Chapter-1: Introduction

Pixel:

A *Pixel* is most widely used to denote the elements of a Digital Image, Which is the smallest element of the digital image. Pixels are arranged in a 2-dimensional grid, represented using squares or circles.

Each pixel is a sample of an original image, where more samples typically provide more-accurate representations of the original image. The *intensity* or *gray level* of each pixel is variable; in color systems, each pixel has typically three or four components such as *RGB* (*red*, *green*, *and blue*) or *CYMK* (*cyan*, *yellow*, *magenta and black*). In the monochrome system, each pixel has only two components *black or white*.

The word *pixel* is based on a contraction of *pix* ("pictures") and *el* ("element").

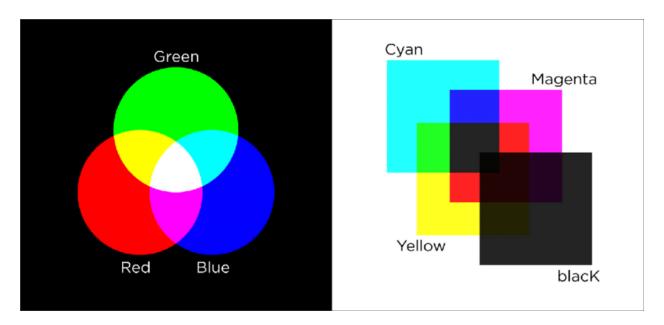


Fig: RGB and CYMK Color mode

Resolution:

The number of pixels in an image is sometimes called the *resolution*, even though this is a bit of a misuse of the term. If we are using the term to describe pixel count, one convention is to express resolution as the *width* by the *height*.

For example: a monitor resolution of 1280×1024 . This means there are 1280 pixels in *width*, and 1024 pixels in *height*.

Another convention is to express the number of pixels as a single number, like a 5 megapixel camera (1 megapixel = 1 million pixels). This means the pixels along the width multiplied by the pixels along the height of the image taken by the camera equals 5 million pixels. In the case of our 1280×1024 monitors, it could also be expressed as $1280 \times 1024 = 1,310,720$, or 1.31 megapixels.

An Image resolution describes the amount of detail that an image contains. The term can be applied to digital images, film images, and prints. The bottom line is that higher resolution means more image detail. Camera manufacturers are always trying to sell you on the number of megapixels.

How many pixels are "enough"?

It depends on what you want to do with the image, and how big you want to enlarge it. If image resolution is low, when we blow it up too much, we start to see the individual pixels. That effect is called "pixelation".

There is software available to help you increase the resolution of your images artificially. It uses an *interpolation algorithm* which essentially fills in extra pixels with a "best guess" at the right color.

For excellent quality prints, you'd ideally like a minimum of 240 pixels per inch in each dimension. This means for a 4"x6" print, you need 240 x 4 pixels in the width, and 240 x 6 pixels in the height. That's 960 x 1440 pixels. Multiplied together, that's 1,382,400 pixels, or approximately 1.4 megapixels. By the same token, to make decent 8"x10" print, you'd need a 4.6 megapixel camera.

Definition of Digital Image:

An image is defined as a two-dimensional function, f(x,y), where x and y are spatial coordinates, and the amplitude of 'f' at any pair of coordinates (x,y) is called the *intensity* or *gray level* of that image at that point. When, x, y, and amplitude values of f are finite, we call it a *digital image*.

In other words, an image can be defined by a two-dimensional array specifically arranged in *rows* and *columns*. Digital Image is composed of a finite number of elements, each of which elements have a particular value at a particular location. These elements are referred to as *picture elements or image elements or pixels*. Typically, pixels are organized in an ordered rectangular array. The size of an image is determined by the dimensions of this pixel array. The *image width is the number of columns, and the image height is the number of rows* in the array. Thus the pixel array is a matrix of *M columns x N rows*. To refer to a specific pixel within the image matrix, we define its coordinate at *x* and *y*. The coordinate system of image matrices defines *x as increasing from top to bottom and y as increasing from left to right*. Compared to normal mathematic convention, the origin is in the top left corner.

Image size specifically describes the number of pixels within a digital image. The real world representation of a digital image requires one additional factor called *resolution*. Resolution is the spatial scale of the image pixels.

For example, an image of 3300x2550 pixels with a resolution of 300 pixels per *inch* (ppi) would be a real world image size of 11" x 8.5". To clarify resolution terms, *ppi* (*pixels per inch*) and *dpi* (*dots per inch*). *ppi* refers to pixel arrays, while *dpi* refers to printer resolution. In reality these two resolution terms are used

Representation of image in matrix form ma rakhan mil

interchangeably. Another resolution term you may encounter is *lpi* (*lines per inch*), which describes halftone resolution (*This is the number of lines of dots in one inch, measured parallel with the screen's angle.*) and is used in magazine and newspaper printing. Many image editing applications default the resolution to *72 ppi*. This is true for saving JPG images.

Types of an digital image

Binary Image: The binary image as its name suggests, contain only two pixel elements, i.e *0* & *1*, where 0 refers to *black* and 1 refers to *white*. This image is also known as *Monochrome*.

Black And White Image: The image, which consists of only black and white color, is called Black and White Image.

Color Image:

8 Bit Color Format: It is the most famous image format. It has 256 different shades of colors in it and commonly known as *Gray-scale Image*. In this format, 0 stands for Black, and 255 stands for white, and 127 stands for gray.

