



# Artificial & Computational Intelligence

**AIMLCLZG557**

## **M2 : Problem Solving Agent using Search**

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**BITS** Pilani  
Pilani Campus

# Course Plan

- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI

## Module 2 : Problem Solving Agent using Search

- A. Uninformed Search
- B. Informed Search
- C. Heuristic Functions
- D. Local Search Algorithms & Optimization Problems

## Learning Objective

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At the end of this class , students Should be able to:

1. Differentiate between uninformed and informed search requirements
2. Apply UCS, GBFS & A\* algorithms to the given problem
3. Prove if the given heuristics are admissible and consistent
4. Design and compare heuristics apt for given problem
5. Apply A\* variations algorithms to the given problem

A\* Search  $h(n) + g(n)$   
 ~~$h(n) + g(n)$~~   
~~good~~

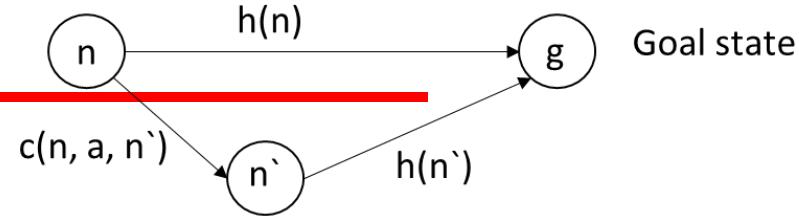
### Optimal on condition

$h(n)$  must satisfies two conditions:

- Admissible Heuristic – one that never overestimates the cost to reach the goal
- Consistency – A heuristic is consistent if for every node  $n$  and every successor node  $n'$  of  $n$  generated by action  $a$ ,  $h(n) \leq c(n, a, n') + h(n')$

$$h(n) = c(n, a, n') + h(n')$$

(1)                          (2)  
 path cost  
 $g(n, n')$



### Complete

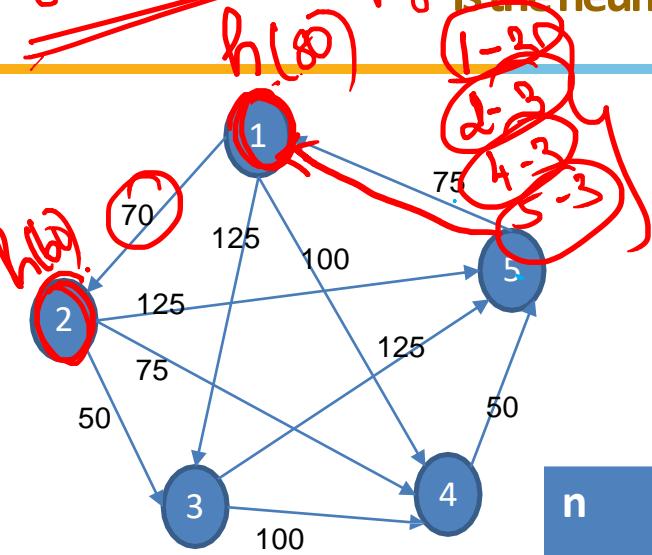
- If the number of nodes with cost  $\leq C^*$  is finite
- If the branching factor is finite
- A\* expands no nodes with  $f(n) > C^*$ , known as pruning

Time Complexity -  $G(b^\Delta)$  where the absolute error  $\Delta = h^* - h$

**A\* Search**

$80 - 60 = 20$ , "less than path cost"  $\rightarrow$  acceptable  
"higher"  $\rightarrow$  Probabilistic

Is the heuristic designed leads to optimal solution?



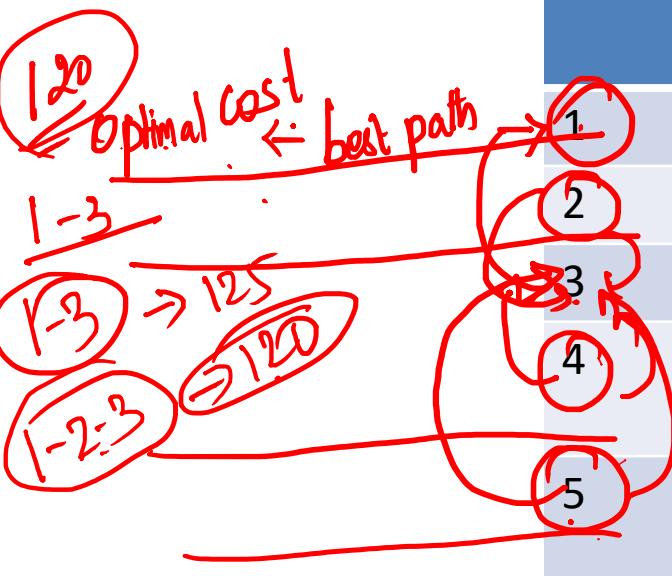
Assuming node 3 as goal, taking only sample edges per node below is checked for consistency

goal oriented

$$h(5) \leq g(5,1) + h(1)$$

$$190 \leq 75 + 80$$

n	h(n)	Is Admissible? $h(n) \leq h^*(n)$	Is Consistent? For every arc $(i,j)$ : $h(i) \leq g(i,j) + h(j)$
1	80	Y	$80 \leq 120$
2	60	N	$60 \leq 50$
3	0	Y	
4	200	Y	
5	190	Y	N (5 → 1): $190 \leq 155$

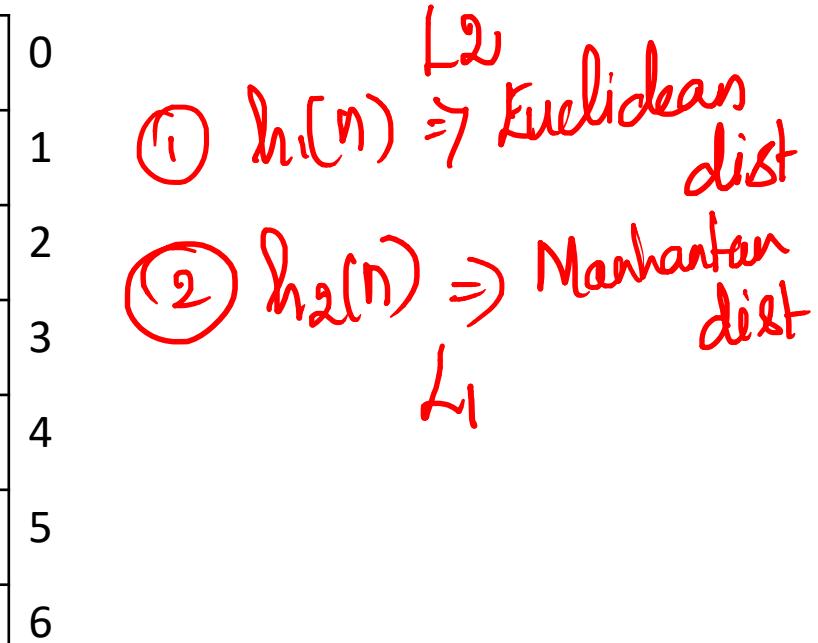


# Path finding Robot – Sample Planning Agent Design

## Successor Function Design

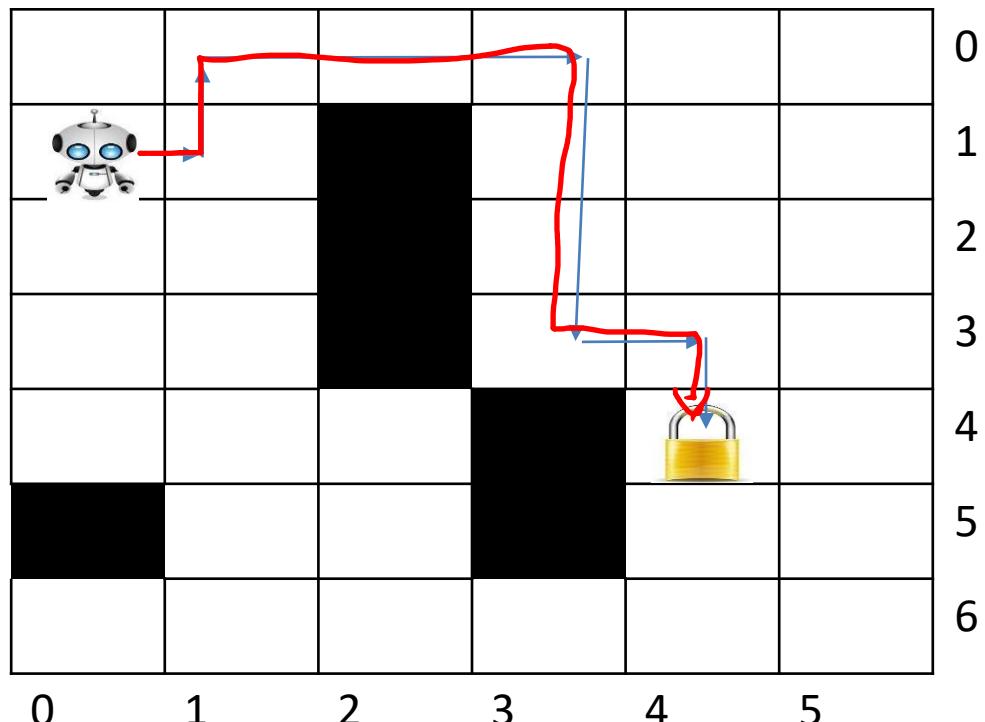
1	2	3	4	5	6
8					
13	14				
19	20				
25	26	27			30
				35	36
37	38	39	40	41	42
0	1	2	3	4	5

N-W-E-S



A\*

Demo



- ①  $A^*$   $h(n) \rightarrow$  good
  - ↓ optimal soln
- ②  $\underline{A^*}$  variation of  $\overline{BFS}$

