Q.1) During the 1980's, the general consensus is the about 5% of the nation's children had autism. Some claimed that increase certain chemical in the environment had led to an increase in autism.

a) Write an appropriate hypothesis test for this situation.

b) Give an appropriate test for this hypothesis, stating what are the necessary conditions for formating the test.

c) A recent study examined 384 children and found that 46 showed signs of autism. Perform a test of the hypothesis and state the P-values.

d) What are the conclusions? State how use the P-value.

⇒ Step 1 :- Establish Null & Alternate hypothesis.

Null hypothesis : 5% of the nation children had autism.

Alternate hypothesis : More than 5% of the nation children had autism

H_0 : $P = 0.05$

H_A : $P > 0.05$

It is a one-tailed test since we are going to checked only one end of the experiment.

Step 2 :- Determine the test
we are going to perform
the Z- test.

Step 3 :- Set value of alpha (α).
Since in this proble the significance value is not given.
Let By Default assumption,

$\alpha = 5\%$

$\alpha = 0.05$

Step 4 :- Establish the Decision Rule,

For Z-critical,
if,
Z-critical < Z-score (Test Score)
we will reject null hypothesis.

For P-values,
if,
P-value < significance value
we reject our null hypothesis.

Step 5 : Gathering Data.

A recent study examined examined 384 children and found that 46 - showed signs of autism.
5% of the nation's children had autism.

Step 6 : Analyze the data.

We know,

$\boxed{p = 0.05}$, $\boxed{n = 384}$

$$z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{p.q}{n}}}$$

Sample proportion,

$$\hat{p} = \frac{46}{384}$$

$\boxed{\hat{p} = 0.11}$

$\boxed{n = 384}$

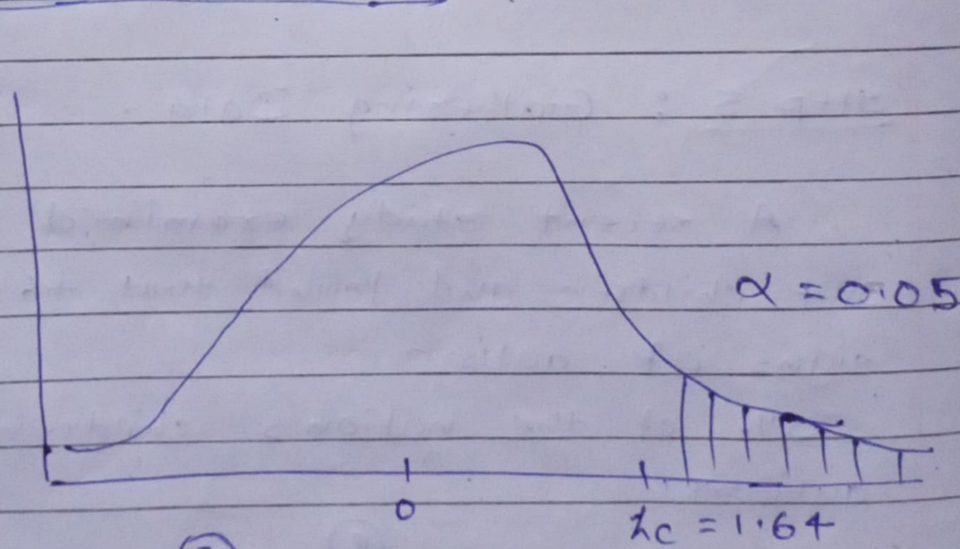
$q = 1 - p$

$\boxed{q = 0.95}$

$$z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{p.q}{n}}}$$

$$= \frac{0.11 - 0.05}{\sqrt{\frac{0.05 \times 0.95}{384}}}$$

$\boxed{z\text{-score} = 5.39}$



③

Step 7 : Take statistical Action

Using z - table,

We have calculated,

$$z\text{-critical} = 1.64$$

$$z\text{-score} = 5.39$$

On the Basis of Decision Rule,

$$z\text{-critial} < z\text{-score}$$

$$1.64 < 5.39$$

∴ We will Reject Null hypothesis.

