End Semester Examination (Session 2018-19)

B. Tech-3rd Semester (EC)

Subject: Numerical methods and Statistical Techniques (MA-1303)

Time: 3 hrs. **Maximum Marks: 60**

Note: Attempt all questions. Each question carries equal marks.

1. (a) Given that

$$u = \frac{5xy^2}{z^3}$$

Find the relative error at $x = y = z = 1$ when the errors in each of x, y, z is 0.001.

(b) Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$ correct to five decimal places.

2. (a) Using Gauss backward interpolation formula, find the population for the year 1936 given that

Year	:	1901	1911	1921	1931	1941	1951
Population (in thousand)	:	12	15	20	27	39	52

(b) (i) Prove that

$$\Delta^3_{bcd}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$$ where Δ is the divided difference operator.

(ii) Show that the n^{th} divided differences

$$[x_0, x_1, \ldots, x_n] \text{ for } u_x = \frac{1}{x} \text{ is } \left[\frac{(-1)^n}{x_0 x_1 \ldots x_n}\right].$$

3. (a) Fit a second degree parabola $y = a_0 + a_1 x + a_2 x^2$ to the data (x_i, y_i):

(1, 0.63), (3, 2.05), (4, 4.08), (6, 10.78).

(b) Obtain the cubic spline in each sub interval for the following data:

x	:	0	1	2	3
y	:	2	-6	-8	2.

with the end conditions $M_0 = M_3 = 0$.

4. (a) Using Bessel's interpolation formula, prove that

$$\frac{d}{dx}(y_x) = \Delta y_{x-1/2} - \frac{1}{24}\Delta^3 y_{x-1/2} + \ldots$$

(b) Evaluate $\int_0^1 \frac{dx}{1+x}$ by dividing the interval of integration into 8 equal parts. Hence find $\log_e 2$ approximately using Simpson's one-third rule.

P.T.O

5. (a) Perform three iterations of Gauss-Seidel method for solving the systems of equations (5)

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 11 \end{bmatrix}$$

$x = 0.9916$
$y = -1.0067$
$z = 1.0069$

Take the components of the approximate initial vector as $x_i^{(0)} = b_i / a_{ii}, i = 1,2,3.$ Compare with the exact solution $X = [1, \ -1, \ 1]^T$ and find the maximum absolute error.

(b) Solve the equations

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

By the method of LU decomposition. (5)

$x = \frac{35}{18} = 1.944$
$y = \frac{22}{18} = 1.611$
$z = \frac{5}{18} = 0.2778$

6. (a) Obtain a regression plane by using multiple linear regression to fit the data given below: (5)

$y = 10 + 2x + 4z$

x	1	2	3	4
z	0	1	2	3
y	12	18	24	30

(b) If the average fraction defective of a large sample of a product is 0.1537. Calculate the control limits given that sub-group size is 2000. (5)

C.L. = 307.4 UCL = 353.70