## Variations of $A^*$

Memory Bounded Heuristics

- ① IDA<sup>K</sup>
- ② R<sub>v</sub>BFS

# Iterative Deepening A\*

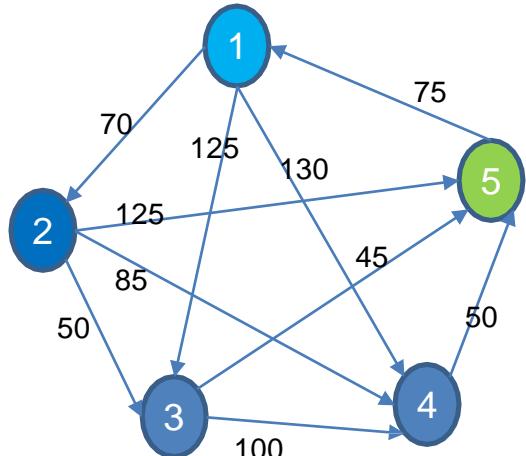
BFS + DFS  $\Rightarrow$  IDA\* ① Limit = 0

innovate

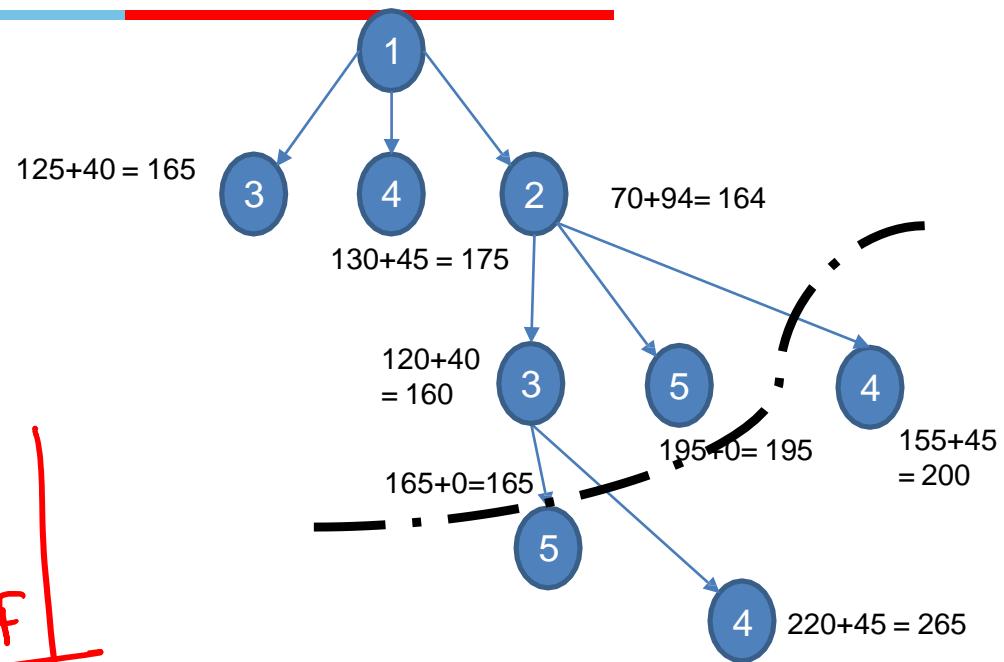
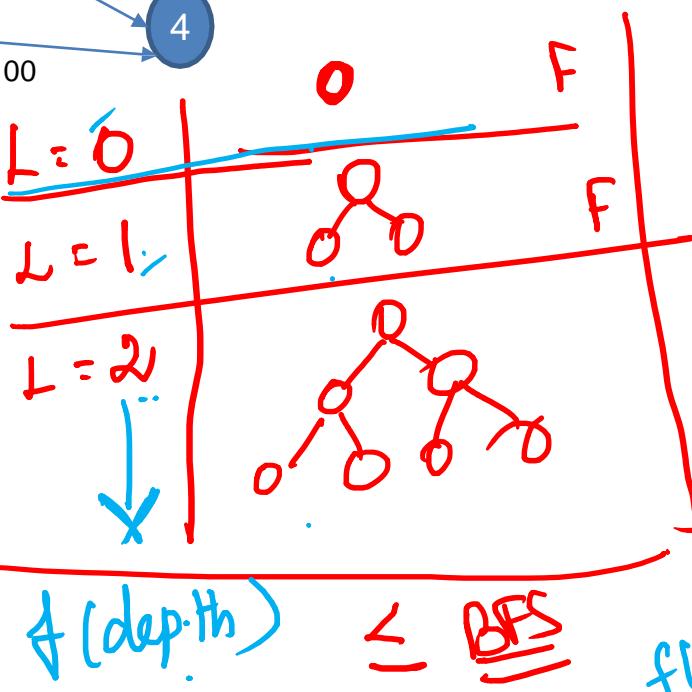
achieve

lead

Set limit for  $f(n)$



n	$h(n)$
1	60
2	94
3	40
4	45
5	0



Cut off value is the smallest of f-cost of any node that exceeds the cutoff on previous iterations

Iterative Limit : Eg

$$\begin{aligned}f(n) &= 180 \\f(n) &= 195 \\f(n) &= 200\end{aligned}$$

eval  $f_n$   
 $f(f(n))$   
 $g(n) + h(n)$

$f(n) \leftarrow A^* + DFS$

$f(n) \leftarrow IDA^* \Rightarrow \text{Limit} \Rightarrow B$

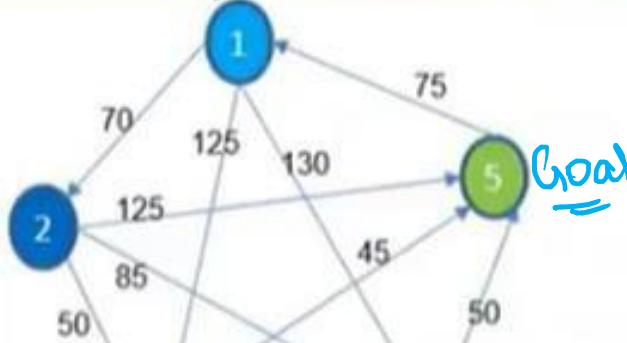
DFS A\*

## Iterative Deepening A\*

$$\text{limit} \rightarrow f(f(n) \rightarrow g(n) + h(n))$$



Start



No Priority Queue

bcoz

DFS

→ last node located in top of the stack.