So,

the More than 5% had autism.

So,

the increase in certain chemical in the environment had led to an increase in autism.

Q.2) A company with a fleet of 150 cars found that the emission system of 7 our of the 22 cars tested failed to meet pollution guidelines.

a) write a hypothesis to test if more than 20% of the entire fleet might be out of compliance.

b) Test the hypothesis based on the binomial distribution and report p-value.

c) Is the test significant at the 10%, 5%, 1% level?

⇒ Step 1 : Establish Null & Alternate hypothesis.

Null hypothesis : 20% of the car failed to meet population guidelines.

Alternate hypothesis :- more than 20% of the faild to meet population guidelines.

$H_0 : P = 0.20$

$H_A : P > 0.20$

It is a one-tailed test since we are going to check only one end of experiment.

Step 2 :- Determine the test.

we are going to perform the z- test.

Step 3 :- set the significance value,

① Significance value = 10%.

$$\alpha = 0.10$$

Step 4 :- Establish Decision Rule,

For z- critical,

z-critical < z-score (Test Score)

we will reject null hypothesis.

For p- value,

if,

p- value < significance level

we will reject our null hypothesis.

Step 5 :- Collecting data.

A company with a fleet of 150 cars found that the emission type system of 7 our of the 22 cars tested failed to meet pollution guidelines.

Step 6 :- Analysis of Data.

For z-score,

$$z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

we know that,

$\boxed{P = 0.20}$, $\boxed{n = 22}$

To calculate $\hat{p}$,

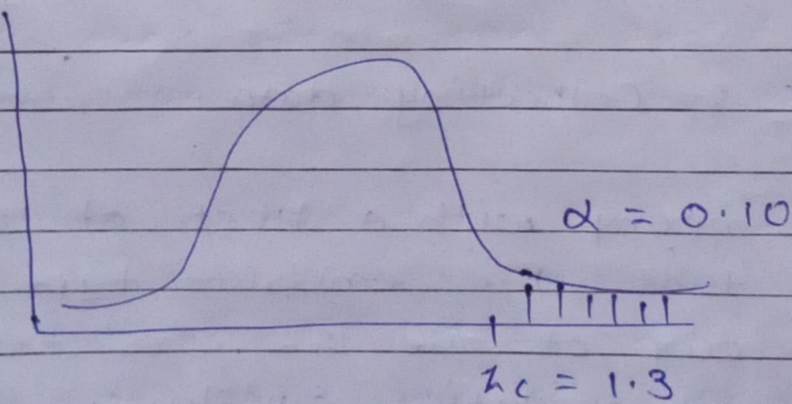
$\hat{p} = \frac{7}{22}$, $q = 1 - p$

$\boxed{q = 0.80}$

$\boxed{\hat{p} = 0.31}$

$$z\text{-score} = \frac{0.31 - 0.20}{\sqrt{\frac{0.20 \times 0.80}{22}}}$$

$\boxed{z\text{-score} = 1.28}$



By z-table,

$\boxed{z_c = 1.3}$

$\boxed{\text{P- values} = 1.003}$

Step 7 :- Take statistical Action,

For z-critical,

z critical > z-score

1.3 > 1.28

we will not reject the null hypothesis,
we will accept the null hypothesis.

For P-values,

P- value > significance value

1.003 > 0.10

we will accepth the null hypothesis.

therefore,

20% of the entire fleet of the cars are failed to meet the population guidelines.

Step 8 :- Determine Business Implication

So, the 20% of the cars are failed to meet the population guidelines, that why we should improve the cars so that they can meet to the population guidelines.

(2) For,

$\alpha = 5\%$

$\boxed{\alpha = 0.05}$

we have calculated,

$\boxed{z\text{-score} = 1.28}$

Using z- table,

$\boxed{z\text{-critical} = 1.64}$

On the Basis of Decision Rule,

z-critical > z-score

we,

1.64 > 1.28

we will accept the null hypothesis.

that,

20% of the entire fleet of the cars are failed to meet the population guidelines.

