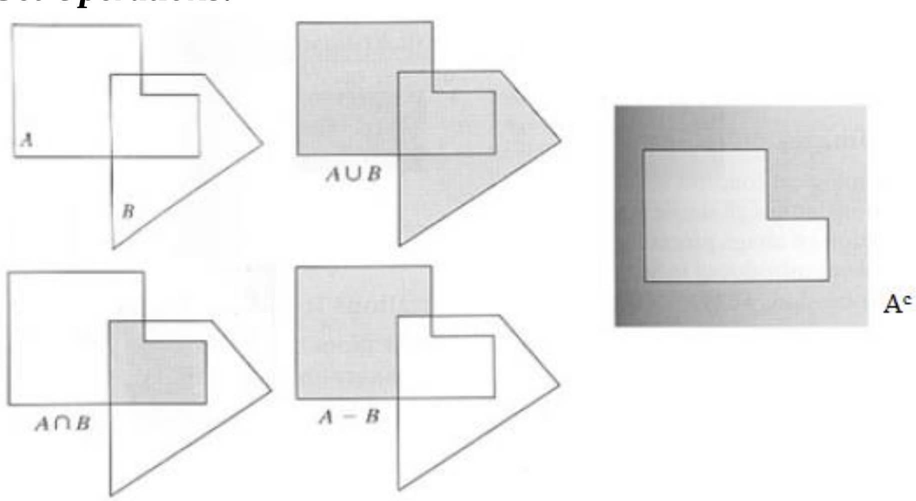
The language of mathematical morphology use set theory to represent objects in an image.

## ***Basic Concepts from Set Theory***

Sets in mathematical morphology represent objects in an image. For example, the set of all white pixels in a binary image is a complete morphological description of the image. In binary image, the sets are members of the 2-D integer values $Z^2$, where element of sets is a tuple *(2-D vector)* of white or black pixels whose coordinates are *(x,y)*. For binary image, let A be a set in $\mathbf{Z^2}$

$a \in A$ ;   $a = (a1, a2)$ is an element of A.

***Set Operations:***



In addition, the concept of set reflection and translation are used in mathematical morphology.

The reflection of a set ***B***, denoted $\hat{B}$ is defined as

$$\hat{B} = \{w \mid w = -b, \quad \text{for} \quad b \in B\}.$$

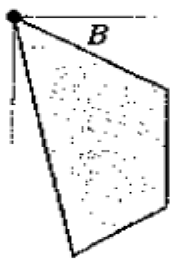



Fig (a)

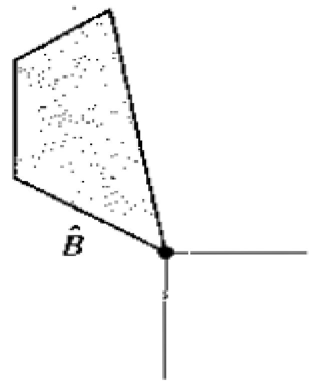



Fig (b)

If ***B*** is the set of pixels representing an object in an image, then $\hat{B}$ is simply the set of points in ***B*** whose (x,y) coordinates in *fig (a)* are replaced by (-x,-y) in *fig (b)*

The translation of a set ***B*** by pixel $z = (z_1, z_2)$, denoted by $(B)_z$ is defined as

$$(B)_z = \{c \mid c = b + z, \quad \text{for} \quad b \in B\}$$

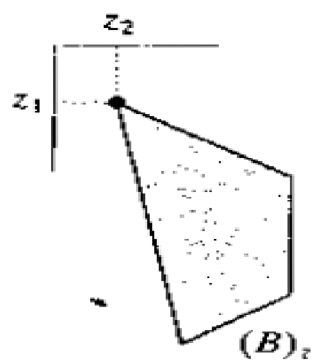



Fig (c)

If ***B*** is the set of pixels representing an object in an image, then $(B)_z$ is the set of points in ***B*** whose *(x,y)* coordinates are replaced by $(x+z_1, y+z_2)$ in *fig (c)*

***Logic operation in binary image:***

Logical operations commonly used are as follows:

