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Department of Computer Science and Engineering
Mid-Sem Examination 2016-17
B.Tech 4th semester(CS/IT), Course Code: CS1402
Course Name: Graph Theory and Combinatorics

Time: 1.5 Hour **MM: 20.**

Note: Attempt all questions and justify your answers with the proper reason.

1) Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i,j): 1 \le i \le 12, 1 \le j \le 12\}$. There is an edge between (a, b) and (c, d) if $|a - c| \le 1$ and $|b - d| \le 1$. What is the number of edges in this graph? [3]

2) Explain under which condition a complete bipartite graph is (i) regular graph (ii) complete graph. Also answers the following: [1+1+1+1=4]
   (iii) How many complete bipartite graphs have k vertices?
   (iv) What is the maximum number of edges in a simple bipartite graph with k vertices?

3) Every Arbitrarily Traceable graphs is Euler graph but every Euler graphs is not Arbitrarily Traceable graphs discuss it with examples. [2]

4) The complement G' of a graph G has the same vertex set as G, but xy is an edge in G' if and only if xy is not an edge in G. Show: [2+1=3]
   a) A graph G is called self-complementary if G and G' are isomorphic. Show: if a graph G on n vertices is self-complementary, then either n or $n - 1$ is divisible by 4.
   b) A cycle on n vertices is isomorphic to its complement. What is value of n? Also draw the diagram for it.

5) The graph Fig(i) shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: {(A, C), (B, C), (B, E), (E, F), (D, F)}. The edge weights of only those edges which are in the MST are given in the figure shown below. What is the Minimum possible sum of weights of all 8 edges of this graph? [3]

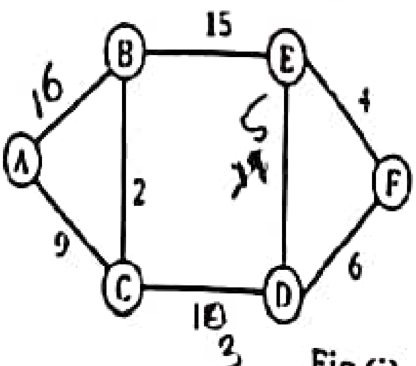
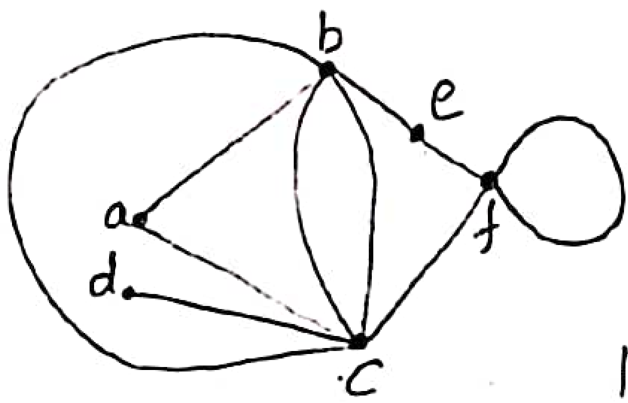


Fig (i)

P.T.O

6) Show $K_{3,3}$ is non-planner and find its crossings. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then find the number of edges in G. [1+1+1=3]

7) Draw Geometric Dual of graph 'G' i.e G*. Fig(ii)[1+1=2]
Also show relationship between:
e, n, f, r, μ of G to (e^*, n^*, f^*, r^*, μ^*) of G*
Where, [e= edges, n= no. of vertices, r=rank, f=region, μ=nullity]



Fig(ii)

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