2. Differentiate between true and pseudo random numbers. What are the basic properties of random numbers? The sequence of numbers 0.37, 0.29, 0.19, 0.88 0.44, 0.63, 0.77, 0.70 0.21, and 0.58 has been generated. Use K-S test to determine if the numbers are uniformly distributed ($D\alpha = 0.41$ for $\alpha = 0.05$ (2 + 2 + 6)

Given sequence of number,

0.37, 0.29, 0.19, 0.88 0.44, 0.63, 0.77, 0.70 0.21, and 0.58

Arranging the given number in ascending order:

0.19, 0.21, 0.29, 0.37, 0.44, 0.58, 0.63, 0.7, 0.77, 0.88

Here, N = 10

Calculation table for Kolmogorov-Smirnov test :

i	$R_{(i)}$	$\frac{i}{N}$	$\frac{i}{N} - R_{(i)}$	$R_{(i)} - \frac{i-1}{N}$
1	0.19	0.1	-	0.19
2	0.21	0.2	-	0.11
3	0.29	0.3	0.01	0.09
4	0.37	0.4	0.03	0.07
5	0.44	0.5	0.06	0.04
6	0.58	0.6	0.02	0.08
7	0.63	0.7	0.07	0.03
8	0.7	0.8	0.1	-
9	0.77	0.9	0.13	-
10	0.88	1.0	0.12	-

Now, calculating

$$D^{+} = \max_{1 \le i \le N}\left\{\frac{i}{N} - R_{(i)}\right\} = 0.13$$

$$D^{-} = \max_{1 \le i \le N}\left\{R_{(i)} - \frac{i-1}{N}\right\} = 0.19$$

$$D = \max(D^{+}, D^{-}) = 0.19$$

Given, Critical value $D_\alpha = 0.41$

Since the computed value, D = 0.19, is less than the tabulated critical value, D_α = 0.41, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is not rejected.

2. Define and develop a Poker test for four-digit random numbers. A sequence of 10,000 random numbers, each of four digits has been generated. The analysis of the numbers reveals that in 5120 numbers all four digits are different, 4230 contain exactly one pair of like digits, 560 contain two pairs, 75 have three digits of a kind and 15 contain all like digits. Use Poker test to determine whether these numbers are independent. (Critical value of chi-square test for α=0.05 and N=4 is 9.49)

The probabilities associated with each of the possibilities is given by

P (four different digits) = $^4C_4$ * (10/10) * (9/10) * (8/10) * (7/10) = 0.504

P (one pair) = $^4C_2$ * (10/10) * (1/10) * (9/10) * (8/10) = 0.432

P (two pair) = ($^4C_2$/2)*(10/10) * (1/10) * (9/10) * (1/10) = 0.027

P (three digits of a kind) = $^4C_3$ * (10/10) * (1/10) * (1/10) * (9/10) = 0.036

P (four digits of a kind) = $^4C_4$ * (10/10) * (1/10) * (1/10) * (1/10) = 0.001

Now the calculation table for the Chi-square statistics is:

Combination(i)	**Observed Frequency(O_i)**	**Expected Frequency(E_i)**	**(O_i-E_i)**	**$(O_i-E_i)^2/E_i$**
Four different digits	5120	0.504*10000 = 5040	80	1.269
One pair	4230	0.432*10000 = 4320	-90	1.875
Two pair	560	0.027*10000 = 270	290	311.481
Three digits of a kind	75	0.036*10000 = 360	285	225.625
Four digits of a kind	15	0.001*10000 = 10	5	2.5
	10000	10000		$\Sigma(O_i-E_i)^2/E_i = 542.75$

$$x_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 542.75$$

and Given $\chi^2_{\alpha, N} = \chi^2_{0.05, 4} = 9.49$

Here the calculated value of chi-square is 542.75 which is greater than the given tabulated value of chi- square so we reject the null hypothesis of independence between given numbers.

2. Use multiplicative congruential method to generate a sequence of three digits random numbers between (0, 1) with X_0=27, a=3 and m=1000. Use any one of the uniformity test to find out whether the generated numbers are uniformly distributed or not? (Critical value for α=0.05 and N=5 is 0.565).