Set limit for  $f(n)$

$$f(n) = g(n) + h(n)$$

$$f(n) = 0 + 60 \Rightarrow 60$$

$$125 + 43 = 168$$

$$=$$

$$130 + 45 = 175$$

$$=$$

$$70 + 92 = 162$$

$$=$$

$$120 + 43 = 163$$

$$=$$

$$165 + 0 = 165$$

$$=$$

$$195 + 0 = 195$$

$$=$$

$$155 + 45 = 200$$

$$=$$

$$1-2 \Rightarrow 162 \leq 60 \text{ F}$$

$$168 \leq 60 \text{ F}$$

$$175 \leq 60 \text{ F}$$

$$162 \leq 162 \text{ True}$$

$$163 \leq 162 \text{ Fah}$$

$$200 \leq " \text{ F}$$

$$195 \leq 162 \text{ F}$$

$$168 \leq 162 \text{ F}$$

$$175 \leq 162 \text{ F}$$

Given

n	$h(n)$
1	60
2	92
3	43
4	45
5	0

$$B=60$$

$$(1: 60)$$

TEST-F

$$(1 2: 162) (1 3: 168) (1 4: 175)$$

$$B=162$$

$$(1: 60)$$

TEST-F

$$(1 2: 162) (1 3: 168) (1 4: 175)$$

TEST-F

$$(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)$$

$$B=163$$

$$(1: 60)$$

TEST-F

$$(1 2: 162) (1 3: 168) (1 4: 175)$$

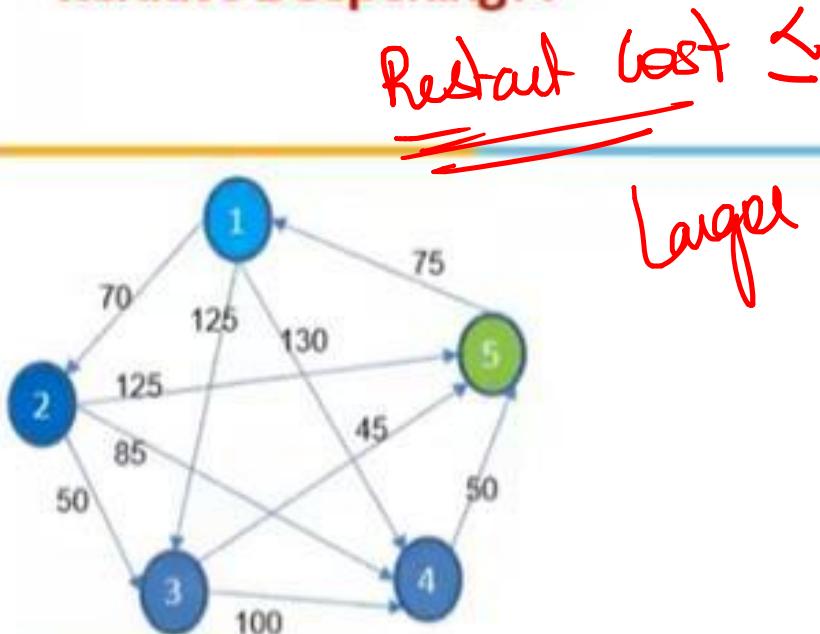
TEST-F

$$(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)$$

TEST-F

## Iterative Deepening A\*

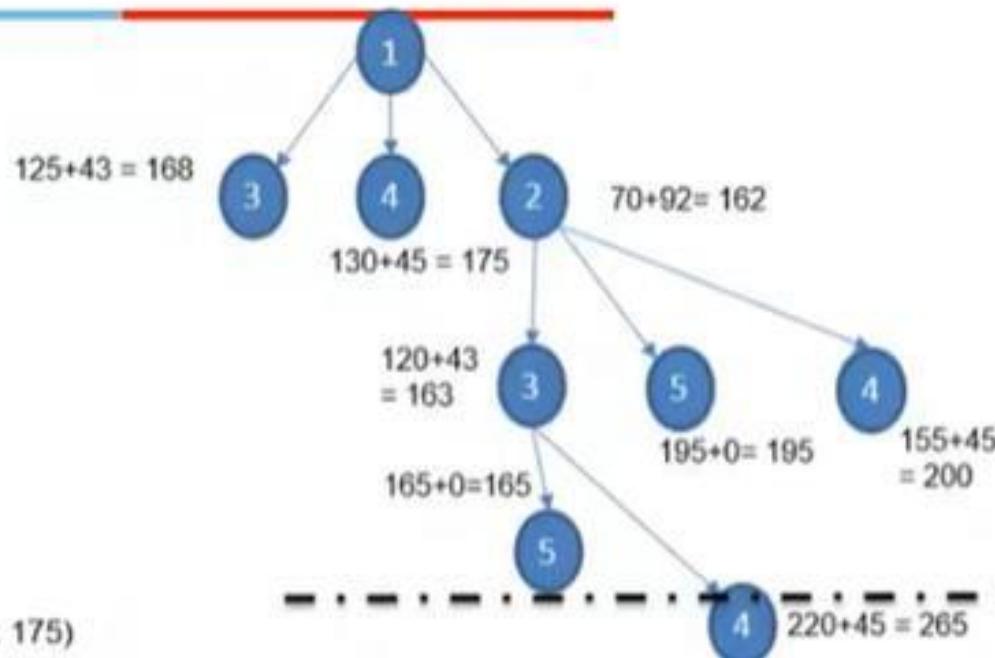
A\*  
BFS



Restart cost  $\leq$

Lager

Set limit for  $f(n)$



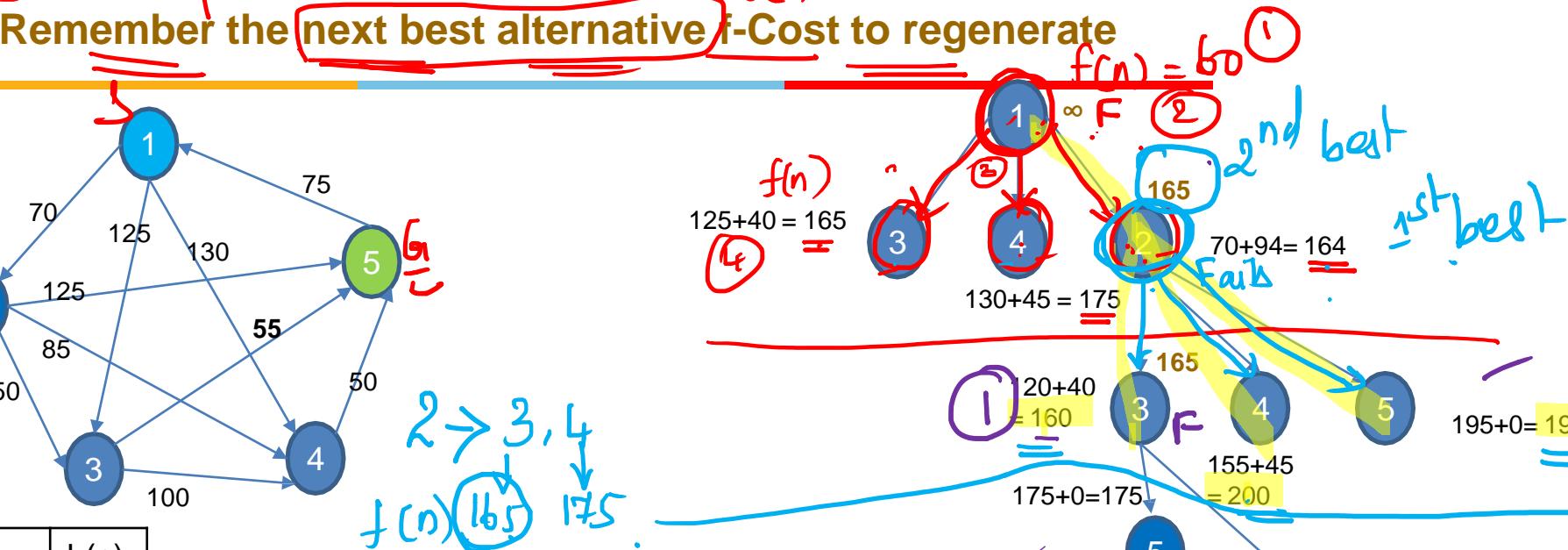
n	$h(n)$	B=163
1	60	(1: 60) TEST-F (1 2: 162) (1 3: 168) (1 4: 175)
2	92	TEST-F (1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175) (1 2 3 5: 165) (1 2 3 4: 265) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
3	43	B=165 (1: 60) TEST-F (1 2: 162) (1 3: 168) (1 4: 175)
4	45	TEST-F (1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
5	0	TEST-F (1 2 3 5: 165) (1 2 3 4: 265) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)

Pawned  
(1 2 3 5: 165)

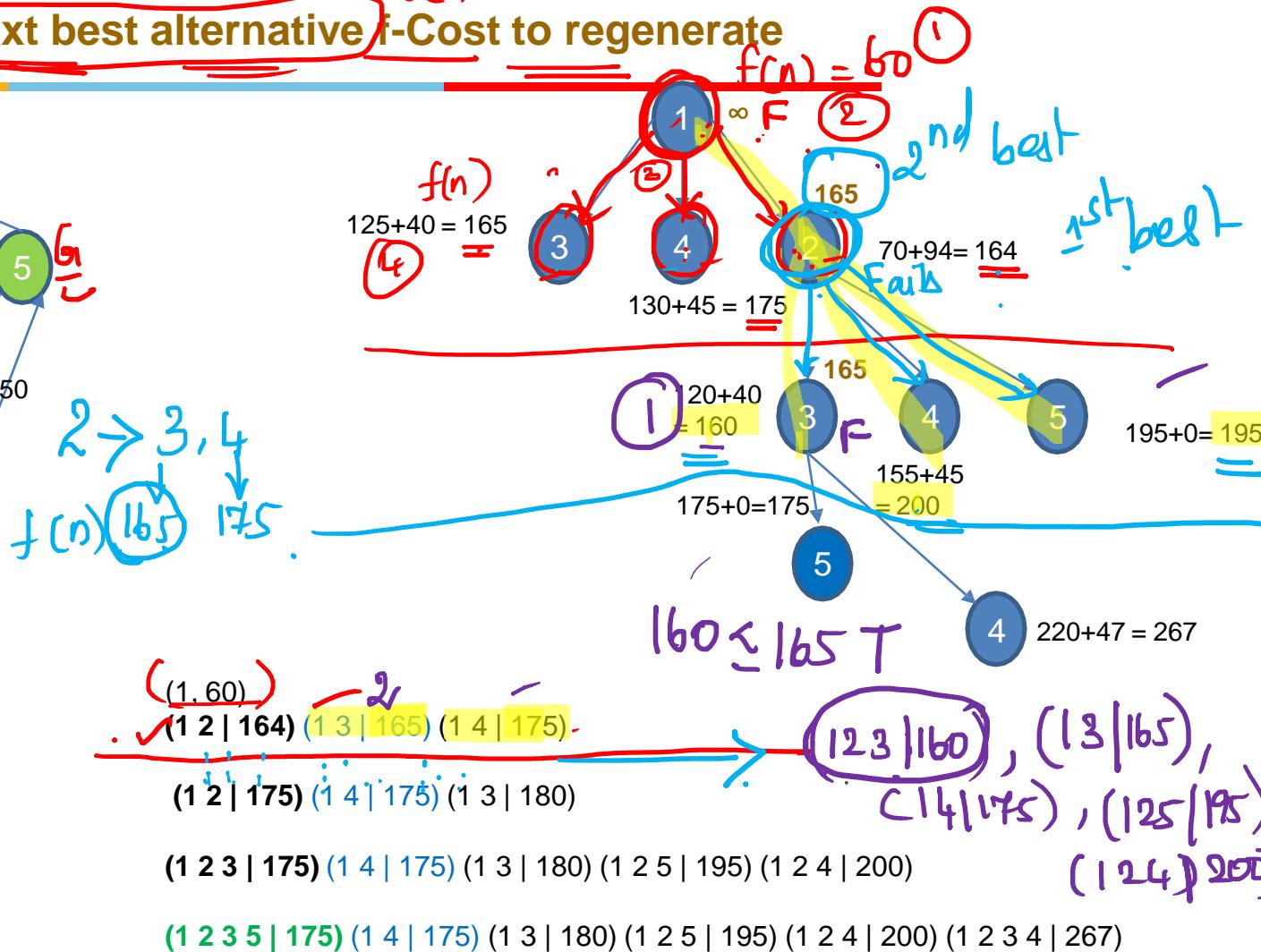
# Recursive Best First Search A\*

① Priority Queue ✓

Remember the next best alternative f-Cost to regenerate



n	$h(n)$
1	60
2	94
3	40
4	45
5	0

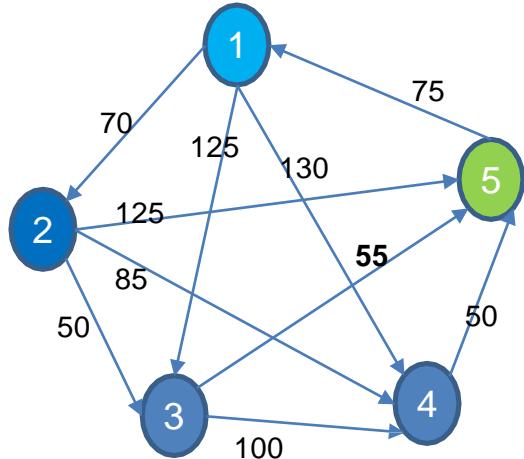


PASS

# Recursive Best First Search A\*



Remember the next best alternative f-Cost to regenerate



n	$h(n)$
1	60
2	94
3	40
4	45
5	0

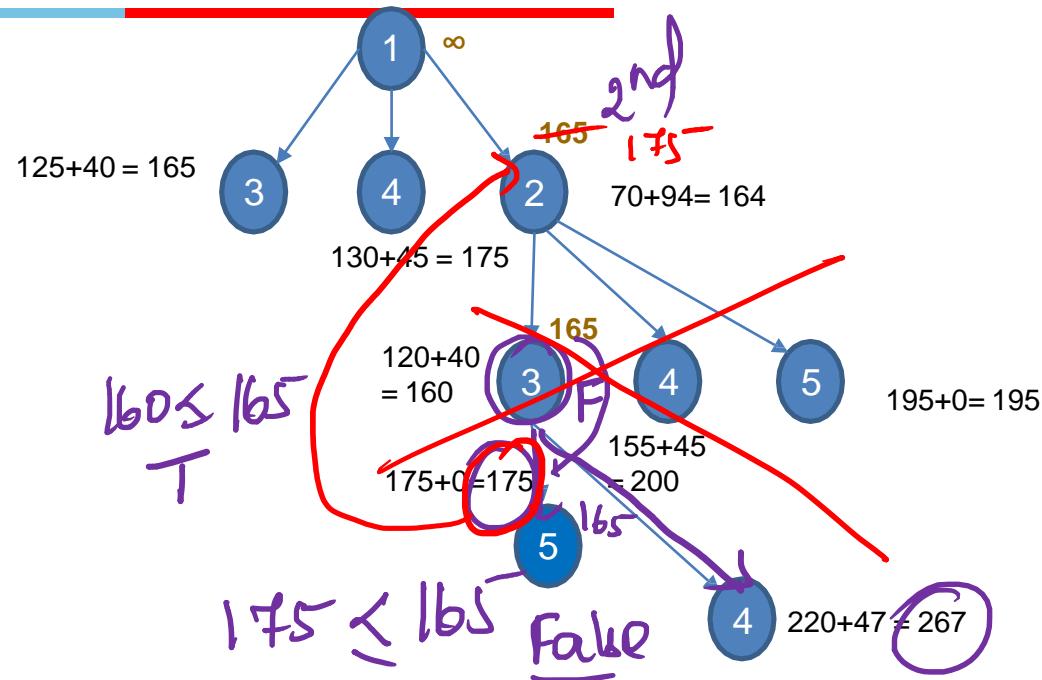
$$(1, 60) \\ (\underline{1} \underline{2} | 164) (\underline{1} \underline{3} | 165) (\underline{1} \underline{4} | 175)$$

$$(\underline{1} \underline{2} | 175) (\underline{1} \underline{4} | 175) (\underline{1} \underline{3} | 180)$$

$$(\underline{1} \underline{2} \underline{3} | 175) (\underline{1} \underline{4} | 175) (\underline{1} \underline{3} | 180) (\underline{1} \underline{2} \underline{5} | 195) (\underline{1} \underline{2} \underline{4} | 200)$$