16 Bit Color Format: It is a color image format. It has 65,536 different colors in it. It is also known as *High Color Format*. A 16 bit format is actually divided into three further formats which are *Red*, *Green and Blue*. That is famous as RGB format.

Digital Image Processing:

Digital Image Processing (**DIP**) is the technology of manipulating groups of pixels to enhance the quality of the image or create different perspectives or to extract information from the image digitally, with the help of computer algorithms.

Image processing mainly include the following steps:

- 1. Importing the image via image acquisition tools
- 2. Analyzing and manipulating the image

- 3. Output in which result can be altered image or a report which is based on 1. image sharpening (filtering technique use gaerw)
 - analyzing that image

- 3.image compression(using diffrn technique like huffmann coding)
- 4. pattern recognition(object detect grna image processing hunxa)

Application of Digital Image Processing:

6.medicial field (gamma ray imaging, CT scan, UV imaging esto ma use)

Image sharpening and restoration: Image sharpening and restoration refers to process images that have been captured from the modern camera to make them a better image or to manipulate those images in way to achieve desired result. It refers to do what Photoshop usually does. This includes zooming, blurring, sharpening, gray scale to color conversion, detecting edges etc.

Medical field: The common applications of DIP in the field of medical are Gamma ray imaging, PET (Positron Emission Tomography which use radiotracer to visualize the changes in tissues) scan, X-Ray Imaging, CT (Computed Tomography) scan, UV (ultraviolet) imaging etc.

Remote Sensing: In the field of remote sensing, the area of the earth is scanned by a satellite or from a very high ground and then it is analyzed to obtain information about it. One particular application of digital image processing in the field of remote sensing is to detect infrastructure damages caused by an earthquake.

Transmission and encoding: Digital Image can transmit from one place to another place using transmission media. Encoding image helps to compress the image to send in lower bandwidth.

Machine/Robot vision: Apart from the many challenges that a robot face today, one of the biggest challenge still is to increase the vision of the robot. Make robot able to see things, identify them, identify the hurdles, it has to take picture and process it.

Color processing: Color processing includes processing of colored images and different color spaces that are used. For example RGB color model, CMYK. It also involves studying transmission, storage, and encoding of these color images.

Pattern recognition: Pattern recognition involves study from image processing and from various other fields that includes machine learning (a branch of artificial intelligence). In pattern recognition, image processing is used for identifying the objects in an image and then machine learning is used to train the system for the change in pattern. Pattern recognition is used in computer aided diagnosis, recognition of handwriting, recognition of images etc.

Video processing: A video is nothing but just the very fast movement of pictures. The quality of the video depends on the number of frames/pictures per minute and the quality of each frame being used. Video processing involves noise reduction, detail enhancement, motion detection, frame rate conversion, aspect ratio conversion, color space conversion etc.

Representation of Digital Image in Spatial Domain Form

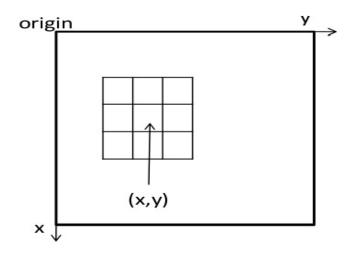
Spatial Domain refers the position of pixels in a plane image itself, and are based on the direct manipulation of pixels in an image.

Spatial Domain process is denoted by equation:

$$g(x,y)=T[f(x,y)]$$

Where, f(x,y) is the input image, g(x,y) is the processed image, T is the operator on f define over some neighborhood of (x,y).

Square or rectangular sub-image area centered at (x,y) is used as neighborhood about a point (x,y).



Gray level transformation is denoted by expression:

$$s=T(r)$$

Where, T is the gray level transformation function, r and s denotes gray level of f(x,y) and g(x,y) respectively at any point (x,y).

Representation of Digital Image in Matrix Form

As we know, images are represented in *rows* and *columns* we have the following syntax in which images are represented

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2) & \dots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ f(M-1,0) & f(M-1,1) & f(M-1,2) & \dots & f(M-1,N-1) \end{bmatrix}$$

In **MATLAB** the start index is from 1 instead of 0. Therefore, f(1,1) = f(0,0).

Fundamental Steps in Image Processing

Image processing is a method to convert an image into digital form and perform some operations on it, in order to get an enhanced image or to extract some useful information from it.

Purpose of Image processing

Visualization: To observe the objects that are not visible.

Image sharpening and restoration: To create a better image.

Image retrieval: To get the image of interest or desirable image.

Measurement of pattern: To measure various objects in an image.

Image Recognition: To distinguish the objects in an image.

The fundamental steps in Digital Image Processing are shown in figure below:

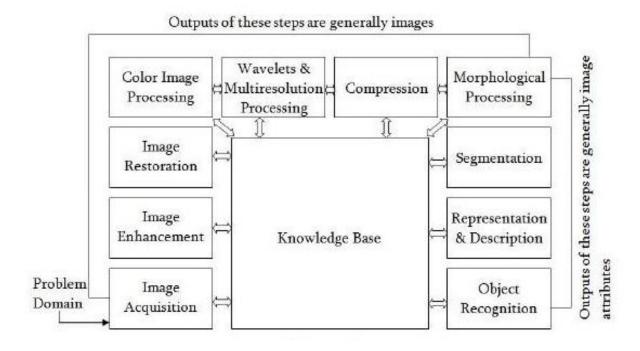


Fig: Fundamental Steps in Digital Image Processing

1. Image Acquisition

- It is basically capturing an image by camera
- Generally, the image acquisition stage involves pre-processing, such as scaling, etc.