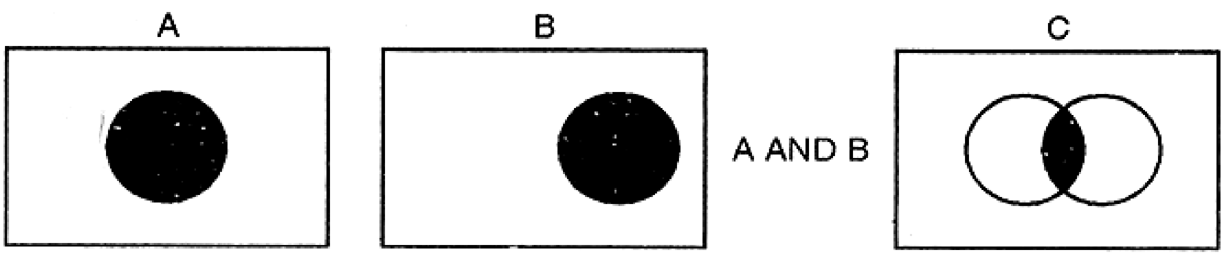
a) AND: *a AND b*

b) OR: *a OR b*

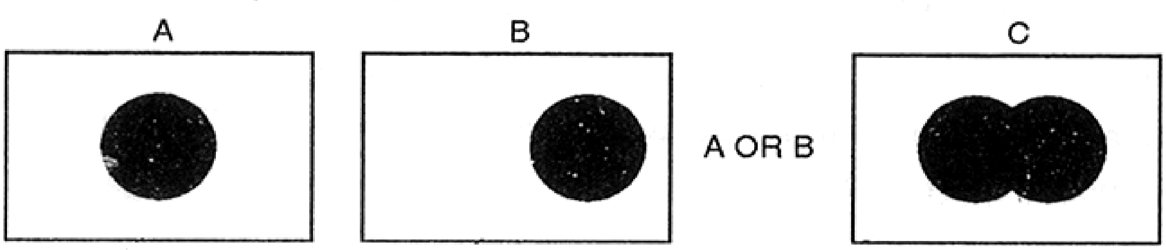
c) COMPLEMENT: *NOT a*

These operations can be combined to form other logic operations. Logic operations actually apply only to binary images *(Image having only two values, 0 and 1).*

***AND operation:***
The AND operator gives out 1 only when both 'a' and 'b' are equal to 1.

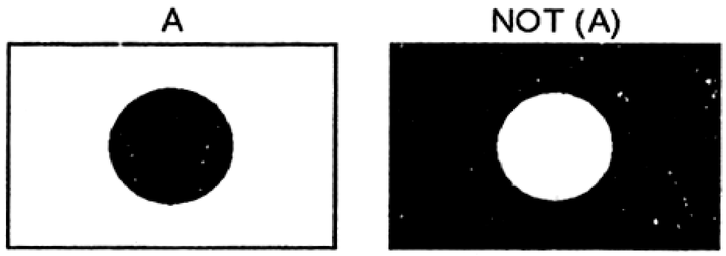



***OR operation:***
The OR operator gives out 1 if either 'a' or 'b' or both are equal to 1.



***NOT operation:***

The COMPLEMENT (NOT) operator gives out 1 when a = 0



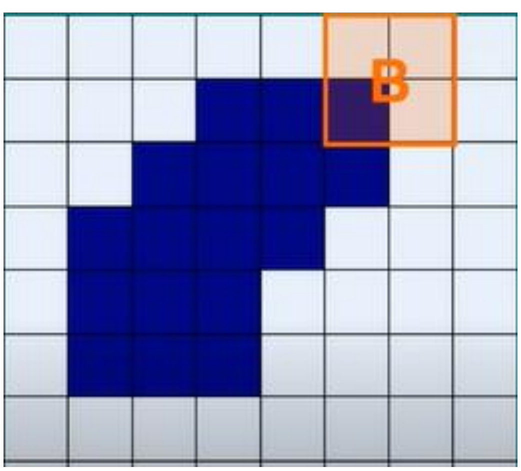

## ***Definition of Hit, Fit and Miss***

In mathematical morphology, a ***structuring element*** (s.e.) is a shape used to interact with a given image with the purpose of drawing conclusion on how this shape hits, fits or miss the shape in the image.

It is typically used in morphological operations such as dilation, erosion, opening, and closing as well as the hit or miss transform.

***Hit:***

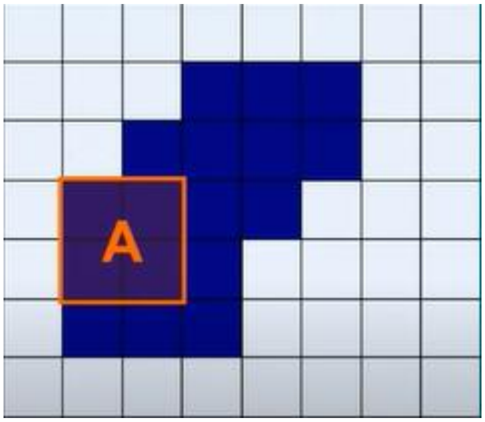
Any ‘ON’ pixels of structuring element covered by ‘ON’ pixels in the image then, it is called ~~Fit~~ hit. Here in figure bellow one pixel of B is covered by “ON” pixel of image.