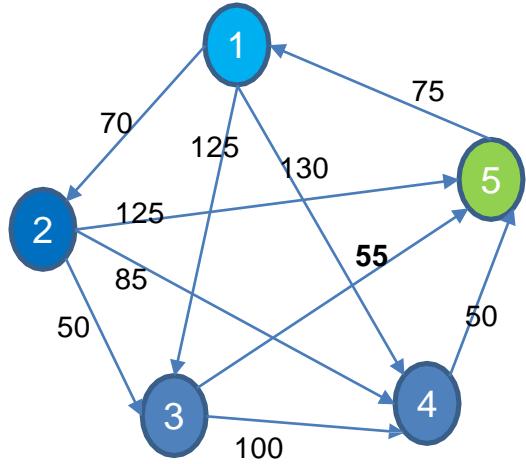
$$(\underline{1} \underline{2} \underline{3} \underline{5} | 175) (\underline{1} \underline{4} | 175) (\underline{1} \underline{3} | 180) (\underline{1} \underline{2} \underline{5} | 195) (\underline{1} \underline{2} \underline{4} | 200) (\underline{1} \underline{2} \underline{3} \underline{4} | 267)$$

PASS

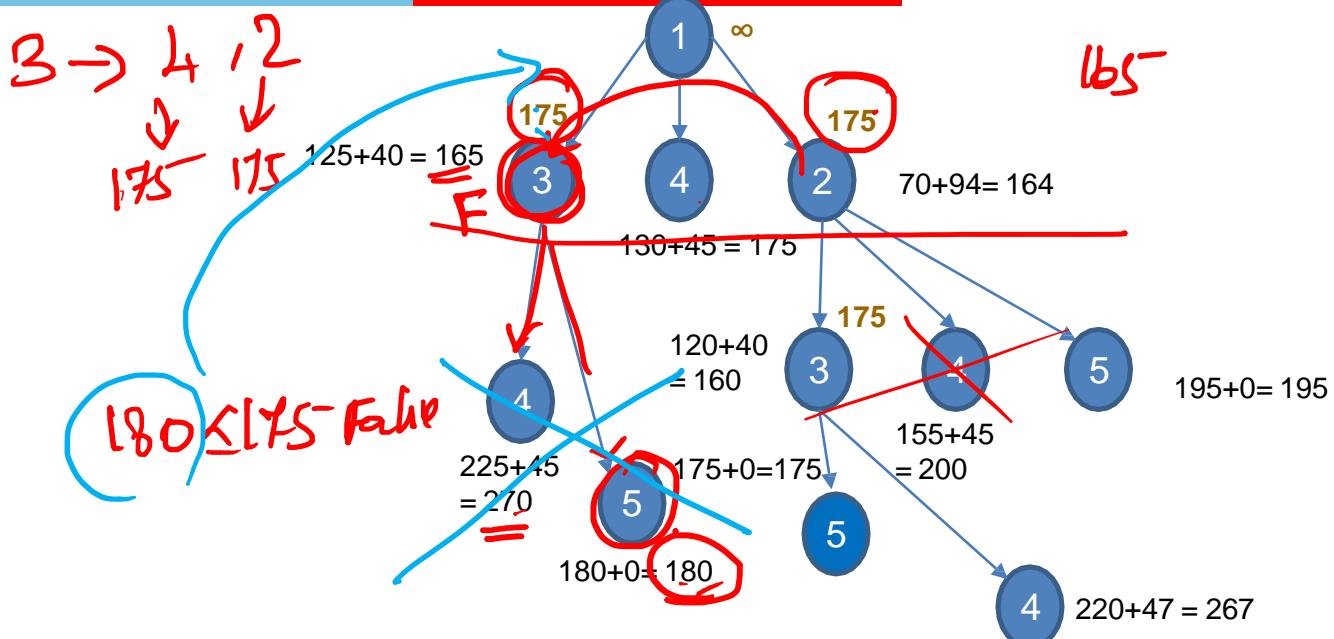


# Recursive Best First Search A\*

Remember the next best alternative f-Cost to regenerate



n	$h(n)$
1	60
2	94
3	40
4	45
5	0



(1, 60)

(1 2 | 164) (1 3 | 165) (1 4 | 175)

(1 2 | 175) (1 4 | 175) (1 3 | 180)

(1 2 3 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200)

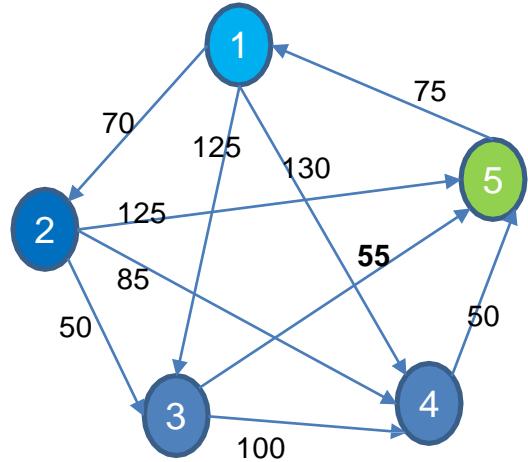
(1 2 3 5 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200) (1 2 3 4 | 267)

PASS

# Recursive Best First Search A\*

Remember the next best alternative f-Cost to regenerate

175  
 ① Recently updated  
 ② Unexplored



n	$h(n)$
1	60
2	94
3	40
4	45
5	0

(1, 60)

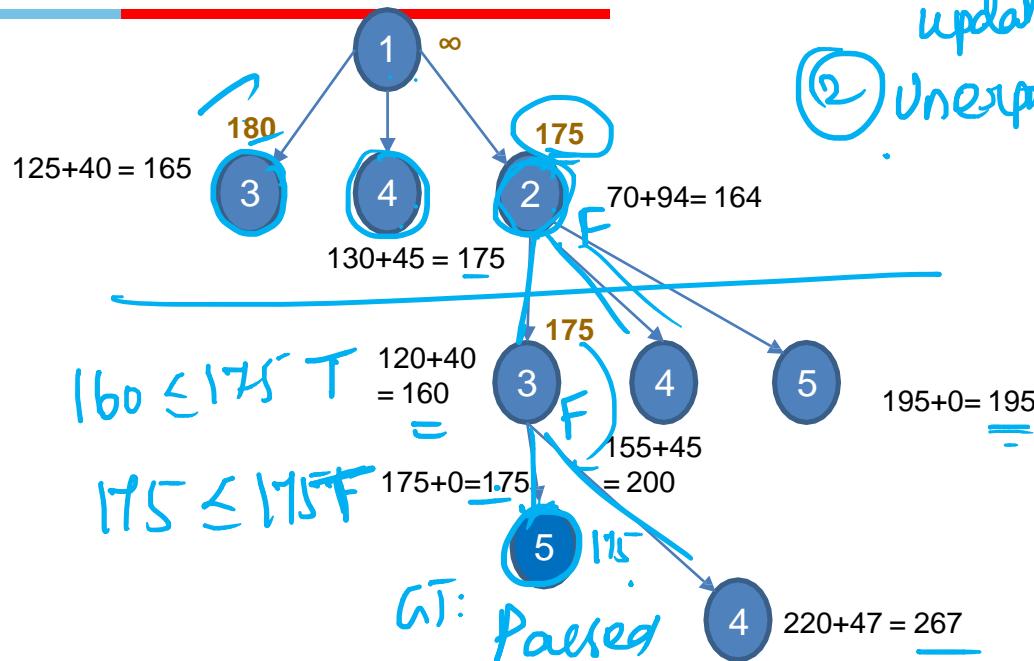
(1 2 | 164) (1 3 | 165) (1 4 | 175)

(1 2 | 175) (1 4 | 175) (1 3 | 180)

(1 2 3 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200)

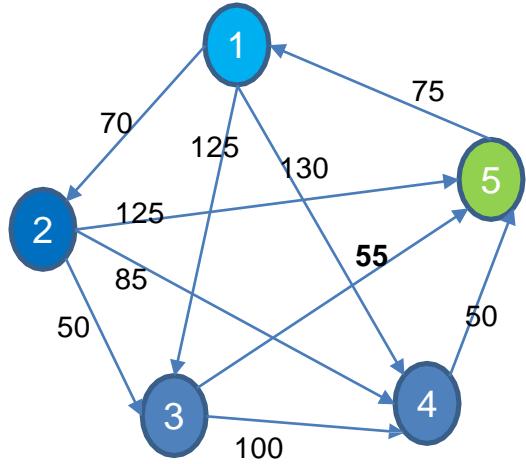
(1 2 3 5 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200) (1 2 3 4 | 267)