2. Image Enhancement

- It is the process of filtering image (removing noise, increasing contrast and brightness etc.) to improve the quality of image so it will be more pleasing to eyes.
- The resulting image will be more suitable than the original image.

3. Image Restoration

• It is the process of improving appearance (*reducing blurring etc.*) of an image by mathematical or probabilistic models.

4. Color Image Processing

- Color image processing is an area that has been gaining its importance because of the significant increase in the use of digital images over the Internet. This may include color modeling and processing in a digital domain etc.
- It use color models like RGB, CYMK elc.

5. Wavelets and Multi-resolution Processing

- Wavelets are the small waves of limited duration which are used to calculate wavelet transform which provides time-frequency information.
- Multi-resolution is the process of representing images in various degrees of resolution.

6. Compression

- Compression deals with techniques for reducing the size of image with minimum reduction of its quality.
- Particularly in the uses of internet it is very much necessary to compress data.

7. Morphological Processing

• Morphological processing deals with the tools for extracting image components, that are useful in the representation and description of shape.

8. Segmentation

• In this stage, an image is a partitioned into its objects.

• Segmentation is the most difficult tasks in DIP. It is a process which takes a lot of time for the successful solution of imaging problems which requires objects to identify individually.

9. Representation and Description

- Representation makes a decision whether the data should be represented as a boundary or as a complete region:
 - ➤ **Boundary Representation** focuses on external shape characteristics, such as corners and borders that separate the different properties of an image.
 - ➤ **Region Representation** focuses on internal properties, such as texture or skeleton shape or similar properties.

10. Object recognition

• Recognition is the process that assigns a label, such as, "vehicle" to an object based on its descriptors.

11. Knowledge Base:

- Knowledge is the last stage in DIP.
- In this stage, important information of the image is located, which limits the searching processes.
- The knowledge base is very complex when the image database has a high-resolution satellite.

Elements of Digital Image Processing

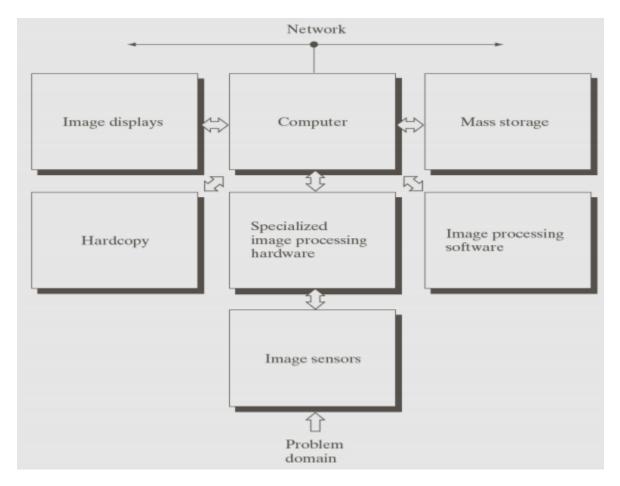


Fig: Elements of Digital Image Processing

Image Sensors:

- Physical device that is sensitive to the energy radiated by the object.
- Digitizer that converts the output of the physical sensing device into digital form.

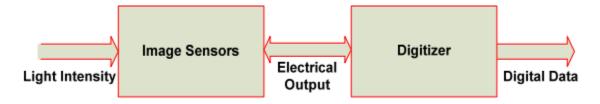


Fig: Digital Camera

Specialized Image Processing Hardware:

• Specialized Image Processing Hardware usually consists of the digitizers and hardware that performs other primitive operations, such as arithmetic logic unit (ALU). Speed is the most important parameter (30 frames /sec).

Specialized Image Processing Software

• Specialized Image Processing Software is specialized modules that manipulate the image and produce the result as desired form.

Computer:

• It contains image processing system

Mass Storage:

- It is a must in image processing applications (image size of 1024x1024 pixels, with intensity level for each pixel: 8 bits, requires one Megabyte for saving)
- Mass Storage categories:
 - ➤ Short-term storage for use during processing.
 - ➤ On-line storage for relatively fast operations.
 - ➤ Archival storage for infrequent access.

Image Displays:

• Flat screen TV monitors.

Hardcopy:

• Devices for recording images: laser printers, film cameras, CD-ROM disk, others.

Networking:

• Transfer image using networking. The key parameter of networking is the bandwidth.

Element of Visual Perception

The field of digital image processing is built on the foundation of mathematical and probabilistic formulation, but human intuition and analysis play the main role to make the selection between various techniques, and the choice or selection is basically made on subjective, visual judgments.

In human visual perception, the eyes act as the sensor or camera, neurons act as the connecting cable and the brain acts as the processor.

The basic elements of visual perceptions are:

- Structure of Eye
- Image Formation in the Eye
- Brightness Adaptation and Discrimination

Structure of Human Eye

Figure bellow shows the horizontal cress section of the human eye. The eye is nearly a sphere, with an average diameter of approximately *20 mm*.

The three members enclose the eyes:

- Cornea and Sclera: outer cover
- Choroid
- Retina

Cornea: it is a strong, transparent tissue that covers the anterior surface of eye.

