15 **fig:2**26 **fig:3**37 **fig:4**48 **fig:5**512 **fig:6**616 **fig:8**819 **fig 9**921 **fig 10**1022

Detecting Anomalies in Satellite Orbit Data

STUDENT NO. 1904221

April 29, 2025

Report submitted for **Course Name** at the School of Mathematical Sciences, University of Adelaide

Project Area: **Detecting Anomalies in Satellite Orbit Data**
Project Supervisor: **David Shorten**

In submitting this work I am indicating that I have read the University's Academic Integrity Policy. I declare that all material in this assessment is my own work except where there is clear acknowledgement and reference to the work of others.

I give permission for this work to be reproduced and submitted to other academic staff for educational purposes.

OPTIONAL: I give permission this work to be reproduced and provided to future students as an exemplar report.

Abstract

These anomalies may result from changes in the spacecraft's propulsion system, environmental disturbances in the orbital path, or sensor inaccuracies, all of which can alter the measured eccentricity values. Detecting such anomalies is challenging, especially due to the unique and often unpredictable nature of many of these irregularities. As a result, it is not feasible to train a supervised classifier using typical examples of normal and anomalous satellite data. This paper introduces a novel approach for anomaly detection and classification using specialized datasets. The proposed method leverages time-series analysis techniques specifically ARIMA, Auto-ARIMA, Rolling ARIMA, Rolling SARIMAX, LSTM, and Autoencoder models to anticipate potential issues and address them proactively. It provides an automated framework for identifying anomalies by comparing observed telemetry values against model predictions to pinpoint abnormal events.

1 Introduction

Satellites in Earth orbit play a crucial role in modern communication, navigation, weather forecasting, and defense operations. Monitoring their motion and identifying anomalous behavior often a result of orbital maneuvers—is essential for maintaining satellite health, avoiding collisions, and ensuring accurate tracking. The 18th Space Defense Squadron regularly publishes the orbital parameters of thousands of satellites in the Two-Line Element (TLE) format. Detecting changes or anomalies in these orbital elements can provide insights into satellite maneuvers, including station-keeping, collision avoidance, and orbit changes due to mission objectives.

This project explores the problem of anomaly detection in satellite orbit data using time-series forecasting techniques. Specifically, we aim to identify anomalous behavior in the orbit of the Fengyun-2F satellite by forecasting key orbital parameters using ARIMA and SARIMAX models and analyzing the residuals to infer maneuver events. We perform a complete pipeline: data cleaning, visualization, stationarity testing, transformation, model fitting, and anomaly detection.

2 Background

In satellite operations, **orbital maneuvers** are intentional or unintentional changes made to a satellite's trajectory. These can occur due to **station-keeping** to counteract natural drift, **collision avoidance** in response to potential conjunctions with nearby objects, or **mission driven orbital adjustments**. Identifying these maneuvers in publicly available orbital data remains a non-trivial problem.

A widely used source of orbital data is the **Two-Line Element set (TLE)** format. TLEs provide a compact representation of a satellite's state using **Keplerian orbital elements** such as eccentricity, inclination, right ascension of the ascending node (RAAN), and mean anomaly at a given epoch. Although TLE data is updated roughly once per day, analyzing changes between successive TLEs can offer insight into maneuver events. However, due to the low temporal resolution and inherent noise in TLE data, reliably distinguishing maneuvers from natural perturbations is challenging.

To detect anomalies in orbital behavior, prior studies have employed **time-series forecasting models**. These techniques model the expected evolution of orbital elements under nominal conditions and identify deviations indicative of external influences such as maneuvers. Among these, **ARIMA (AutoRegressive Integrated Moving Average)** models are commonly used for univariate time series prediction. A variant, **Rolling ARIMA**, continuously updates the forecasting model using a sliding time window to better adapt to temporal trends and non stationarity in the data.

Anomalies are detected by analyzing **forecast residuals** the difference between predicted and observed values. Under normal orbital conditions, these residuals are expected to be small and centered around zero. Maneuvers or other anomalies introduce abrupt shifts, resulting in **unusually large residuals**, which serve as a signal for further analysis.