PASS

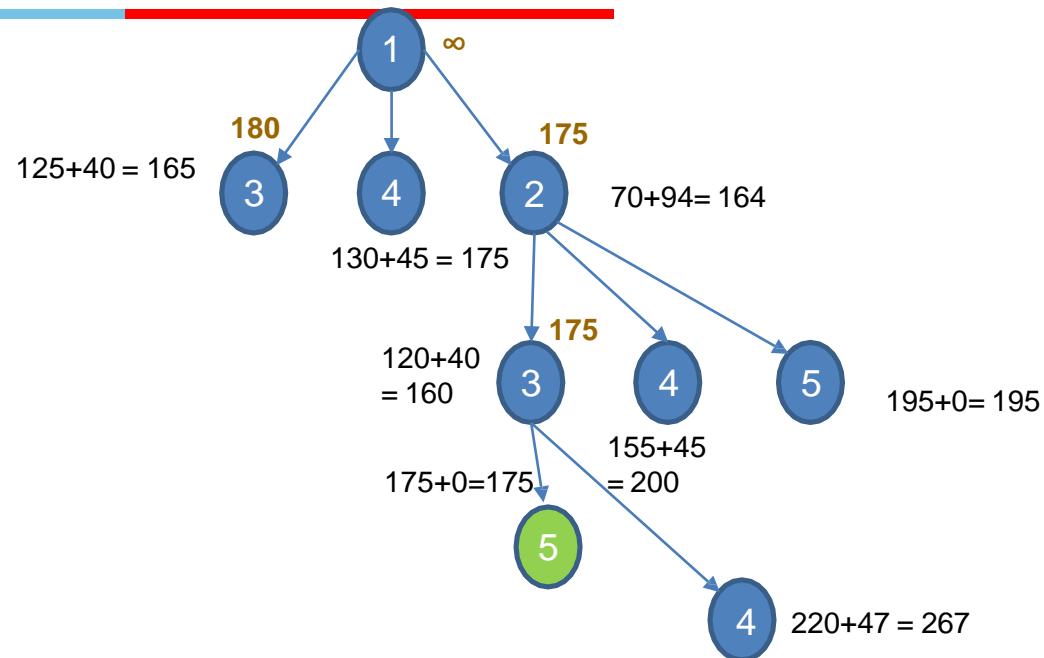


# Recursive Best First Search A\*

Remember the next best alternative f-Cost to regenerate



n	$h(n)$
1	60
2	94
3	40
4	45
5	0

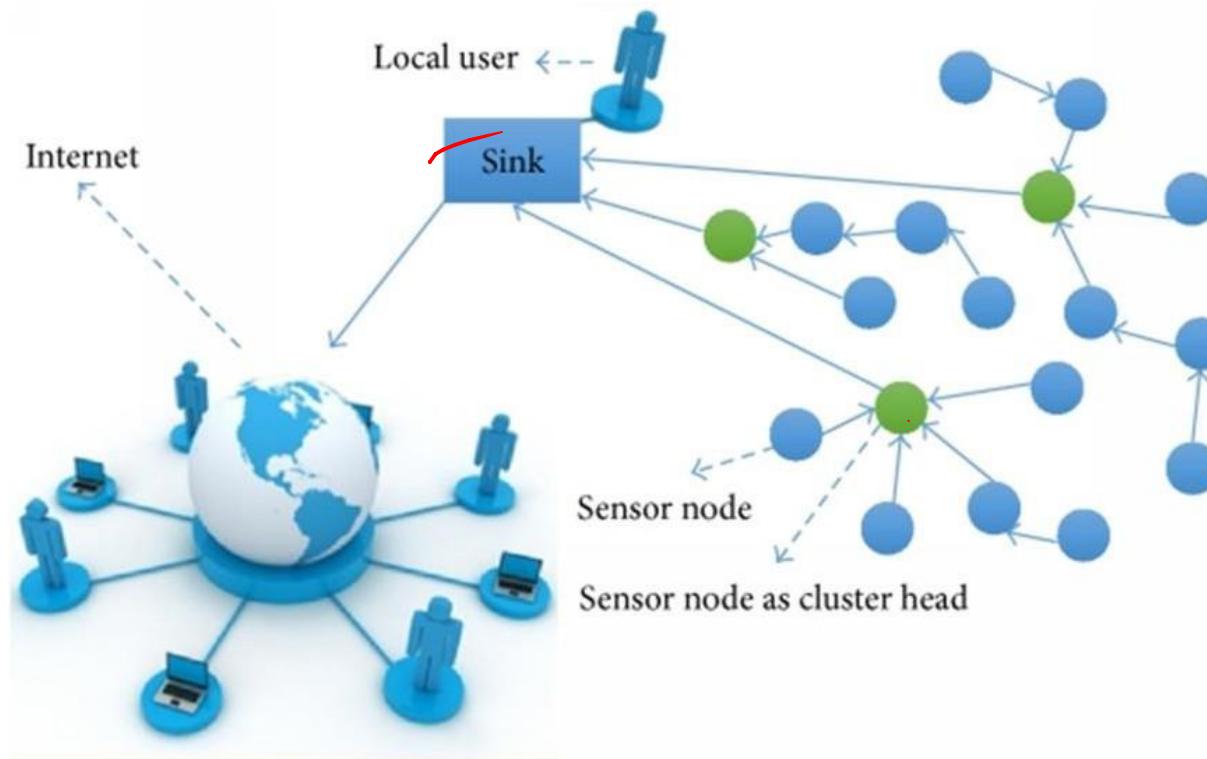


If the current best leaf value > best alternative path  
 Best leaf value of the forgotten subtree is backed up to the  
 ancestors

Else  
 Recursion unwinds  
 Continue expansion

Space Usage =  $O(bd)$  very less

# Case Study – Search in Network Routing



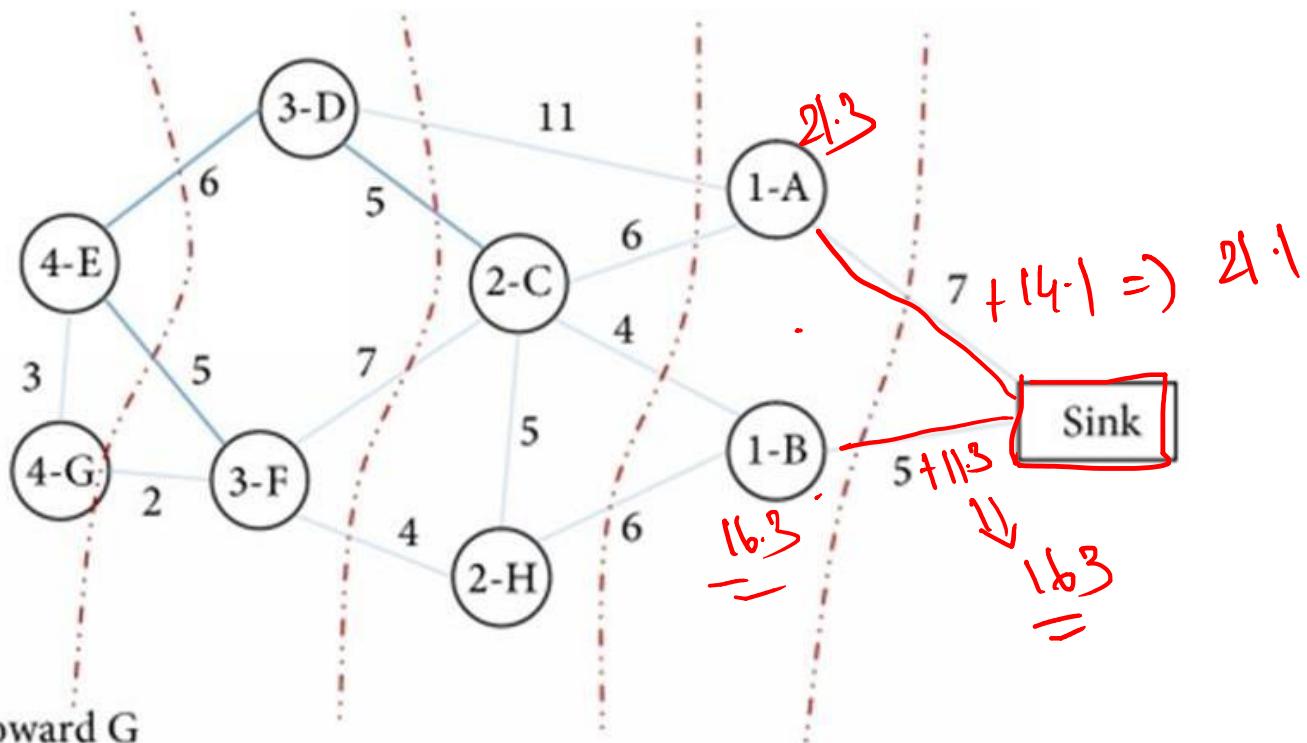
Source Credit :

AR-RBFS: Aware-Routing Protocol Based on Recursive Best-First Search Algorithm for Wireless Sensor Networks

<https://doi.org/10.1155/2016/8743927>

# Case Study – Search in Network Routing

A	14.1
B	11.3
C	8.2
H	6.6
F	2
E	3
D	4.8

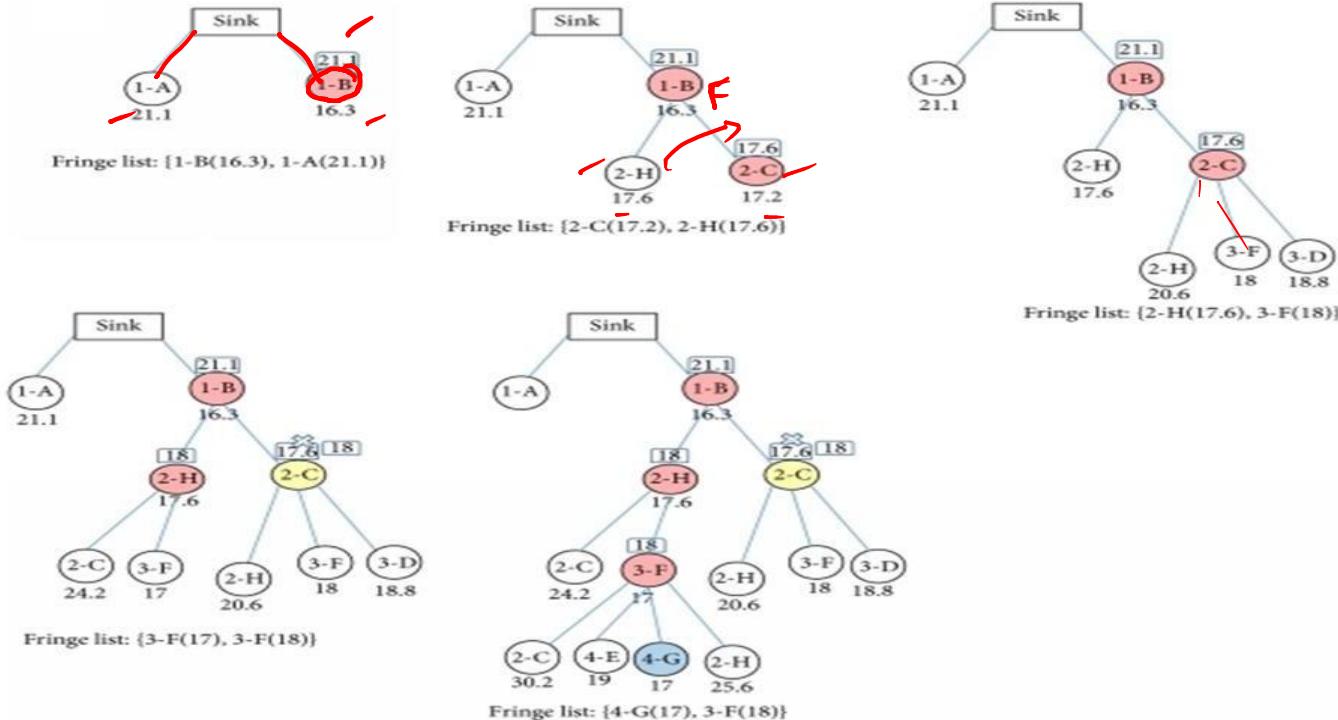


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# Case Study – Search in Network Routing



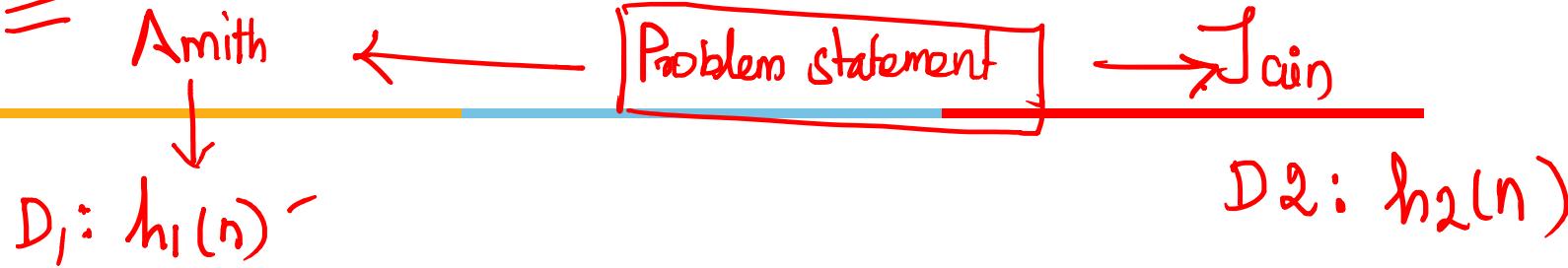
Fringe list is a sorted array  
 Length of the list always is two → energy and memory saving  
 (1) Best → select and (2) best → maintain in selected node

- Selected node
- Cancelling the selected node
- Goal node (destination node)

Source Credit :

AR-RBFS: Aware-Routing Protocol Based on Recursive Best-First Search Algorithm for Wireless Sensor Networks  
<https://doi.org/10.1155/2016/8743927>