Sclera: it is an opaque (*non-transparent*) membrane (*layer*) that encloses the remainder of the optic globe.

Choroid: it lies directly below the sclera. This membrane contains a network of blood vessels that serve as the major source of nutrition to the eye. The choroid coat is heavily pigmented (*natural coloring of animal tissues*) and hence helps to reduce the amount of extraneous (*irrelevant*) light entering the eye and the backscatter within the optic globe.

The choroid is divided into parts *Ciliary body* and *Iris*, which controls the amount of light that enters the eye. The central opening of the Iris (*pupil*) is diameter approx. 2 to 8 mm. The front of the Iris contains the visible pigment of the eye and back contains a black pigment.

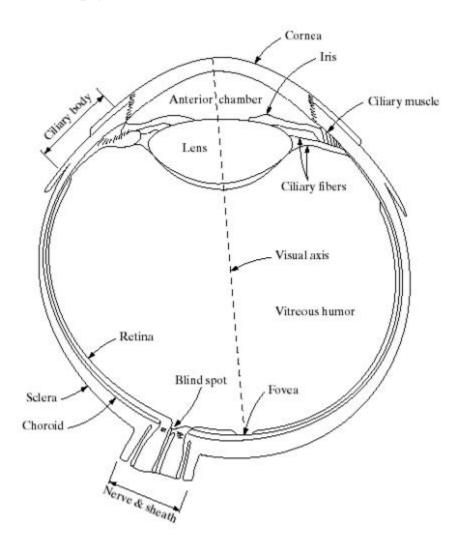


Fig: Simplified diagram of a Human Eye

Lens: it is made up of concentric (surrounded by the circle) layers of fibrous cells and is suspended by fibers that attach to the Ciliary body. It contains 60 to 70% water, about 6% fat and mere protein than any other tissues in the eye. The lens is slightly yellow in color and absorbs approx. 8% of the visible light spectrum, with relatively higher absorption at shorter wavelengths. Both infrared and ultraviolet

light are absorbed by proteins within the lens and in excessive amounts can damage the eye.

Retina: The innermost member of the eye is retina, which lies inside of the wall's entire posterior (*back position*) portion. When eye is focused, light comes from the object outside the eye is imaged on the retina.

There are two classes of receptors: *Cones* and *Rods*

Cones: the numbers of cones in each eye between the 6 to 7 million. They are located primarily in the central portion of the retina, called **Fovea** and are highly sensitive to color. The muscle controlling the eye rotates the eyeball until the image of an object of interest falls on the fovea. Cone vision is called **Photopic** or **bright-light** vision.

Rods: the numbers of rods in each eye between the 75 to 150 million, and they are distributed all over the retina surface. Rods serve to give overall picture of the field of view. They are not involve in the color vision and are sensitive to low levels of illumination (*lighting/brightness*).

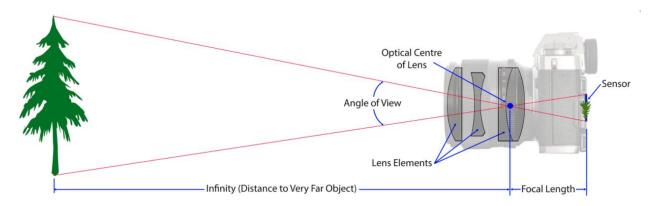
For example: objects that appear brightly colored in day-light but seen colorless in moon-light because only the rods are stimulated. This phenomenon is known as *Scotopic* or *dim-light* vision.

Fovea: it is a circular indentation in the retina of about 1.5 mm diameter. And can also take as square sensor array of 1.5mm x 1.5mm. The density of *Cones* in that area is approx. 150,000 elements per mm².

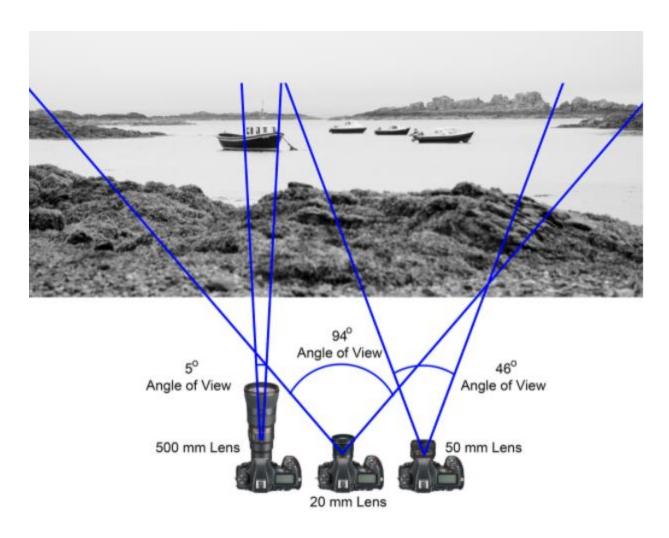
Blind Spot: it is the area where *Cones* and *Rods* are absence.

Image Formation in the Eye

In an ordinary photographic camera, the lens has a fix focal length (*distance between the optical center of the lens and the camera's sensor or film plane*), and focusing at various distances is achieved by varying the distance between the lens and the imaging plane, where the film is located. This property is provided by the camera lens which has variable focal length.



