

Linear Algebra

Shaurya Prakash
II Semester , Ph.D
Instructor: Dr. S.Karthikeyan

Department of Computer Science,
BHU , Varanasi

Transpose

Interchange row and columns.

Uses : To make matrix operation compatible.

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 5 \\ 5 & 8 & 9 \end{pmatrix}^T = \begin{pmatrix} 3 & 4 & 5 \\ 2 & 7 & 8 \\ 1 & 5 & 9 \end{pmatrix}$$

Why do we go for Transposition?

$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \times \begin{pmatrix} 3 & 4 \\ 5 & 6 \\ 4 & 8 \end{pmatrix} = \text{Not Compatible..}$$



$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \times \begin{bmatrix} 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix} = \begin{pmatrix} 18 & 26 & 34 \\ 25 & 36 & 47 \\ 39 & 56 & 73 \end{pmatrix}$$

Transpose

$$\begin{pmatrix} 2 & 3 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \times \begin{pmatrix} 3 & 4 \\ 5 & 6 \\ 4 & 8 \end{pmatrix} = \text{Not Compatible..}$$



$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \begin{pmatrix} 3 & 4 \\ 5 & 6 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 43 & 66 \\ 55 & 84 \end{pmatrix}$$

Example

```
In [1]: 1 import numpy as np  
        2 arr =np.array([[2,3,5],[3,4,6]])  
        3 brr =np.array([[3,4,5],[4,6,8]])
```

```
In [2]: 1 np.matmul(arr.T,brr)
```

```
Out[2]: array([[18, 26, 34],  
               [25, 36, 47],  
               [39, 56, 73]])
```

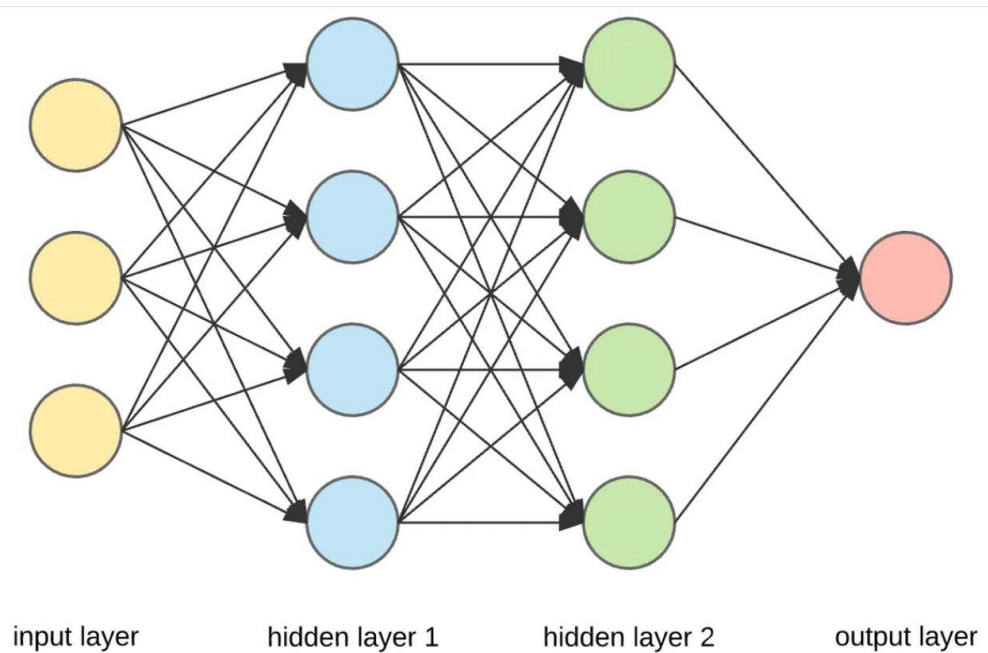
```
In [3]: 1 np.matmul(arr,brr.T)
```

```
Out[3]: array([[43, 66],  
               [55, 84]])
```

```
In [ ]: 1 |
```

Uses of Transpose

Evaluation of weight matrix and aggregate value:



Using Transpose To Develop a Model for Neural Networks

$$\begin{array}{ccc} A^{[1]} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} & W^{[1]} = \begin{bmatrix} w_{11} & w_{21} & w_{31} & w_{41} \\ w_{12} & w_{22} & w_{32} & w_{42} \\ w_{13} & w_{23} & w_{33} & w_{43} \end{bmatrix} & b^{[1]} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \\ (3 \times 1) & (3 \times 4) & \end{array}$$

Note : A and W cannot be multiplied directly.

$$z^{[L]} = W^{[L]T} * A^{[L-1]} + b^{[L]}$$

Using Transpose To Develop a Model for Neural Networks : Change \mathbf{W} to \mathbf{W}^T

$$\mathbf{A}^{[1]} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{W}^{[1]T} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \end{bmatrix} \quad \mathbf{b}^{[1]} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$\mathbf{z}^{[L]} = \mathbf{W}^{[L]T} * \mathbf{A}^{[L-1]} + \mathbf{b}^{[L]}$$

Using Transpose To Develop a Model for Neural Networks : **Plug it into expression.**

$$W^{[1]T} = \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

Vector(4x1)

Vector(4x3)

Vector(3x1)