In this case,

The car manufactures should improve the cars effeciency that they can meet the population guidelines.

(3) For,

$$\alpha = 1\%$$

$$\boxed{\alpha = 0.01}$$

Since it is a one tailed test

we have already calculated.

$$\boxed{z\text{-score} = 1.28}$$

By using z-table,

$$\boxed{z\text{-critical} = 2.33}$$

So,

on the Basis of Decision Rule,

z-critical > z-score

2.33 > 1.28

we will accept the null hypothesis.

that,

20% of entire fleet of cars are failed to meet the population guidelines.

Q.3) National Data in the 1960 showed that about 44% of the adult population had never smoked.

a) State a null and alternate to test that the fraction of the 1995 of adults that had never smoked had increased.

b) A national random sample of 891 adults were interviewed and 463 stated that they had never smoked. Perform a z-test of the hypothesis and given an appropriate P-value.

c) Create a 98% confidence interval for the proportion of the adults who had never been smoked.

⇨

Step 1 :- Establish Null & Aternate hypothesis.

Null hypothesis :- 44% of the adult population had never smoked.

Alternate hypothesis :- more than 44% of the adult population had never smoked.

H_0 : $P = 0.44$

H_A : $P > 0.44$

It is a one-tailed test, since we are testing only one end of the experiment.

Step 2 :- Determine the test :

we are going to perform the
Z - test.

Step 3 :- Set the value of significance level.

Since, confiden level is 98%.
then,

$\alpha = 2\%$

$\alpha = 0.20$

$\boxed{\alpha = 0.02}$

Step 4 :- Establish the Decision Rule

For Z - critical,
if
Z - critical < Z - test (Test score)
we will reject null hypothesis.

For P - values,
if
P - value < significance value
we will reject the null hypothesis.

Step 5 :- Gathering the Data :

A national random sample of 891 adults
were interviewed and 463 stated that

they had never smoked.

Step 6 :- Analysis of the Data.

For z- score,

$$z\text{-score} = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

we know that,

$\boxed{p = 0.44}$, $\boxed{n = 891}$

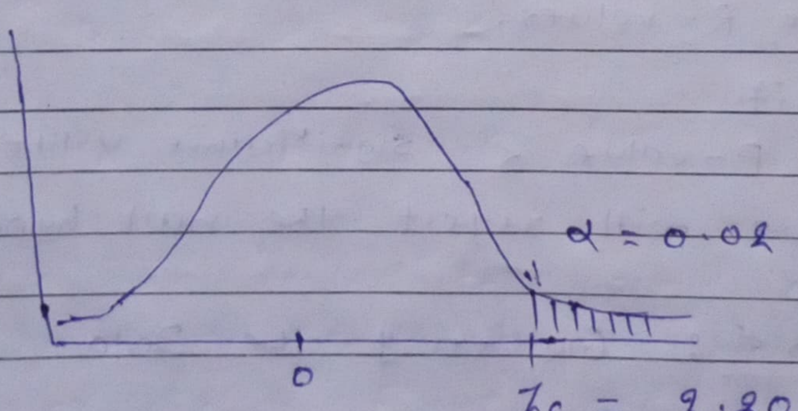
$q = 1 - p$, $\hat{p} = \frac{463}{891}$

$= 1 - 0.44$

$\boxed{q = 0.56}$ $\boxed{\hat{p} = 0.519}$

$$z\text{-score} = \frac{0.519 - 0.44}{\sqrt{\frac{0.44 \times 0.56}{891}}}$$

$$\boxed{z\text{-score} = 4.75}$$



By Using z table,

$$\boxed{z\text{-critical} = 2.20}$$

Step 7 : Take statistical Action.

We have calculated,

$$\boxed{z\text{-score} = 4.75}$$

$$\boxed{z\text{-critical} = 2.20}$$

On the basis of Decision Rule,

$$z\text{-critical} < z \text{ score}$$

$$2.20 < 4.75$$

we will reject the null hypothesis.

So,
the more than 44% of the adult population had never smoked.