Lens focal length tells us the *angle of view* (how much of the scene will be captured) and the *magnification* (how large individual elements will be). Longer the focal length (which is Zoom In), narrower the angle of view and the higher the magnification. Shorter the focal length (which is Zoom Out), the wider the angle of view and the lower the magnification.



But, in the human eye, the distance between the lens and imaging region (*the retina*) is fixed, and the focal length needed to achieve proper image is obtained by varying the shape of the lens. The fibers in the *Ciliary body* accomplish this, by flattering or thickening the lens for distance or near objects respectively.

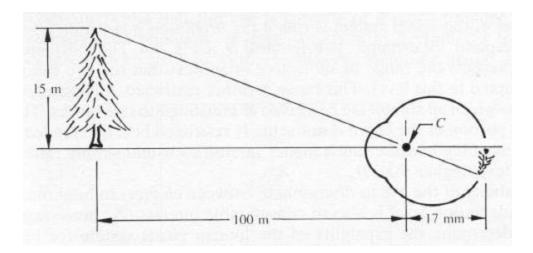


Fig: Graphical representation of the eye looking at the tree

Pint C is the Optical Center of the lens.

Distance between the center of the lens and the retina along the visual axis is approx. 17mm. The range of focal length is approx. 14mm to 17mm. If the distance of object from center of lens is grater then 3m then focal length will be 17mm and less then 3m focal length will be 14mm.

New, we can calculate the retinal image of the object:

Here, in the above figure, a person is looking at the tree of 15m height from the distance of 100m.

Let's consider height of the retinal image is x,

$$\frac{15}{100} = \frac{x}{17}$$

$$x = 2.55mm$$

Hence, the retinal image of the tree in Fovea of the eye is $2.55 \ mm$.

Brightness Adaptation and Discrimination

The digital images are displayed as a discrete set of intensities (*brightness*), the human eye's ability to discriminate between different intensity levels is an important consideration in image processing. The range of light intensity level to which the human visual system (*HVS*) can adapt is order of 10¹⁰ from the *Scotopic threshold* to the *glare limit*. The subjective brightness (*intensity perceived by HVS*) is a log of intensity incident on the eye.

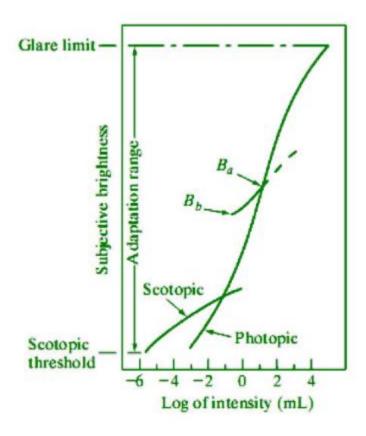


Fig: Range of subjective brightness sensation in a particular adaptation level

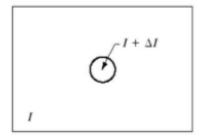
Here, *Scotopic* is dim-light vision or *cone* vision and *Photopic* is bright-light vision or *rod* vision.

Above figure shows the graph of light intensity versus subjective brightness. The long solid curve represents the range of intensity to which HVS can adapt. But HVS cannot operate over such a range simultaneously; it accomplishes this large

variation by changing its overall sensitivity, this phenomenon known as *brightness adaptation*.

The total range of intensity level the eye can discriminate simultaneously is smaller as compare to total adaptation range. For any given set of condition, the current sensitivity level of visual system is called *brightness adaptation level*. In the above graph B_a and B_b are the different brightness adaptation level in different condition.

The ability of the eye to discriminate between change in light intensity at any specific adaptation level is depends on weber ratio. The ratio of increment of illumination of intensity to background of intensity illumination is called weber ratio i.e ($\Delta I/I$). Where, ΔI is the change in intensity and I is background intensity.



In figure, circle is an area where change in intensity (ΔI) is high, so brightness is discriminated by HVS.

If the ratio $(\Delta I/I)$ is small, then small percentage of change in intensity is needed. This indicates good brightness adaptation.

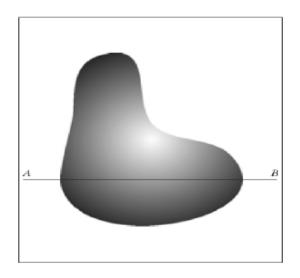
If the ratio $(\Delta I/I)$ is large, then large percentage of change in intensity is needed. This indicates poor brightness adaptation.

Sampling and Quantization

When we acquire the image, we have to convert sense data in to digital form to store in digital platform. The output of most sensors is a continuous voltage waveform whose amplitude and spatial behavior are related to the physical phenomenon being sensed. To create a digital image, we need to convert the continuous sensed data in to digital form. This involves two processed *Sampling* and *Quantization*.

Basic Concept in Sampling and Quantization

The basic idea behind Sampling and Quantization is illustrated in figure bellow:



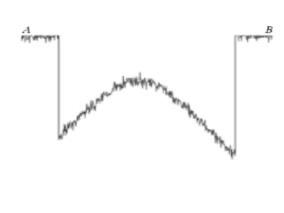


Fig (a): Continuous Image

Fig: (b): A scan line from A to B

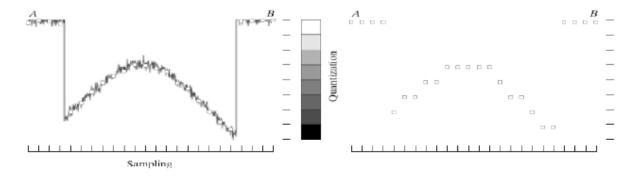


Fig (c): Sampling and Quantization

Fig (d): Digital Scan Line

Figure: (a) shows a continuous image, f(x, y), that we want to convert to digital form. An image may be continuous with respect to the x- and y- coordinates and also in amplitude. To convert it to digital form, we have to sample the function of both coordinates and in amplitude. Digitizing the values of coordinate is called *Sampling*. Digitizing the amplitude is called *Quantization*.