Vector(4x1)

$$z^{[L]} = W^{[L]T} * A^{[L-1]} + b^{[L]} \text{ (Successfully computed !!)}$$

Example

```
In [21]: 1 W = np.random.rand(3,4)
          2 X = np.random.rand(3,1)
          3 b=  np.random.rand(4,1)
          4 c=  np.random.rand(3,1)
          5
          6 W,X,b
```

```
Out[21]: (array([[0.899255  , 0.52149056, 0.75988558, 0.46614012],
                  [0.24637199, 0.49076638, 0.83288382, 0.51380146],
                  [0.38120651, 0.16910539, 0.93104312, 0.54162068]]),
          array([[0.60389517],
                  [0.45503992],
                  [0.60113909]]),
          array([[0.91962447],
                  [0.39762239],
                  [0.68937966],
                  [0.69974358]]))
```

```
In [22]: 1 # Here W is 4X3 weight matrix.
          2 #Step1 : take transpose of W.
          3
          4 Y = np.matmul(W.T,X)+b
          5 Y
```

```
Out[22]: array([[1.80394745],
                  [1.03752217],
                  [2.0869527 ],
                  [1.54063289]])
```

Sanity Check (For each layer)

- Number of outputs = Number of neurons.
- $4 = 4$
- Number of Inputs = Number of Columns in W^T .
- $3 = 3$
- Number of outputs = Number of bias vector element.
- $4 = 4$
- Number of rows in W^T = Number of neurons.
- $4 = 4$

Inverse of Matrices

Invertible Matrices ?

*Simple **telltale signs** for invertible matrices !!*

- Matrix is full rank.
- The determinant of A is not zero.
- The matrix A has non-zero singular values.

Inverse of Matrices

- The inverse of a square matrix, sometimes called a reciprocal matrix, is a matrix such that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \text{ (Identity Matrix)}$$

the matrix inverse is

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

Inverse of Matrices

- $(AB)^{-1} = B^{-1}A^{-1}$
- $AA^{-1} = I$ (Important)
- $A^T = A^{-1}$, $A^TA = I$ (Important : **symmetric matrices**)
- Inverse of Diagonal Matrices

$$D^k = \begin{bmatrix} d_1^k & 0 & \dots & 0 \\ 0 & d_2^k & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & d_n^k \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 1/d_1 & 0 & \dots & 0 \\ 0 & 1/d_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1/d_n \end{bmatrix}$$

What is a Matrix?

- It is a transformation.
- Inverse will exist if a non singular matrix A has a unique matrix A^{-1} such that $AA^{-1} = I$.
- Relation between $A, A^{-1} \sim x, f(x)$.
- This concept is exploited in *change of basis transformation*.

Inverse of Matrices : Uses

Change of Basis :

- vector X has basis $u_1, u_2 \dots u_n$.
- vector Y has basis $v_1, v_2 \dots v_m$.
- Transformation A such that, $X = \sum x_i u_i$, $Y = \sum y_j v_j$

$$AX=Y$$

- vector X has basis $t_1, t_2 \dots t_n$.
- vector Y has basis $w_1, w_2 \dots w_m$.
- Transformation A' such that, $X = \sum x'_i t_i$, $Y = \sum y'_j w_j$

$$A'X' = Y'$$

Inverse of Matrices : Uses

- Change the each basis component to target component
- $t_i = \sum t_{ji} v_j$, $w_i = \sum w_{ji} u_j$
- Basis for $B_t = [t_1, t_2, \dots]$, $B_w = [w_1, w_2, \dots]$

$$A'X' = Y'$$

$$AB_t X' = B_w Y'$$

$$B_w^{-1} AB_t = A'$$

Example

A vector is rotated through α degrees. Find transformation matrix. (NND Matrix representation page 6.6)

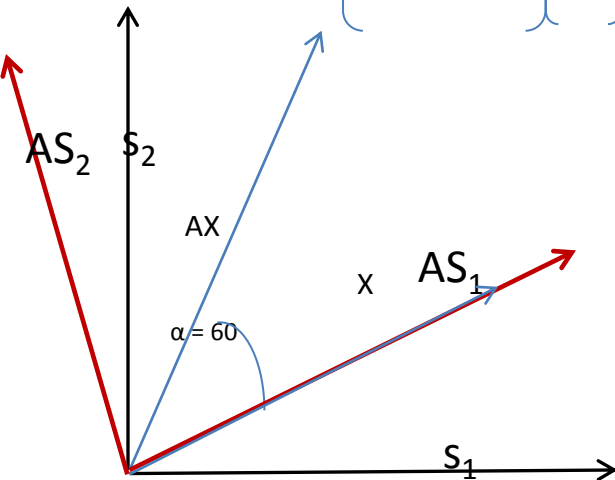
- X be a vector , AX be the transformed vector.
- If we rotate s_2 α degrees anticlockwise , transformation of s_2 .
- If we rotate s_1 α degrees anticlockwise , transformation of s_1 .

$$AS_1 = s_1 \cos(\alpha) + s_2 \sin(\alpha)$$

$$AS_2 = s_1 \cos(90+\alpha) + s_1 \sin(90+ \alpha)$$

Combining $AS = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$

Here Matrix is rotation transformation.



Inverse of Matrices : Uses

SVD

Eigen Values and Vectors

Examples

Eigen Values and Vectors :Uses

- Learning Rates

Example

Practical Examples