Department of Computer Science and Engineering
Motilal Nehru National Institute of Technology Allahabad
B.Tech III Semester CSE + IT
Foundation of Logical Thought

Note: All questions are compulsory. Assume any missing data and mention it at the top of the answer.

Time: 3 hrs

M.M 60

Ques 1 Justify whether commutative, associative, absorption and idempotent laws hold for all elements x, y, z of a lattice L. — 2.5*4=10 marks

Ques 2 — 5*2=10 marks

a) Suppose that $(S, \preccurlyeq_1)$ and $(T, \preccurlyeq_2)$ are posets. State whether $(S \times T, \preccurlyeq)$ is also a poset where $(s,t) \preccurlyeq (u,v)$ if and only if $s \preccurlyeq_1 u$ and $t \preccurlyeq_2 v$.

b) How many reflexive relations are there on a set with n elements? Also, how many symmetric relations are there on a set with n elements? Justify your answer in each case.

Ques 3 A knight on a chess board can move one space horizontally (in either direction) and two spaces vertically (in either direction) or two spaces horizontally (in either direction) and one space vertically (in either direction). Suppose that we have an infinite chess board, made up of all squares (m, n) where m and n are non-negative integers that denote the row number and the column number of the square respectively. Use Mathematical induction to show that a knight starting at $(0,0)$ can visit every square using a finite sequence of moves. — 10 marks

Ques4 — 5*2 =10 marks

a) *If $\forall y \exists x\, P(x,y)$ is true, does it necessarily follow that $\exists x \forall y\, P(x,y)$ is also true? Justify your answer.*

b) Use rules of inference to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$ and $\sim R(a)$, where a is in the domain are true, then the conclusion $\sim P(a)$ is true.

Ques 5 — 5*2=10 marks

a) Let $(H,.)$ be a subgroup of a group $(G,.)$. Let $N = \{x | x \in G,\ xHx^{-1} = H\}$. Show that $(N,.)$ is a subgroup of (G,.).

b) Let $(A,\bullet)$ be a semigroup. Furthermore, for every a,b in A, if $a \neq b$, *then* $a \bullet b \neq b \bullet a$ i.e. if $a \bullet b = b \bullet a$, then $b = a$.

1. Show that for every a in A, $a \bullet a = a$
2. Show that for every a,b,c in A, $a \bullet b \bullet c = a \bullet c$

Ques 6 — 5*2=10 marks

a) Write a pseudo code to find the reverse complementary of DNA sequence. DNA primarily consists of 4 hydrocarbons: Cytosine [C], Guanine [G], Adenine [A], Thymine [T]. DNA bases pair up with each other, A with T and C with G to form units called base pairs. The complement value of each are {A: T, T: A, G: C, C: G}, i.e. A replaced by T and vice versa, C replaced by G and vice versa.

b) Kushagra was given a task to color a tree in such a way that no two adjacent nodes have the same color. But Kushagra is not artistic and decides to color them with numbers. He decides to make it a challenge and find the minimum numbers he can color the graph with, such that the sum is minimum. Given a tree, help Kushagra determine the minimum color sum that can be achieved when the tree is properly colored.