Step 8 :-

So, The more than 44% of the adult population never smoked.

Q.4) One of the lenses in your supply is suspected to have a focal length f of 9.1 cm rathan than the 9cm claimed by the manufacturer.
Heres S.1 is the distance from the lens to the object and s_2 is the distance from the lens to the real image of the object.
The distances s_1 and s_2 are each independently measured 25 times. The sample mean of the measurement is $\bar{s}_1 = 26.6$ centimeters and $\bar{s}_2 = 13.8$ centimeters, respectively. The standard deviation of the measurement is is 0.1cm for s_1 and 0.5cm for s_2.

a) Write the appropriate test hypothesis test for this situation.

b) Use this to devise a z-test for the hypothesis and report a p-value for the test.

⇨ Step 1 : Establish Null & Alternate Hypothesis.

Null Hypothesis :- The distance from the lens to the object and distance from the lens to real image is same.

Alternate Hypothesis :- The distance from the lens to the object and distance from the lens to real image is not same.

H_0 ; $\mu_A = \mu_B$

H_A ; $\mu_A \neq \mu_B$

Step 2 :- Determine the Test.

We are going to perform the z - test.

Step 3 :- Set the significance value.

As the significance value is not given in the problem we are going to take the default significance value.

i.e. $\alpha = 0.05$

Since it is a two tailed test and we are going to check the two left & Right end of the experiment so,

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Step 4 :- Establish the Decision Rule.

For critical value,

if

z-critical < z-score

we will reject the Null Hypothesis.

For P-values,

if

P-values < significance value

we will reject the Null Hypothesis.

Step 5 :- Collecting the data:

The distance each s_1 & s_2 are measured independently 25 times. The sample mean of the measurement is $\bar{s}_1 = 26.6$ centimeters and $\bar{s}_2 = 13.8$ centimeters respectively. The standard deviation of the measurement is 0.1cm for s_1 and 0.5cm for s_2.

Step 6 :- Analysis of the data.

since we are going to perform the z-test for two samples.

$$\alpha = 0.05$$

$$\boxed{\frac{\alpha}{2} = 0.025}$$

For Distance from the lens to object s_1,

$$\bar{s}_1 = 26.6\text{cm}$$

$$\sigma_1 = 0.1\text{cm}$$

$$n_1 = 25$$

For Distance from lens to Real image S_2,

$\overline{S_2} = 13.8$

$\sigma_2 = 0.5$ cm

$n_2 = 25$

For Z-score,

$$Z\text{-score} = \frac{\overline{S_1} - \overline{S_2}}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}}$$