S<sub>1</sub>

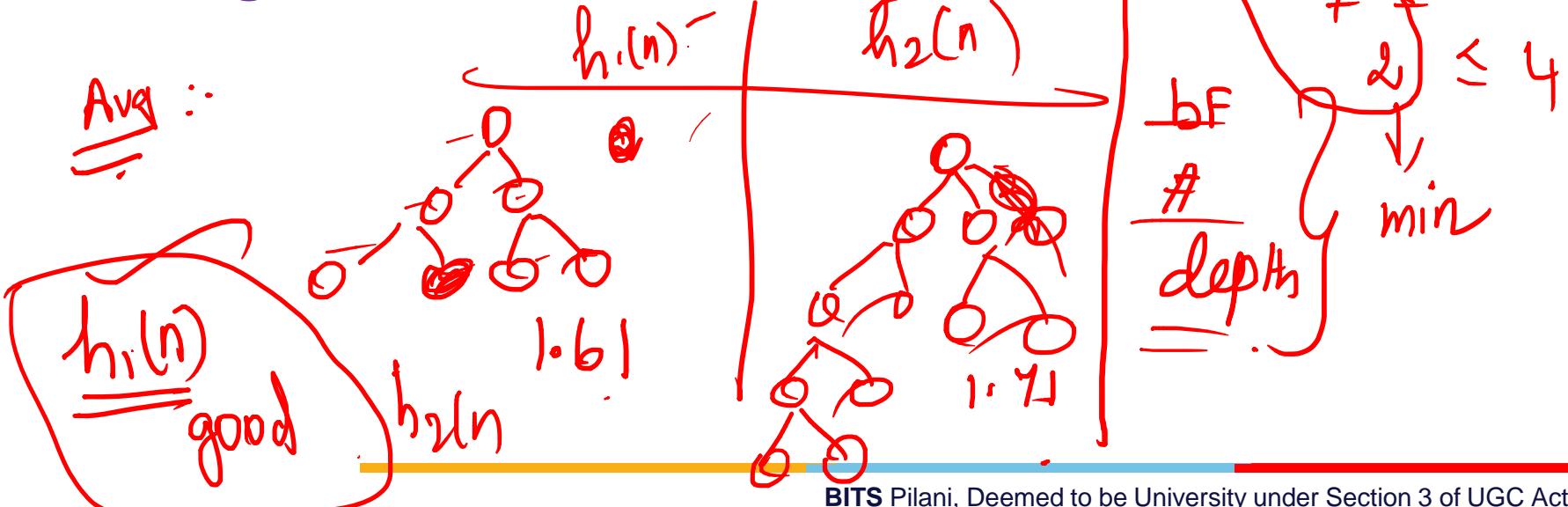


Information

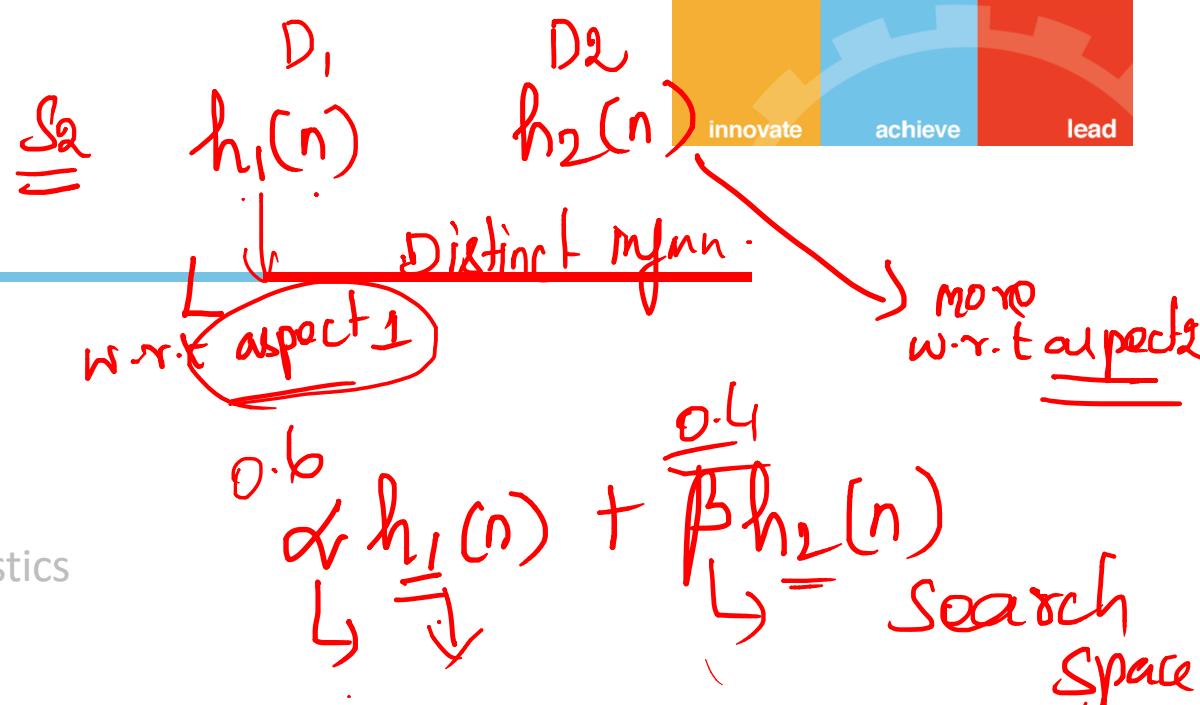
- \* branching -
- \* # nodes generated -
- \*  $h_1(n)$

## Design of Heuristics

Avg. :-



## Heuristic Design



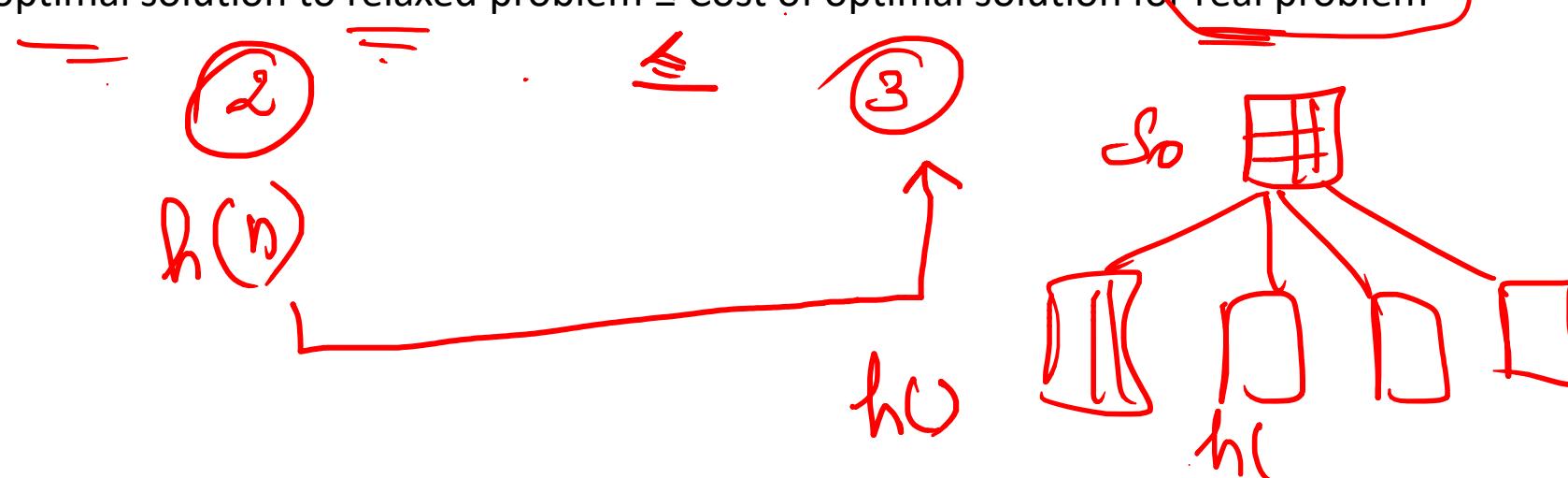
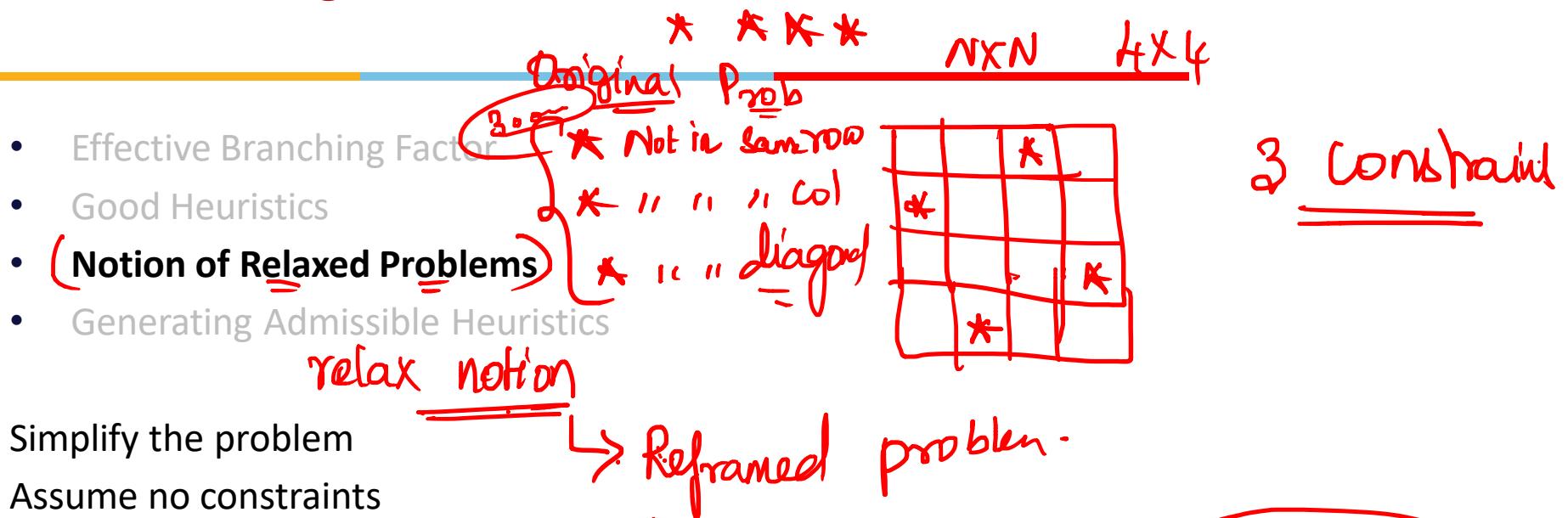
Effective branching factor ( $b^*$ ):

