Motilal Nehru National Institute of Technology, Allahabad-211004
B. Tech. III Semester
End Semester Examination, 2016-2017
Subject: Mathematics-III

Subject Code: MA1302

Time: 3hours

Maximum Marks: 60

Note: Attempt any six questions. Each question carries equal marks.

Q.1 (a) Solve the following differential equation in series

$$(1-x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0,$$ where n is a real number.

(b) Prove that the Legendre's polynomial $P_n(x)$ satisfy the following relation

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \begin{cases} 0, & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$

Q.2 (a) Prove that the Legendre's polynomial $P_n(x)$ can also be expressed in the form

$$P_n(x) = \frac{1}{2^n\, n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

(b) Prove that the eigen functions of Sturm-Liouville problem are orthogonal.

Q.3 (a) Prove that the function $u(x, y) = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function $f(z) = u(x, y) + i\,v(x, y)$.

(b) Evaluate $\displaystyle\int_{0}^{\infty} \frac{1 - \cos x}{x^2}\, dx$, using the calculus of residue.

Q.4 (a) State and prove Cauchy's integral formula.

(b) Evaluate $\displaystyle\int_{0}^{\infty} \frac{\cos mx}{x^2 + a^2}\, dx$, where $m \geq 0$, $a > 0$, using the calculus of residue.

P.T.O.

Q5. (a) (i) Under the transformation $w = \dfrac{1}{z}$, find the image of $y - x + 1 = 0$.

(ii) Find the bilinear transformation which mapsthe points $z = 0, -1, i$ onto $z = i, 0, \infty$.

(b) Define moment generating function (m.g.f.). Find the m.g.f. of the random variable x having the probability function given by

$$f(x) = \begin{cases} x, & \text{when } 0 \le x_i < 1 \\ 2-x, & \text{when } 1 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Also, find the mean and variance of x.

Q6. (a) The probability density function for continuous random variable x is given by

$$f(x) = \begin{cases} (1/2)\sin x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and variance.

(b) A die is tossed twice. A 'success' is getting an odd number on each toss. Find the probability distribution of the number of success

Q7. (a) Show that the mean, median and mode coincide in the case of the Normal Distribution. Also, find the point of inflexion of the Normal Distribution.

(b) Find the mean, variance and standard deviation of the Binomial Distribution. Determine the Binomial Distribution whose mean is 9 and whose standard deviation is 3/2.