The one-dimensional function shown in Fig (b) is a plot of amplitude (gray or intensity level) values of the continuous image along the line segment AB as shown in fig (a). The random variations are due to image noise. To sample this function, we take equally spaced samples along line AB, as shown in fig (c). The spatial location of each sample is given by a vertical tick mark in the bottom part of the figure. The samples are shown as small white squares superimposed (placed) on the function. The set of these discrete locations gives the sampled function. However, the values of the samples still span (vertically) a continuous range of gray-level values. In order to form a digital function, the gray-level values also must be converted (quantized) into discrete quantities. The right side of fig (c) shows the gray-level scale divided into eight discrete levels, ranging from black to white. The vertical tick marks indicate the specific value assigned to each of the eight gray levels. The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample. The assignment is made depending on the vertical proximity (closeness) of a sample to a vertical tick mark. The digital samples resulting from both sampling and quantization are shown in *fig* (d). Starting at the top of the image and carrying out this procedure line by line produces a two-dimensional digital image.

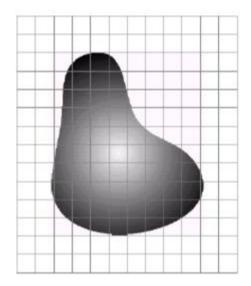


Fig: Continuous image projected in to sensor array

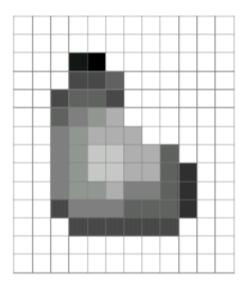


Fig: Result of image sampling and quantization

Representing Digital Imags

Let f(x,y) represent continuous image function of two continuous variables x and y. where, x represent the rows and y represents the columns in the corresponding x- and y- coordinates, and f contain the real number, which is the value of gray level or intensity. We convert this function in to digital image by sampling and quantization.

Suppose, we take samples from the image in to a 2-D array, f(x, y), containing M rows and N columns, where f(x, y) are discrete coordinates. Coordinate can be denote as:

$$x=0,1,2,...,M-1$$
, and $y=0,1,2,...,N-1$

The value of the digital image at origin is f(0,0), next sample in first row is f(0,1) and so on.

In general, the value of the image at any coordinates (x,y) is denoted by f(x,y), where x and y are the integers. The section of the real plan spanned by the coordinates of an image is called **Spatial Domain**, with x and y being referred to as **Spatial Variables** or **Spatial Coordinates**.

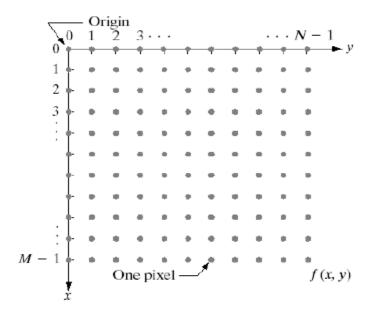
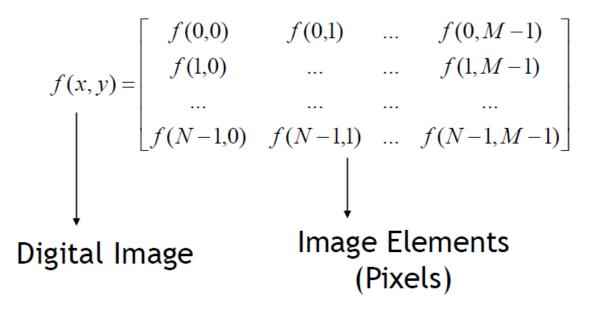


Fig: Coordinate convention of digital image

In this figure function f(x,y) provide the position of each element of image with corresponding grey level value.

We can represent the digital image in matrix form as:



Each element of this matrix is called an *image element*, *picture element*, *pixel or pel*.

Expressing sampling and quantization in mathematical terms, let \mathbf{Z} and \mathbf{R} denotes set of integers (*coordinates*) and set of real numbers (*gray level*) respectively.

The sampling is Cartesian product of coordinate i.e. \mathbb{Z}^2 , which is the set of all ordered pair of elements (z_i, z_j) , z_i , and z_j are the integers from \mathbb{Z} .

Hence, f(x,y) is a digital image, if (x,y) are integers from \mathbb{Z}^2 and f is a function that assign an gray level value denoted by R to each distinct pair of coordinate (x,y), this function assignment is called quantization process.

If the gray level also the integers then, R replace by Z and digital image becomes 2-D function whose coordinates and amplitude (intensity level) value are integers.

This digitization process required values for M, N and the number L of discrete intensity level. Where, M, N are the positive integers. And the number of intensity

level is the integer power of 2, i.e. $L=2^k$. Where, k is the number denote intensity level.

