Motilal Nehru National Institute of Technology Allahabad
Department of Computer Science and Engineering
End-Sem Examination 2016-17
B.Tech 3rd semester(CS/IT), Course Code: CS1304
Course Name: Foundations of Logical Thought

Time: 3 Hour **MM: 60.**

Note: Attempt all questions and justify your answers with the proper reason.

1) Let E, F and G be finite sets. Let $X=(E\cap F)-(F\cap G)$ and $Y=(E-(E\cap G))-(E-F)$ Then show X=Y. (Using Venn diagram and also Boolean algebra). [2+2=4]

2) Solve the following using Quantifiers:[1+2=3]
   a) Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. Then state the statement for the following formula $\forall x\exists y\exists t(\neg F(x,y,t))$
   b) Convert the following sentences into quantifiers
      I. The only even prime is 2
      II. There is one and only even prime

3) Define the injective, serjective, and bijective function. Let f: A→B be an injective (one to one) function. Define g:$2^A\rightarrow 2^B$ as: g(C)={f(x)x∈C}, for all subsets C of A. Define h: $2^B\rightarrow 2^A$ as : h(D)={x|x∈A,f(x) ∈D}, for all subset D of B. Then justify $g(h(D))\subseteq D$.[3+2=5]

4) Define reflexive closure. The binary relation: S={(x,y) | y=x+1 and x,y∈{0,1,2,....}} Then prove reflexive transitive closure of S is {x,y}| y≥x and x,y∈{0,1,2,....}}.[1+4=5]

5) Suppose A is a finite set of n elements. [2+2+2+2=8].
   a) How many elements are there in the largest equivalence relation on A
   b) What is the rank of the largest equivalence relation on A
   c) How many elements are there in the smallest equivalence relation on A
   d) What is the rank of the smallest equivalence relation on A

6) What is partial order? Determine whether *R* is a partial order relation on *A*.[1+2+2=5]
   a) Let *A* = *{(x, y) | x, y* integers*}*. The rule to define a relation R on A is given by: $(a, b)R(c, d) \leftrightarrow a\le c$ or $b\le d$
   b) Let *A* = *{(x, y) | x, y* integers*}*. The rule to define a relation R on A is given by: $(a, b)R(c, d) \leftrightarrow a=c$ or $b=d$

(E ∩ F) – (E ∩ F ∩ G)

P.T.O

7) Using the given Fig 1, Hasse diagram of a partially ordered set, find the following:[1+1+2+2=6]

a) Greatest and least element

b) Maximal and minimal elements

c) Lower bound & Greatest lower bound of {a, e, g}, {b, c, f}

d) Upper bound & Least upper bound of {h, i. g}, {f, i}

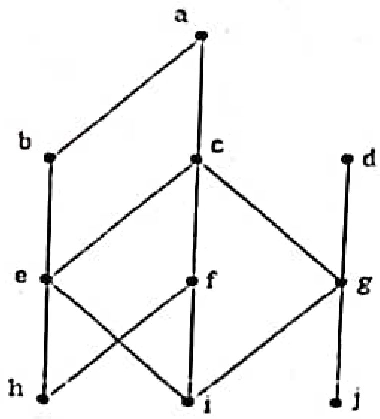

Fig1

8) Define Ring, Integral domain, fields, with help of suitable examples.
And also show that (A,*) is a group if set A= $\{0^0, 60^0, 120^0, 180^0, 240^0, 300^0\}$ where * is represent the rotation with angle of member set A.[6+2=8]

9) Let (A,*) be a semigroup, furthermore, for every a, &b in A, if a $\neq$ b, the $a*b \neq b*a$. [1+2+2=5]

a) Show that for every a in A is $a*a=a$

b) Show that for every a,b in A is $a*b*a=a$

c) Show that for every a, b, c in A is $a*b*c=a*c$

10) Let R be binary relation on set A and suppose $R^s=R^t$ for some s &t with s<t. Let p=t-s. then prove the following:[1+1+2=4]

a) $R^{s+k}=R^{t+k}$ for all $k \geq 0$

b) $R^{s+kp+i}=R^{s+i}$ for all k, i>0

c) Let $S=\{R^0, R^1......R^{t-1}\}$. Then every power of R is an element of S, such that $R^q \in S$ for all $q \in N$

11) Prove the given identity [2+2=4]

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2,$$

Also find the coefficient of the term x^{23} in $(1+x^5+x^9)^{100}$.

12) The composition table of a cyclic group shown below. Find the generators [3]

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$x^{20}[1+x^9]$

—x—x— End —x—x—