Motilal Nehru National Institute of Technology Allahabad
Department of Computer Science and Engineering
End-Sem Examination 2016-17
Programme- B.Tech 4^{th} semester(CS-IT), Course Code: CS1402
Course Name: Graph Theory and Combinatorics

Time: 3 Hour MM:60.

Note: Attempt all questions

J. Anushka prasad

1) Solve following questions
   a) Prove that simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges.
   b) The 2^n vertices of graph G correspond to all subsets of a set of size n, for n≥6. Two vertices of G are adjacent if and only if the corresponding subsets intersect in exactly two elements. What is the number of connected components in G? [3+3=6]

2) What is the difference between matching and covering of a graph? Why dimer covering is often referred to as a perfect matching? How many perfect matchings are there in a complete graph of six vertices? [2+2+2=6]

3) Discuss the relationship between fundamental circuits and cut-sets. "With respect to a given spanning tree T, a chord c_i that determines a fundamental circuit Γ occurs in every fundamental cut-set associated with the branches in Γ and in no other", Explain it with suitable example .[2+2=4]

4) Define branch and chord in Graph G? If we have a farm consisting of six walled plots of land as shown in Fig(i) and these plots are full of water, how many walls have to be broken so that all the water can be drain out? Also make diagram after breaking the walls. [2+2=4]

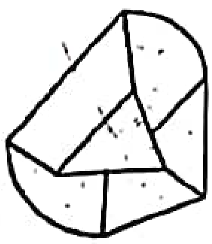
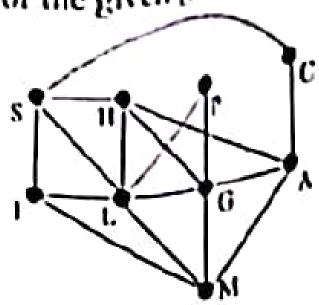


Fig (i)

5) What is Hamiltonian circuit? Twenty members of a club meet each day for lunch at round table. They decide to sit such that every member has different neighbors at each lunch. How many days can this arrangement last? [1+3=4]

6) Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then what is the number of distinct cycles of length 4 in G? [4]

7) For the given graph fig(ii), find out



•Fig (ii)

a) Chromatic no and chromatic partitioning
b) Maximal independent set and coefficient of internal stability
c) Minimal dominating set and dominating number [2+2+2=6]

8) Derive the Chromatic polynomial for the given graph Fig (iii) (disconnected).

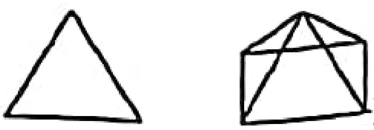


Fig (iii) [4]

9) Find out the complexity through recurrence relation by substitution method.

a) $T(n) = \begin{cases} T(\sqrt{n}) + 1, & n > 2 \\ a, & n = 2 \end{cases}$

b) $T(n) = \begin{cases} 4T(n/2) + n^2, & n > 1 \\ a, & n = 1 \end{cases}$

[3+3=6]

10) State the Pigeon-hole principle. Solve the following problem:
A bag contains 100 apples, 100 bananas, 100 oranges, and 100 pears. If I pick one piece of fruit out of the bag every minute, how long will it be before I am assured of having picked at least a dozen pieces of fruit of the same kind? [1+3=4]

11) Imagine a prison consisting of 64 cells arranged like the squares of an 8-by-8 chessboard. There are doors between all adjoining cells. A prisoner in one of the corner cells is told that he will be released, provided he can get into the diagonally opposite corner cell after passing through every other cell exactly once. Can the prisoner obtain his freedom?Explain. [4]

12) Solve following permutation and combinations.

a) How many integers between 0 and 10,000 have only one digit equal to 5.

b) A football team of 11 players is to be selected from a set of players, 5 of whom can play only in the backfield, 8 of them can only play on the line, 2 of them can play either in the backfield, or on the line. Assuming a football team has 7 men on the line and 4 men in the backfield determine the number of football teams possible. [4+4=8]

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