$X_0 = 27,$

$\alpha = 3,$

$m=1000$

We have,

For multiplicative congruential method:

$X_{i+1} = (\alpha X_i) \bmod m$

& $R_i = X_i/m,$

The sequence of random numbers are calculated as follows:

$X_0 = 27$

$R_0 = 27/1000 = 0.027$

$X_1 = (\alpha X_0) \bmod m = (3*27) \bmod 1000 = 81 \bmod 1000 = 81$

$R_1 = 81/1000 = 0.081$

$X_2 = (\alpha X_1) \bmod m = (3*81) \bmod 1000 = 243 \bmod 1000 = 243$

$R_2 = 243/1000 = 0.243$

$R_2 = 243/1000 = 0.243$

$X_3 = (\alpha X_2) \bmod m = (3*243) \bmod 1000 = 729 \bmod 1000 = 729$

$R_3 = 729/1000 = 0.729$

$X_4 = (\alpha X_3) \bmod m = (3*729) \bmod 1000 = 2187 \bmod 1000 = 187$

$R_4 = 187/1000 = 0.187$

Therefore,
The sequence of random numbers are 0.027, 0.081, 0.243, 0.729, 0.187

Now, to find out whether these random numbers are uniformly distributed or not using Kolmogorov--Smirnov test
Arranging the above random numbers number in ascending order:

0.027, 0.081, 0.187, 0.243,, 0.729

i	$R_{(i)}$	$\frac{i}{N}$	$\frac{i}{N} - R_{(i)}$	$R_{(i)} - \frac{i-1}{N}$
1	0.027	0.2	0.173	0.027
2	0.081	0.4	0.319	-
3	0.187	0.6	0.413	-
4	0.243	0.8	0.557	-
5	0.729	1	0.271	-

Now, calculating

$$D^{+} = \max_{1 \le i \le N} \left\{ \frac{i}{N} - R_{(i)} \right\} = 0.557$$

$$D^{-} = \max_{1 \le i \le N} \left\{ R_{(i)} - \frac{i-1}{N} \right\} = 0.027$$

$$D = \max(D^{+}, D^{-}) = 0.557$$

Given, Critical value $D_\alpha = 0.565$

Since the computed value, D = 0.557, is less than the tabulated critical value, D_α = 0.565, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is not rejected.

2. Use multiplicative congruential method to generate a sequence of three digits random numbers between (0, 1) with X_0=27, a=3 and m=1000. Use any one of the uniformity test to find out whether the generated numbers are uniformly distributed or not? (Critical value for α=0.05 and N=5 is 0.565).

$X_0 = 27,$

$\alpha = 3,$

m=1000

We have,

For multiplicative congruential method:

$X_{i+1} = (\alpha X_i)$ mod m

& $R_i = X_i/m$,

The sequence of random numbers are calculated as follows:

$X_0 = 27$

$R_0 = 27/1000 = 0.027$

$X_1 = (\alpha X_0)$ mod m = (3*27) mod 1000 = 81 mod 1000 = 81

$R_1 = 81/1000 = 0.081$

$X_2 = (\alpha X_1)$ mod m = (3*81) mod 1000 = 243 mod 1000 = 243

$R_2 = 243/1000 = 0.243$

$X_3 = (\alpha X_2)$ mod m = (3*243) mod 1000 = 729 mod 1000 = 729

$R_3 = 729/1000 = 0.729$

$X_4 = (\alpha X_3)$ mod m = (3*729) mod 1000 = 2187 mod 1000 = 187

$R_4 = 187/1000 = 0.187$

Therefore,
The sequence of random numbers are 0.027, 0.081, 0.243, 0.729, 0.187

Now, to find out whether these random numbers are uniformly distributed or not using Kolmogorov--Smirnov test
Arranging the above random numbers number in ascending order:

0.027, 0.081, 0.187, 0.243,, 0.729

Here, N = 5

i	$R_{(i)}$	$\frac{i}{N}$	$\frac{i}{N} - R_{(i)}$	$R_{(i)} - \frac{i-1}{N}$
1	0.027	0.2	0.173	0.027
2	0.081	0.4	0.319	-
3	0.187	0.6	0.413	-
4	0.243	0.8	0.557	-
5	0.729	1	0.271	-

Now, calculating

$$D^{+} = \max_{1 \leq i \leq N} \left\{ \frac{i}{N} - R_{(i)} \right\} = 0.557$$

$$D^{-} = \max_{1 \leq i \leq N} \left\{ R_{(i)} - \frac{i-1}{N} \right\} = 0.027$$

$$D = \max(D^{+}, D^{-}) = 0.557$$

Given, Critical value $D_{\alpha} = 0.565$

Since the computed value, D = 0.557, is less than the tabulated critical value, D_{α} = 0.565, the hypothesis of no difference between the distribution of the generated numbers and the uniform distribution is not rejected.