If the algorithm generates  $N$  number of nodes and the solution is found at depth  $d$ , then

$$N + 1 = 1 + (b^*) + (b^*)^2 + (b^*)^3 + \dots + (b^*)^d$$

# Heuristic Design

## N Queen Prob      4 Queen





# Design of Heuristics

## N-Queen

	Q		
			Q
Q			
		Q	

4 Queen

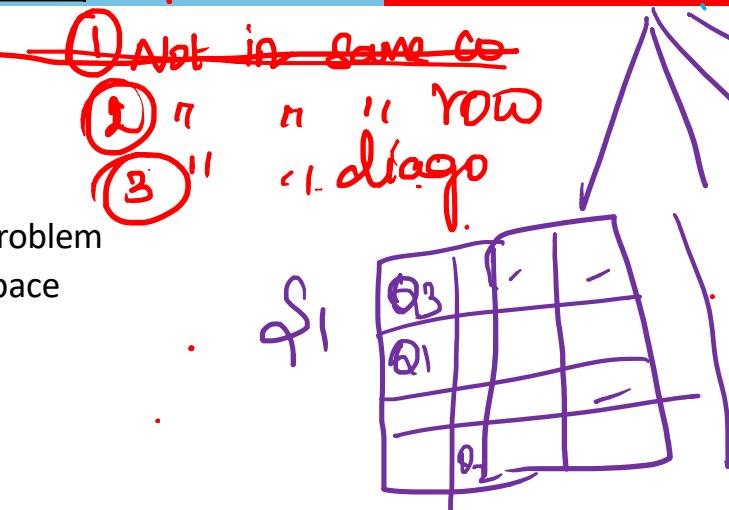
3 constraint

Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
Q <sub>1</sub>	X	X	X
X	X	X	Q <sub>3</sub>
X	X	X	X



Initial State

- Construct the search tree by considering one row of the board at a time
- State space graph of relaxed problem is a super graph of original state space because of removal of restrictions



Place : 3rd Col

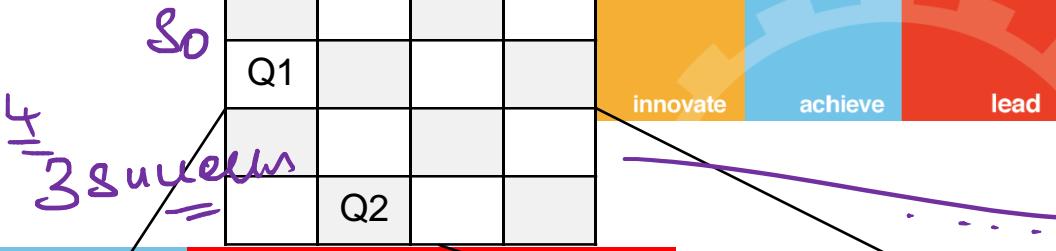
Desi



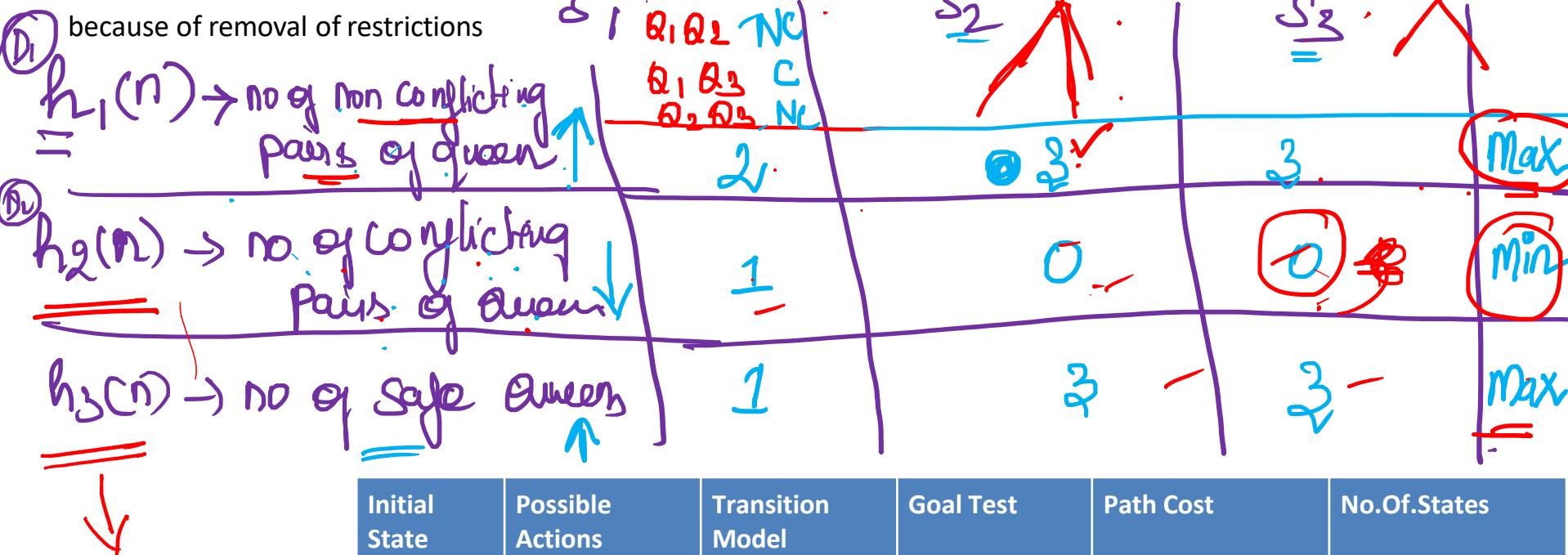
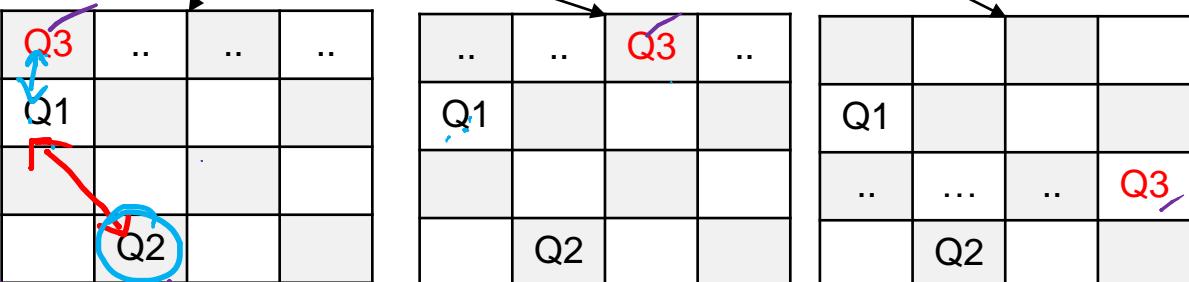
Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
$< X_i, Y_i >$	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	$n!$

# N-Queen

	Q		
			Q
Q			
		Q	

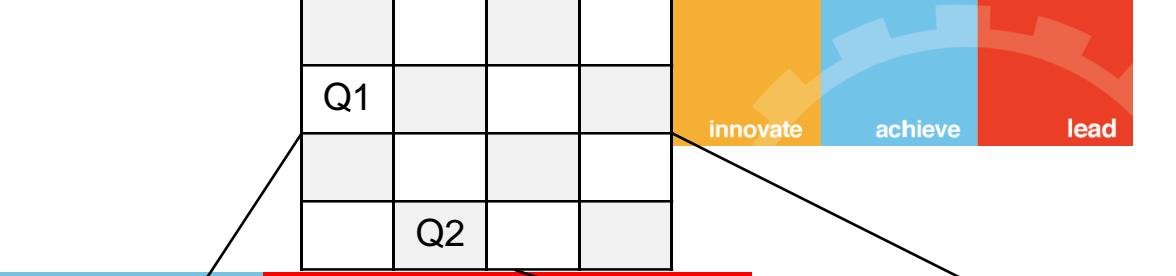
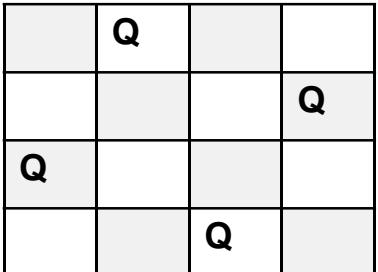


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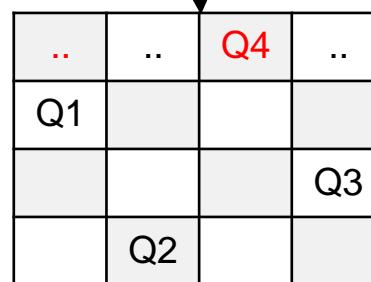
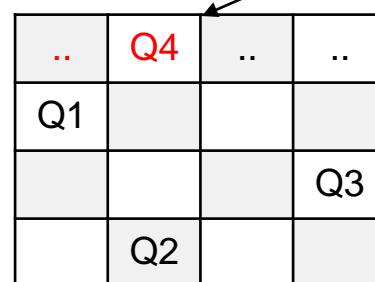
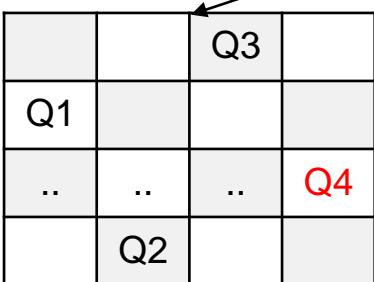
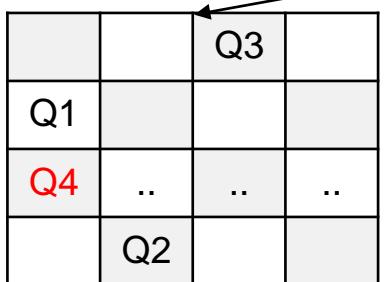
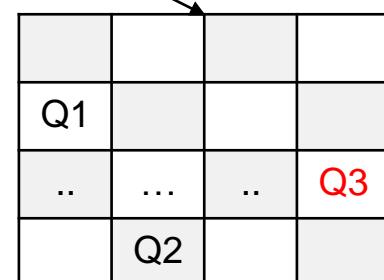
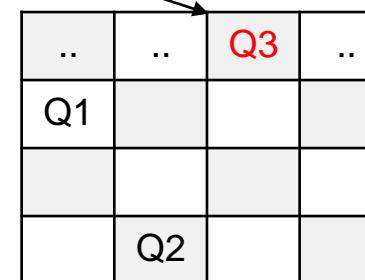
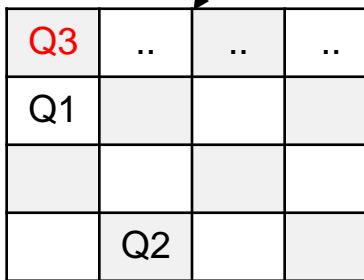


Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
$< X_i, Y_i >$	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	$n!$

# N-Queen



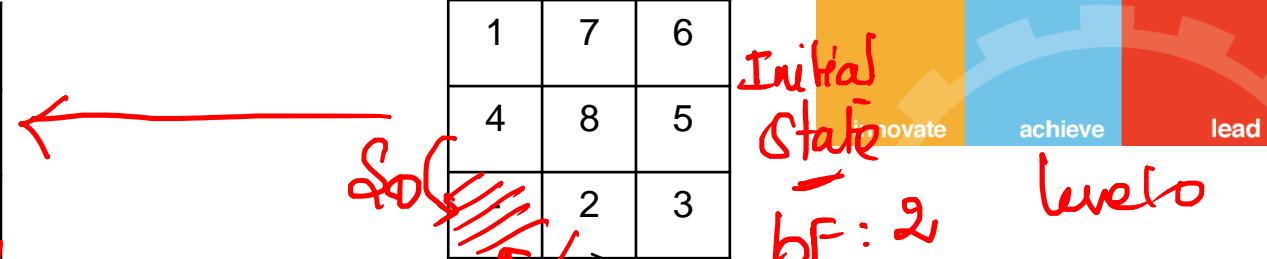
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Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
$< X_i , Y_i >$	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	$n!$

Goal State  
N-Tile

-	1	2
3	4	5
6	7	8



Constraint:

Swap → adjacent cell

$h_1(n) \rightarrow$  Manhattan dist q. labeled cell

$h_1(n) \rightarrow$  Man dist q. empty tile

$h_3(n) \Rightarrow$  # of misplaced cell

→ 9 → 8 labeled  
1 empty

$h_2(n) + h_3(n)$

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of STATES
<LOC, ID>	Move Empty to near by Tile		ID=LOC+1	Transition + Positional + Distance + Other approaches	9!

