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(End Semester Examination 2016-2017)

Subject: Analysis of Algorithms(CS1401) Duration: 3 HRS Max Marks: 60

Note: Be specific and to-the-point in your answers. Make assumptions wherever necessary and quote it. All questions are compulsory. Neat answers, without cuttings will be appreciated. It is advisable to design the solution in rough before writing the final answers. Answer the questions serially.

Q1. [15 marks] A student, studying 3 subjects (Automata, Algorithms & Networks), has to participate in a quiz. S/He has left with *4* hours only for preparation. We are given a certain probability *p*, of failing in a particular subject if S/He studies that subject for a particular number of hours.

Hours for study	Probability (p) that S/He fails		
	Automata	Algorithms	Networks
0	0.80	0.75	[illegible]
1	0.70	0.70	0.70
2	0.65	0.67	0.60
3	0.62	0.65	0.55
4	0.60	0.62	0.50

Example: If S/He devotes *1* hour for Automata, *1* hour for Algorithm & *2* hours for Network then his failure probability will be *.294 (= .7 * .7 * .6).*

Finding a preparation strategy ({subject *s*, number of hours to devote for subject *s*}) is a dynamic programming problem. Give an algorithm with its complexity analysis so as to minimize the failure probability by finding the optimal preparation strategy.

Q2. [15 marks] Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that *1* U.S. dollar buys *64.27* Indian Rupees, *1* Indian Rupee buys *1.73* Japanese Yen, *1* Japanese Yen buys *1.37* Sri Lankan Rupees and *1* Sri Lankan Rupee buys *0.01* U.S. Dollars. Then, by converting currencies, a trader can start with *1* U.S. dollar and buy $64.27\times1.73\times1.37\times0.01\approx1.52$ U.S. dollars, thus turning a profit of *52%*.

Suppose that we are given n currencies $c_1, c_2, \ldots, c_n$ and an $n\times n$ table of R exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j. Give an efficient algorithm to determine whether there exists a sequence of currencies $\langle c_{i1}, c_{i2}, \ldots, c_{ik}\rangle$ such that $R[i_1; i_2] \times R[i_2; i_3] \ldots R[i_{k-1}; i_k] \times R[i_k; i_1] > 1$. Also analyze the running time of your algorithm.

Q3. [6 marks] In context of Towers-of-Hanoi algorithm answer the following questions:

a. Let $f(n)$ be the number of single-disc moves this algorithm makes to solve the n-disc problem. Write a recurrence for $f(n)$.
b. Derive the complexity notation for $f(n)$.

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Towers-of-Hanoi (A; B; n; C)
    if (n == 1)
        MOVE (A;B)
    else
        Towers-of-Hanoi (A; C; n-1; B)
        MOVE (A;B)
        Towers-of-Hanoi(C; B; n -1;A)
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Q4. [6 marks] Answer the following questions while considering the recurrence $T(n)$, $T(n) = \begin{cases} 0 & if\ n = 1 \\ 2T\left(\frac{n}{2}\right) + \log_2 n & if\ n > 1 \end{cases}$ where n is a power of 2.

a. Use master method and find a working solution.
b. Prove by induction that the exact solution is of the form: $T(n) = An + B\log_2 n + C$ and thus find the constants A, B and C.

Q5. [6 marks] How many spurious hits does the *Rabin-Karp* matcher encounter in the text $T = 3141592653589793$ while searching for pattern $P = 26$ with working modulo $q = 11$?

Q6. [6 marks] Prove that the complexity of *BUILD-MAX-HEAP* over an array of size n is $O(n)$.

Q7. [6 marks] Explain asymptotic notations and their utility in algorithm analysis.