$$= \frac{26.6 - 13.8}{\sqrt{\frac{(0.1)^2}{25} + \frac{(0.5)^2}{25}}}$$

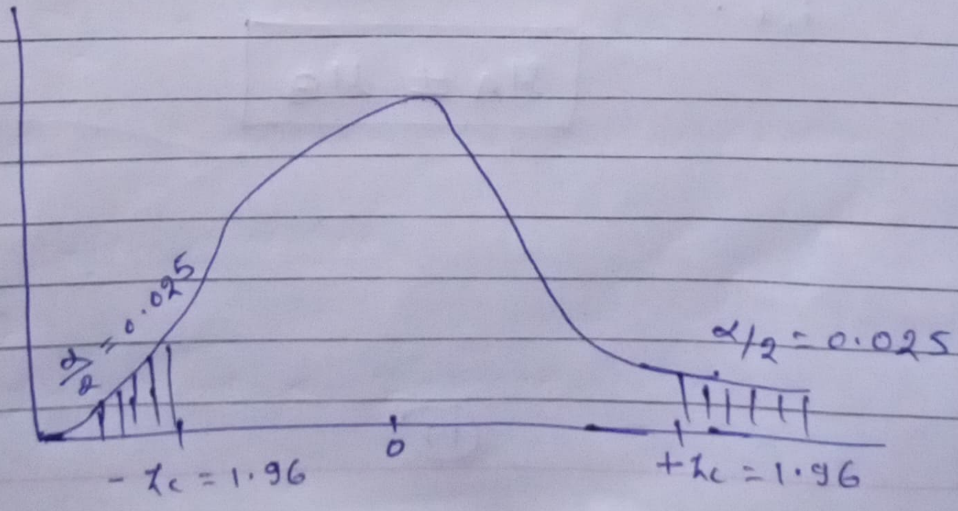
$$= \frac{12.8}{0.102}$$

$$\boxed{Z\text{-score} = 125.51}$$

Now Using Z-table,

$$\boxed{Z\text{-critical} = 1.96}$$

$$\boxed{p\text{-value} = 0.0000}$$



Step 7 :- Taking Statistical Action

On the basis of Decision Rule,

For critical value,

if

z-critical < z-score

1.96 < 125.51

we will reject the Null hypothesis.

For critical value,

if

p-value < significance value

0.0000 < 0.05

we will reject the null hypothesis.

On the basis of Result of the hypothesis test we can say that,
The distance from the lens to the object and distance from the lens to the real image are not same.

i.e.

$$\boxed{\mu_A \neq \mu_B}$$

Q.5) The body temparature in degrees Fahrenheit of 52 randomly chosen healthy adults in measured with the following summary of data.

$n = 52$, $\bar{x} = 98.2846$, $s = 0.6824$

a) Are the necessary conditions for constructing a valid t-interval satisfied?

b) Find a 98% confidence interval for the mean body temparature of 98.6° Fahrenheit and use the information above to the evaluate a test with significance level of $\alpha = 0.02$

⇨ Step 1 :- Establish Null & Alternate hypothesis.

Null hypothesis : Mean Body temparature is 98.6.

Alternate hypothesis :- Mean Body temparature is not equal to 98.6.

H_0 ; $\mu = 98.6$

H_A ; $\mu \neq 98.6$

It is two-tailed test since we are going to test it on two ends or left & Right end of the test.

$$\boxed{\frac{\alpha}{2} = \frac{0.02}{2} = 0.01}$$

Step 2 :- Determine the test.

we are going to perform the t- test.

Step 3 :- Set the significance value.

$\boxed{\alpha = 0.02}$ -- Given

Step 4 :- Establish Decision Rule.

For critical value,

if

t-critical < t-score

we will reject the null hypothesis.

For p-value,

if

p-value < significance level

we will reject the null hypothesis.

Step 5 :- Collecting the data.

The body temparature in degrees Fahrenheit of 52 randomly chosen healthy adults in measured with the following summary of data.

$n = 52$, $\bar{x} = 98.2846$, $S = 0.6824$

Step 6 :- Analysis of data.

Since, we testing only one-random variable and its standard davition is given.

$$t\text{-score} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where, $n = 52$, $DF = n - 1$

$\bar{x} = 98.2846$ $= 52 - 1$

$s = 0.6824$ $\boxed{DF = 51}$

$\mu = 98.6$

$$\therefore t\text{-score} = \frac{98.2846 - 98.6}{\frac{0.6824}{\sqrt{52}}}$$

$$\boxed{t\text{-score} = 3.333}$$

Sinc it is two tailed test.