B vneko euta structure element ho

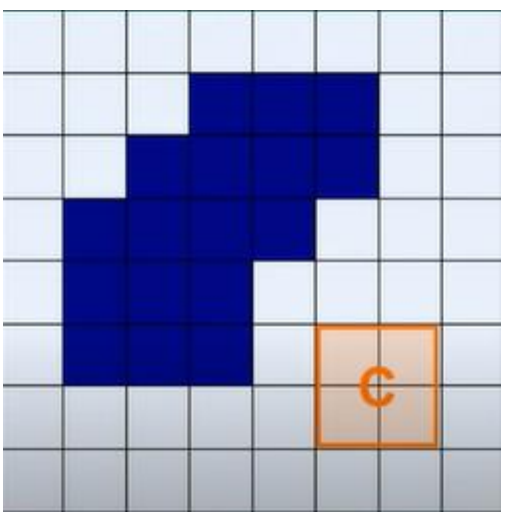
***Fit:***

All ‘ON’ pixels of structuring element covered by ‘ON’ pixels in the image then, it is called Fit. Here in figure bellow all pixel of A is covered by “ON” pixel of image.




***Miss:***

No ‘ON’ pixels of structuring element covered by any ‘ON’ pixels in the image then, it is called Miss. Here in figure bellow no pixel of C is covered by “ON” pixel of image.



## ***<u>Dilation and Erosion</u>***

### *Dilation:*

This is the mathematical morphological operation which adds the pixel to the boundaries of the object in an image. In this process the binary image is expanded from its original image.

Mathematically the Dilation operation is represented by following equation

$$A \oplus B = \left\{ Z \mid (\hat{B})_z \bigcap A \neq \varphi \right\}$$

*Where, A is original image and B is structuring element*

This equation says that, the common pixel of A and B should not be zero. Which means B should be hit or fit to A.

Following rule should be flowed to perform this operation

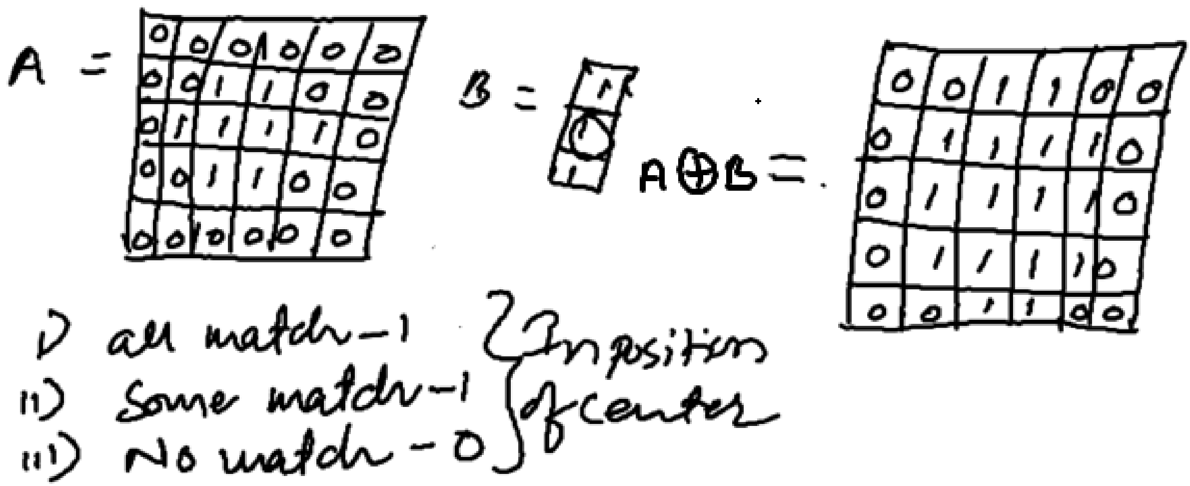
1. If structuring element is ***hit or fit*** then place ***1*** in the center position of structuring element.
2. If structuring element is ***miss*** then place ***0*** in the center position of structuring element.

**Example:** Given the image A and Structuring element B

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculate the dilation of this image.

***Solution:***




***Erosion:***

This is the mathematical morphological operation which removes the pixel from the boundaries of the object in an image. In this process the binary image is shrinks from its original image.

Mathematically the Erosion operation is represented by following equation

$$A \ominus B = \{Z \mid (B)_Z \subseteq A\}$$

This equation says that all the pixel of B should be belongs to A that means B should be fit to A.

Following rule should be flowed to perform this operation

1. If structuring element is ***fit*** then place ***1*** in the center position of structuring element.
2. If structuring element is ***hit or miss*** then place ***0*** in the center position of structuring element.

**Example:** Given the image A and Structuring element B

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculate the erosion of this image.

***Solution:***

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad A \ominus B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

i) all match – 1
ii) Some match – 0
iii) No match – 0

} in position of center

## Opening and Closing:

***Opening:***

It is the process of erosion followed by dilation. It is denoted by symbol **"o"**

Mathematically opening is represented by following equation

$$A \circ B = (A \ominus B) \oplus B$$

***Closing:***

It is the process of dilation followed by erosion. It is denoted by symbol **“.”**

Mathematically opening is represented by following equation

$$A \cdot B = (A \oplus B) \ominus B$$

**Assignment:** Given image A and structuring element B

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Calculate Opening and Closing of this image.

**End of Unit -5**