The number, b, of bits required to store a digital image is:

$$b = M \times N \times k$$

When, M=N, this equation becomes

$$b = N^2 k$$

Following table show the number of bits required to store square image with various values of N and k. The number of intensity levels L corresponding to each value of k is shown in parenthesis, i.e. k(L).

N/k	1(L=2)	2(L=4)	3(L = 8)	4(L=16)	5(L = 32)	6 (L=64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Intensity Resolution

Spatial resolution is a measure of the smallest discernible (*noticeable*) detail in an image. Quantitatively, spatial resolution can be measure in *line pair per unit distance*, and *dots (pixel) per unit distance*. So, the image resolution is defined as number of discernible line pairs per unit distance. Here line may be white or black lines. Dots per inch (dpi) also a measure of image resolution, which is commonly used in printing and publishing industry.

Image resolution is depends on sample number and gray level numbers. More these parameters are increased, closer the digitize image to original image.

But, when value of M, N and k are increase, storage and processing requirements are also increased.

We can get different version of image with different resolution by varying the value of M, N and K

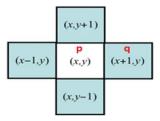
Some Basic Relationships

There are several important relationships between pixels in a digital image. A digital image is denoted by f(x, y). Here, a particular pixel is denoted by lowercase letters such as p and q.

Neighbors of a Pixel

A pixel p, at coordinates (x, y) has four neighbors as two horizontal and two vertical neighbors whose coordinates are given by

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$



This set of pixels are called the **4-neighbors** of p, is denoted by $N_4(p)$

Each pixel is a unit distance from (x, y), and some of the neighbors of p lie outside the digital image, if (x, y) is on the border of the image.

The *four diagonal* neighbors of p as $N_D(p)$ have coordinates

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

(x-1,y+1)	(x,y+1)	q (x+1,y+1)
(x-1,y)	p (x,y)	(x+1,y)
(x-1,y-1)	(x,y-1)	(x+1,y-1)

These points, together with the 4-neighbors, i.e. $N_4(p) + N_D(p) = N_8(p)$ are called the 8-neighbors of p.

As before, some of the points in $N_D(p)$ and $N_8(p)$ falls outside the image, if (x, y) is on the border of the image.

Adjacency, connectivity, Regions and boundaries

Adjacency:

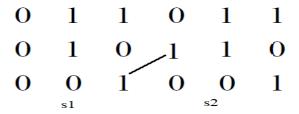
Let V be the set of gray-level values used to define adjacency. In a binary image, $V=\{1\}$, if we are referring to adjacency of pixels with gray-level value 1.

In a gray scale image, the idea is the same, but set V typically contains more elements. For example, in the adjacency of pixels with a range of possible gray-level values 0 to 255, set V could be any subset of these 256 values. We consider three types of adjacency:

- **4-adjacency:** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- 8-adjacency: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
- *m-adjacency* (*mixed adjacency*): Two pixels *p* and *q* with values from *V* are m-adjacent if
 - i. q is in $N_4(p)$, or
 - ii. q is in $N_D(p)$ and the set $(N_4(p) \cap N_4(p))$ (i.e. common adjacent) has no pixels whose values are from V.

4	-adj		8-adj						
0	1	0			1	1	1		
1	p1	1			1	p1	1		
0	1	0			1	1	1		
		0	1	0		0	0	1	
		0	p1	0		0	p1	0	
		0	0	0		0	0	0	
m-adj						r	n-adj		

Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.



Path:

A path (*curve*) from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels:

- (x0,y0), (x1,y1), ..., (xn,yn)
- Where, (x0,y0)=(x,y), (xn,yn)=(s,t)
- (xi,yi) is adjacent to (xi-1,yi-1) or (xi+1,yi+1) for $(1 \le i \le n)$; **n is the length** of the path.
- If (xO, yO) = (xn, yn): This is called *Close Path*

		p(x0, p(x,y	y0))	p(x0	,y0)	close path	
0	0	1	1	0	1	p(xn,yn)	
0	0	1	1	0	0		
0	0	1	1	0	0]	
0	0	1	1	0	0		
0	0	1	1	0	0		
0	0	1	1	0	0		
		q(s,t p(xr	_				

There are three types of path: 4-path, 8-path, m-paths, which are depending on the type of adjacency.

Let, S is the set of pixel and p,q are the member of S, then q is connected to p in S, if there is a path from p to q consisting entirely of pixels in S.

Connectivity:

It is an important concept in digital image processing.

It is used for establishing boundaries of objects and components of regions in an image.

Two pixels are said to be connected:

- if they are adjacent in some sense (neighbor pixels,4/8/m-adjacency)
- if their gray levels satisfy a specified criterion of similarity (*equal intensity level*) which create a path between two pixels.

There are three types of connectivity on the basis of adjacency. They are:

- 1. **4-connectivity:** Two or more pixels are said to be 4-connected if they are 4-adjacent with each other's.
- 2. **8-connectivity:** Two or more pixels are said to be 8-connected if they are 8-adjacent with each other's.
- 3. **m-connectivity:** Two or more pixels are said to be m-connected if they are m-adjacent with each other's.