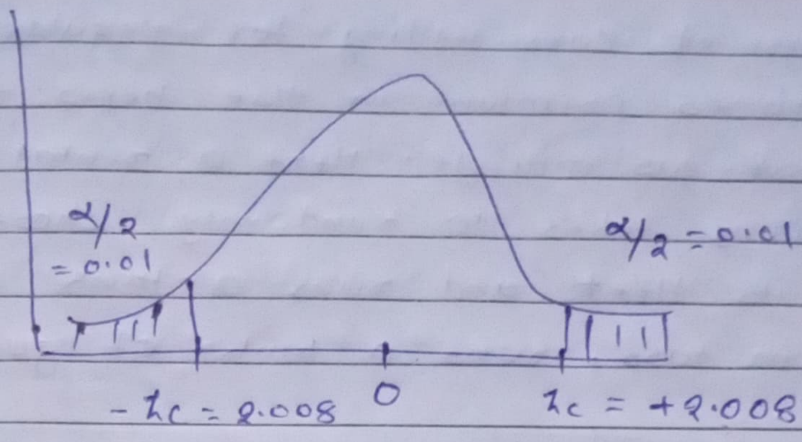
$$\alpha = \alpha/2 = \frac{0.02}{2}$$

$\boxed{\alpha = 0.01}$, $\boxed{DF = 51}$

Now, Using t-table,

$$\boxed{t\text{-critical} = 2.008}$$

$$\boxed{P\text{-values} = 0.0016}$$



Step 7 : Take statistical Action.

Based on Decision Rule,
For critical values,
if

$t_{critical} < t_{score}$

$2.008 < 3.333.$

we will reject null hypothesis.

For p- values,
if

p- values < significance level

$0.0016 < 0.02$

we will reject null hypothesis.

so,
the mean body temparature is not equal to 98.6°F.

Q.6) Drivers of cars calling for regular gas sometimes premium in the hopes that it will improve gas mileage. Here a rental car company takes 10 randomly chosen car in the its fleet and runs a tank of gas according to coin toss, runs a tank of gas of each type.

Car #	1	2	3	4	5	6	7	8	9	10
Regular	16	20	21	22	23	22	27	25	27	28
Premium	19	22	24	24	25	25	26	26	28	32

a) write an appropriate hypothesis test for this situation and state the testing procedure appropriate to this circumstance.

b) compute the necessary summary statistics for the test in part (a).

c) Perform t-test and report the p-value.

d) compare your result to that of a two sample t-test.

⇒ Step 1 : Establish Null & Alternate hypothesis.

Null Hypothesis : there is no differce between the premium & Regular gas tank in term of mileage.

Alternate Hypothesis :- There is differce between the mileage of regular and premium gas tank.

H_0 ; $\mu_A = \mu_B$

H_A ; $\mu_A \neq \mu_B$

It is two - tailed test, since we are going to check it on two ends of the experiment.

Step 2 : Determine the test.

we are going to perform the t - test.

Step 3 : set the significance value.

As the significance value is not given so we take the default value of significance level.

$$\boxed{\alpha = 0.05}$$

Since,

it is a two-tailed test.

$$\boxed{\alpha/2 = 0.025}$$

Step 4 : Establish the Decision Rule.

For critical value

if,

t-critical < t-test

we will reject the Null hypothesis.

For p-value,
if
p-value < significance value
we will reject the Null hypothesis.

Step 5 :- Collecting the Data.

The rental car company takes 10 randomly chosen cars in the its fleet and runs a tank of gas according to a coin toss, runs a tank of gas of each type.

Car #	1	2	3	4	5	6	7	8	9	10
Regular	16	20	21	22	23	22	27	25	27	28
Premium	19	22	24	24	25	25	26	26	28	32

Step 6 : Analysis of Data

since it a t-test for two sample random variable.

$$df = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}\right]}$$

$$t\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For sample of Regular tank,