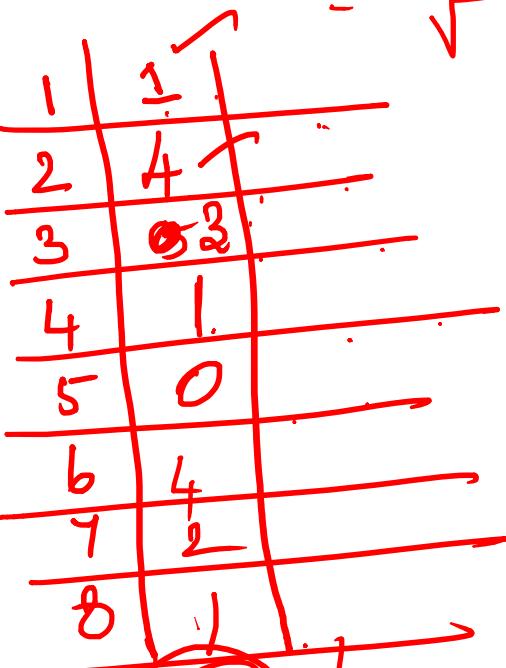
## N-Tile

-	1	2	-
3	4	5	.
6	7	8	.

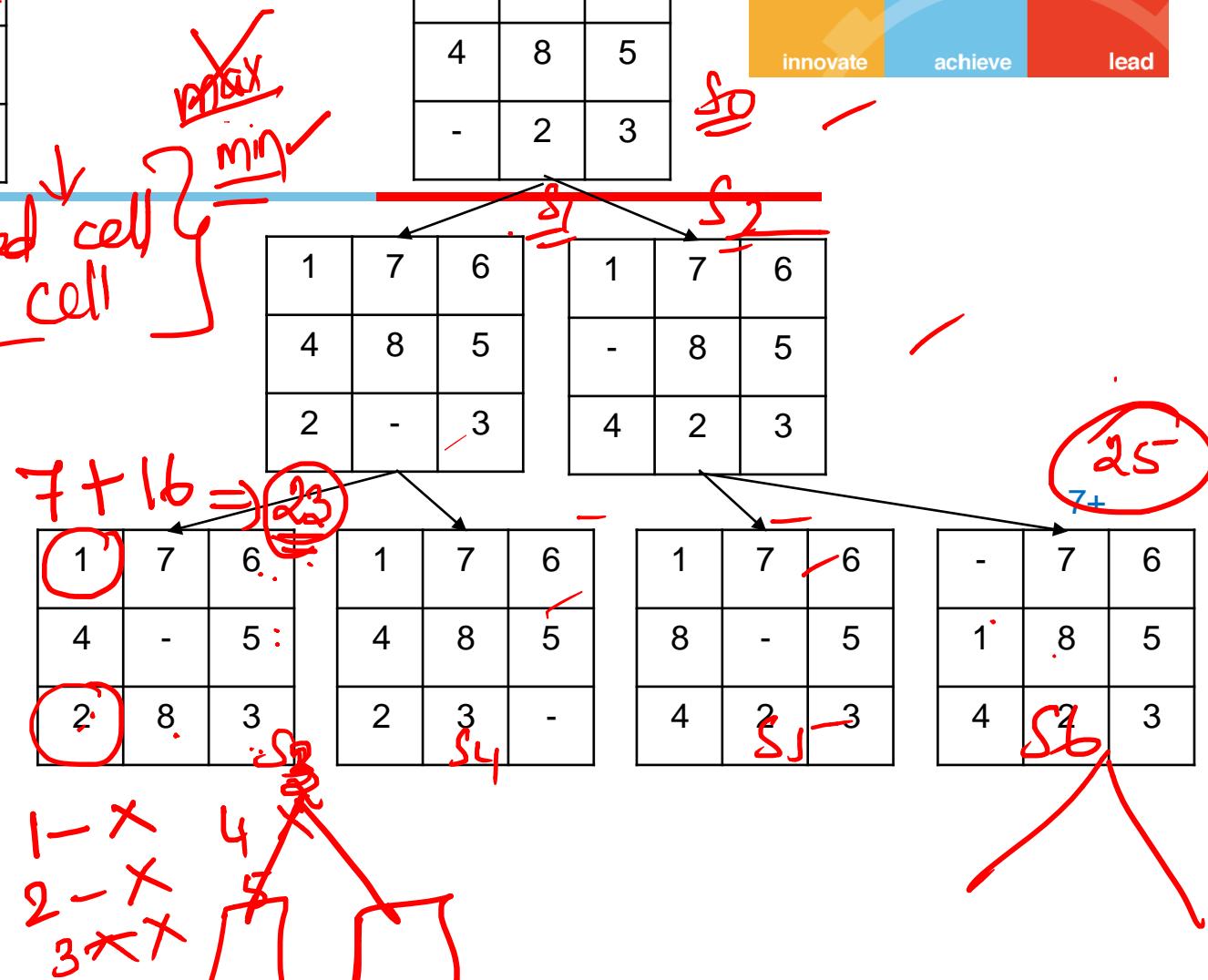
1	7	6
4	8	5
-	2	3



$h_3(n) \Rightarrow$  no of misplaced cell  
 $h_2(n) = m \cdot \text{dist labeled cell}$

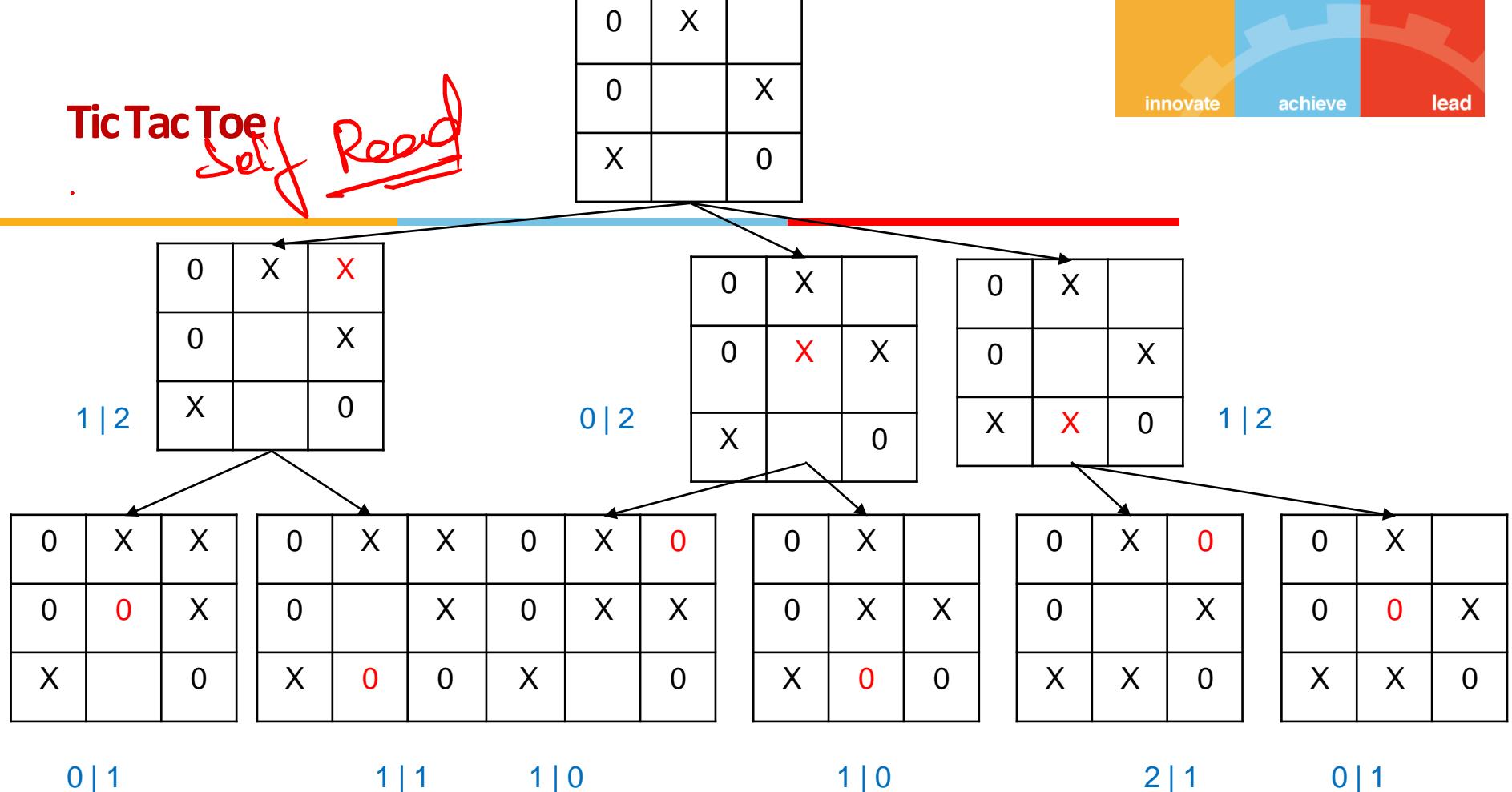


Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of STATES
<LOC, ID>	Move Empty to near by Tile		ID=LOC+1	Transition + Positional + Distance+ Other approaches	9!



# Tic Tac Toe

*Self Road*



Opposite Win | Player Win

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
([X <sub>ij</sub> ], [Y <sub>ij</sub> ])	Place a coin in unoccupied (i,j)		N : i's N : j's N : i=j	No.of.Steps + Opp.Win + (N-1-Curr.Win)	19,683=3 <sup>9</sup>

## Learn from experience

Trail / Puzzle	X1(n) : No.of.Misplaced Tiles	X2(n): Pair of adjacent tiles that are not in goal	X3(n): Position of the empty tile	.....h'(n)
Example 1	7	10	7	.....
Example 2	5	6	6	.....
.....	..	..	..	.....

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

Create a suitable model:

$$h(n) = c_1 \cdot X_1(n) + c_2 \cdot X_2(n) + \dots$$