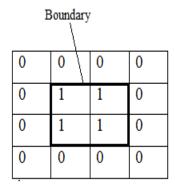
4-	8-connectivity								
0	1	0			1 <	1	1		
1 _	p1-	-1			1 -	p1-	-1		
0	1	0			1 /	1	1		
		0	1 ,	0		0	0	_1	
		0	p1	0		0	p1	0	
		0	0	0		0	0	0	
		m-c	connec	tivity		m-c	onnec	tivitv	

Regions and Boundaries:

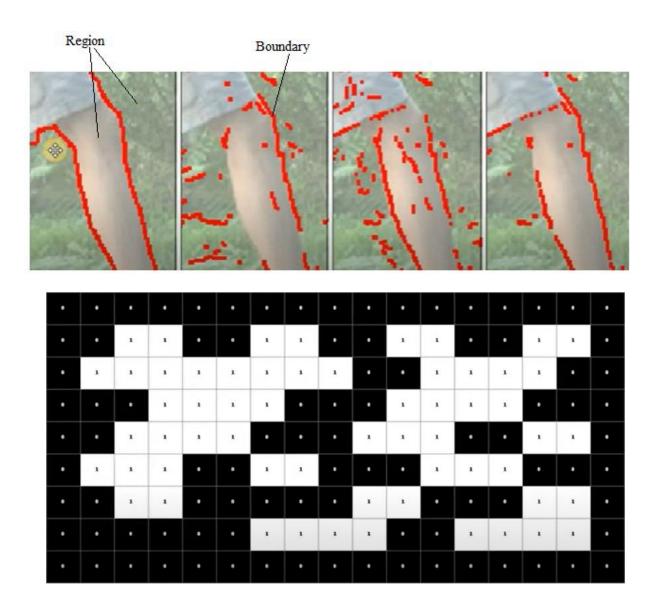
Region: Let $V=\{1\}$, and R is the subset of pixel in an image, then, we can call R is a region if p and q in are connected.

Region									
0	0		0	0					
0	1	1	1	0					
0	1		1	0					
0	0		0	0					

Boundary: The boundary (also called border or contour) of a region R is the set of pixels in the region that have one or more neighbor is **not** in a member of R.



Inner Boundary										
0	0	0	0	0	0	0	0			
0	0	0	0	0	0	0	0			
0	0	0	0	Ì <mark>0</mark>	0	0	0			
0	0	1	1	1	0	0	0			
0	0	1	1	1	0	þ	0	-Outer Boundary		
0	0	1	1	1	0	0	0			
0	0	0	0	0	0	0	0			
0	0	0	0	0	0	0	0			



Distance Measure between Pixels:

For pixels p, q, z with coordinates (x,y), (s,t), (u,v), D is a distance function or metric if:

- $D(p,q) \ge 0 (D(p,q)=0 \text{ if } p=q)$
- $\bullet \quad D(p,q) = D(q,p) \qquad \text{p bata q ma auna jati distance lagxa teti nae distance q bata p ma auna lagxa}$
- $D(p,z) \le D(p,q) + D(q,z)$

There are four types of distance measure

Euclidean Distance (D_e) :

The Euclidean distance between pixel p and q is defined as:

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$
 (which is the Pythagoras's theorem i.e. $c^2 = a^2 + b^2$)

City Block Distance (D_4) :

 $D_4(p,q)$ = The City Block distance between pixel p and q is defined as:

$$D_4(p,q) = |x\text{-}s| + |y\text{-}t|$$

Chessboard Distance (D_8) :

The Chessboard distance between pixel p and q is defined as:

$$D_8(p,q) = max(|x\text{-}s|,|y\text{-}t|)$$

- 2 2 2 2 2
- 2 1 1 1 2
- 2 1 0 1 2
- 2 1 1 1 2
- 2 2 2 2 2

Mix Distance (D_m) :

The mix distance between pixel p and q is depends up on the path to travel from p to q. hence, to find the distance between p and q we have to consider an vector set of pixel.

For example: if the vector set of pixel is $v = \{1, 2\}$, then to find the distance between p and q we can only make a path between pixel value 1 and 2.

Example: Suppose p (0,0) and q (4,4) are two pixel of an image, calculate the distance measure D_4 , D_8 , D_m , D_e the set V is given $V=\{2,4\}$ and $V=\{2,6\}$

2	3	2	6	1
6	2	3	6	2
5	3	2	3	5
2	4	3	2	2
4	5	2	3	6

Here, pixel p(x,y) = p(0,0) and pixel q(s,t) = q(4,4)

Euclidean Distance $D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$

$$= [(0-4)^2 + (0-4)^2]^{1/2}$$

$$= [(-4)^2 + (-4)^2]^{1/2}$$

$$= [16+16]^{1/2}$$

$$= [32]^{1/2}$$

City Block Distance $D_4(p,q) = |x-s| + |y-t|$

$$= |0-4| + |0-4|$$

$$=4+4$$

$$= 8$$

Chessboard Distance $D_8(p,q) = \max(|x-s|,|y-t|)$

$$= \max(|0-4|, |0-4|)$$

$$= \max(|-4|, |-4|)$$

$$= max(4,4)$$

=4

Mix distance (D_m) :

For
$$V = \{2,4\}$$

 D_m = there is no path between p and q because q (6) in not included in $V = \{2, 4\}$

For
$$V = \{2, 6\}$$

$$D_m = 6$$

Class work: Calculate the D4, D8 and De of pixel p(1,2) and q(4,1)

2	3	2	6	1
6	2	3	6	2
5	3	2	3	5
2	4	3	2	2
4	5	2	3	6

End of Unit-1