$$\mu_A = \bar{x}_1 = 023.1$$

$$n_1 = 10$$

$$s_1 = 3.72 \quad , \quad \bar{x}_1 = \mu_A = 23.1$$

For sample of Premium tank,

$$\bar{x}_2 = \mu_B = 25.1$$

$$n_2 = 10$$

$$s_2 = 3.44$$

$$DF = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}\right]}$$

$$= \cancel{\frac{(53.36 + 60)^2}{\frac{(53.36)^2}{9} + \frac{(60)^2}{9}}}$$

$$= \frac{(1.38 + 1.18)^2}{\frac{(1.38)^2}{9} + \frac{(1.18)^2}{9}}$$

$$= \frac{6.55}{0.36}$$

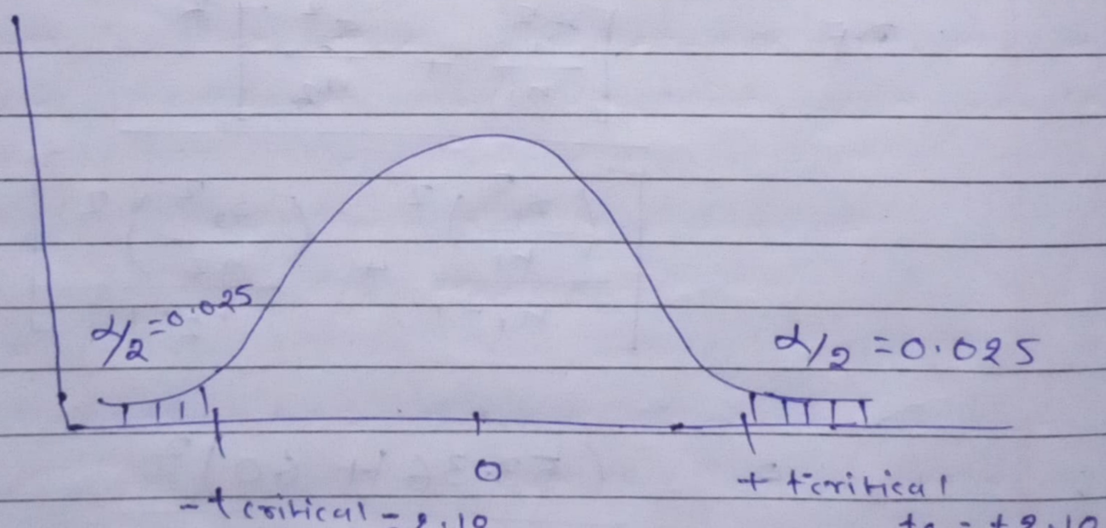
$$= 18$$

$$\therefore \boxed{DF = 18}$$

$$t\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{23.1 - 25.1}{\sqrt{\frac{(3.72)^2}{10} + \frac{(3.44)^2}{10}}}$$

$$\boxed{t\text{-test} = 1.24}$$



By using t-table,

$$\boxed{t\text{-critical} = 2.10}$$

$$\boxed{P = 0.22}$$

Step 7 :- Taking Statical Action

On the basis of Decision Rule.

For t-critical,

if

t-critical < t-score

Reject Null hypothesis.

t-critical > t-score

2.10 > 1.24

we will accept the Null hypothesis.

For p-value,

if

p-value > significance value

0.22 > 0.05

we will accept the Null hypothesis.

So,

On the basis of Decision Rule
we can conclude that
there is no difference between
the mileage of regular and
premium tank of the car.

i.e.

$$\mu_A = \mu_B$$

Q.7) In this problem, we will examine the sugar content of several national brands of cereals, here measured as precentage of weight.

Children	40.3	55	45.7	43.3	50.3	45.9	53.5
	43	44.2	44	33.6	55.1	48.8	50.4
	37.8	60.3	46.6	47.4	44.		
Adult	20	30.2	2.2	7.5	4.4	22.2	16.6
	14.5	21.4	3.3	10.0	1.0	4.4	1.3
	8.1	6.6	7.8	10.6	10.6	16.2	14.5
	4.1	15.8	4.1	2.4	3.5	8.5	4.7

a) Give a summary of two data set.

b) Create side by side boxplots and interpret what you see.

c) Use R to create a 95% confidence interval for the difference in mean sugar content and explain your result.

⇨ Step 1 : Establish Null & Alternat hypothesis.

Null Hypothesis : Sugar content of brand of cerals for children and adult are same.

Alternate Hypothesis : sugar content of brand of cerals for children and adult are not same.

H_0 ; $\mu_A = \mu_B$

H_A ; $\mu_A \neq \mu_B$

Since we are going to check it on left and Right both ends of the experiment it is a two-tailed test.

Step 2 :- Determine the test.

Since we are comparing the mean of two samples.

We are going to perform the t-test.

Step 3 :- set the significance value.

As the confidence level is given 95%. therefore, the significance level is 5%.

$\alpha = 5\%$

$$\boxed{\alpha = 0.05}$$

Since, it is a two tailed test.

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

Step 4 : Establish the Decision Rule.

For Critical value,
if,
t-critical $<$ t-test
we will reject the Null hypothesis.

For P-values,
if
P-value $<$ significance level
we will reject the Null hypothesis.

Step 5 :- Collecting the data.

Sugar content of several national brand of cereals, were measured as a percentage of weight.

Children	40.3	55.0	45.7	43.3	50.3	45.9	53.5
	43.0	44.2	44.0	33.6	55.1	48.8	50.4
	37.8	60.3	46.6	47.4	44.0		
Adult	20.0	30.2	2.2	7.5	4.4	22.2	16.6
	14.5	21.4	3.3	10.0	1.0	4.4	1.3
	8.1	6.6	10.6	10.6	16.2	14.5	4.1
	15.8	4.1	2.4	3.5	8.5	7.8	4.7
	18.4						

Step 6 :- Analysis of Data.

Since it is a t-test for two sample random variable.

$$df = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}\right]}$$

$$t\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For Sample of Children,

$\bar{x}_1 = \mu_A = 46.8$

$n_1 = 19$

$\bar{s}_1 = 6.41$

For Sample of Adult,

$\bar{x}_2 = \mu_B = 10.16$

$n_2 = 29$

$\bar{s}_2 = 7.47$

$$DF = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\left[\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}\right]}$$

$$= \frac{(2.16 + 1.92)^2}{\frac{(2.16)^2}{18} + \frac{(1.92)^2}{28}}$$

$$= \frac{16.64}{0.38}$$

$$\boxed{dF = 43}$$

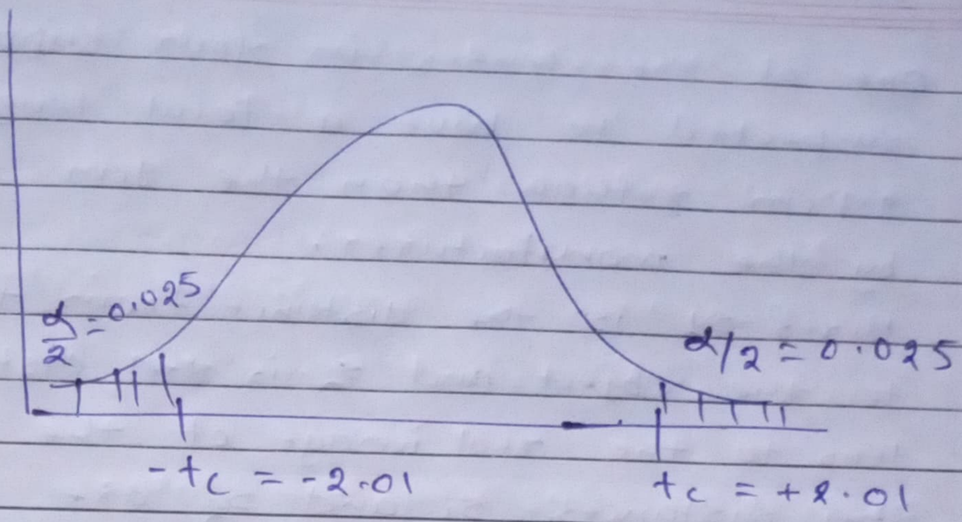
Now,

$$t\text{-test} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{6.41 - 7.47}{\sqrt{\frac{(6.41)^2}{19} + \frac{(7.47)^2}{29}}}$$

$$= \frac{36.38}{1.96}$$

$$\boxed{t\text{-test} = 18.10}$$



By using ~~t~~ - table,

$$\boxed{t_c = 2.01}$$

$$\boxed{P\text{-value} = 0.0001}$$

Step 7 : Take Statistical Action.
On the basic of Decision Rule,

For Critical value,

t-critical < t-test

2.01 < 18.10

we will reject the Null hypothesis.

For P-values,

P-value < significance value

0.0001 < 0.05

we will reject the Null hypothesis.

so,
the sugar content in different brand of cereals for children and Adult are not same or equal.