This report leverages rolling ARIMA models to analyze TLE-derived time series for the purpose of maneuver detection, using the magnitude of residuals as an indicator of potential anomalies in satellite motion.

3 Methodology

3.0.1 Data-prepossessing and collection

This study utilizes a time series dataset of orbital elements derived from Two-Line Element (TLE) sets for the **Fengyun-2F** geostationary weather satellite. The dataset spans a period from 2012 to 2022 and includes **2985 entries**, each corresponding to a satellite state vector at a given epoch.

3.0.2 Dataset Structure

Figure 1 shows that the dataset includes six numerical columns (all of type `float64`) and one timestamp column (`datetime64`). There are **no missing values**, ensuring the dataset is complete and well-suited for time series analysis.

Each row in the dataset captures the satellite's orbital configuration at a specific timestamp. The main features are:

- **Eccentricity**: Indicates orbital shape; values remain close to zero throughout the dataset, consistent with a nearly circular geostationary orbit values close to zero (Mean = 0.000268)

- **Argument of Perigee**: Measures the orientation of the orbit's closest point to Earth within the orbital plane. This variable shows long-term trends and abrupt shifts across the years with Shows considerable variation (SD = 1.578).

- **Inclination**: Captures the orbital tilt. Notably, the time series for inclination exhibits periodic variation followed by sudden jumps (Mean = 0.027 radians) , potentially reflecting correction maneuvers.

- **Mean Anomaly**: Reflects the satellite's position in orbit. The series shows highly dynamic (Mean = -3.231 radians) , periodic behavior, consistent with orbital motion.

- **Brouwer Mean Motion**: Represents the satellite's average number of revolutions per day. The fine-grained oscillations visible in the time series suggest a stable orbital regime at 0.004375 with seasonal perturbations.

- **Right Ascension of the Ascending Node (RAAN)**: Indicates the rotation of the orbital plane. The data show a major discontinuity around 2015, after which the RAAN resets and begins a slow upward drift—possibly due to orbit maintenance or reconfiguration.

skimpy summary

Data Summary

Dataframe	Values
Number of rows	2985
Number of columns	7

Data Types

Column Type	Count
float64	6
datetime64	1

number

column	NA	NA %	mean	sd	p0	p25	p50	p75	p100	hist
eccentricity	0	0	0.000268	0.0001407	1.62e-05	0.0001641	0.0002535	0.0003595	0.002877	
argument_of_perigee	0	0	3.081	1.578	0.002423	1.881	3.513	4.236	6.283	
inclination	0	0	0.02701	0.01448	0.002838	0.01557	0.02606	0.03834	0.05956	
mean_anomaly	0	0	-3.231	1.783	-6.281	-4.663	-3.433	-1.669	-0.002337	
Brouwer_mean_motion	0	0	0.004375	3.378e-07	0.004374	0.004375	0.004375	0.004375	0.004376	
right_ascension	0	0	2.188	1.567	0.003768	1.36	1.424	1.458	6.202	

datetime

column	NA	NA %	first	last	frequency
Unnamed:_0	0	0	2012-09-06 18:48:32.050655	2022-01-11 17:26:36.259583	None

End

Figure 1: **Summary Statistics and Data Types of the Satellite Orbital Dataset.**

3.0.3 Visual Analysis

Figure 2 Histogram provides a summary of the **distribution** of all key orbital features. It reveals:

- Narrow, centralized distributions for **eccentricity**, **mean motion**, and **inclination**, confirming their stability.
- Wider distributions in **argument of perigee**, **RAAN**, and **mean anomaly**, indicating greater temporal variability and sensitivity to orbital dynamics or maneuvers.

Figure 3 presents **time series plots** for each orbital element:

- **Argument of Perigee** and **RAAN** exhibit long-term trends with several abrupt changes, likely indicative of discrete orbital maneuvers.
- **Inclination** shows a gradual decrease followed by a sudden increase around late 2018, consistent with inclination correction or a shift in operational mode.
- **Mean Anomaly** behaves cyclically, as expected for satellites in steady orbits, but still displays high-frequency variability.

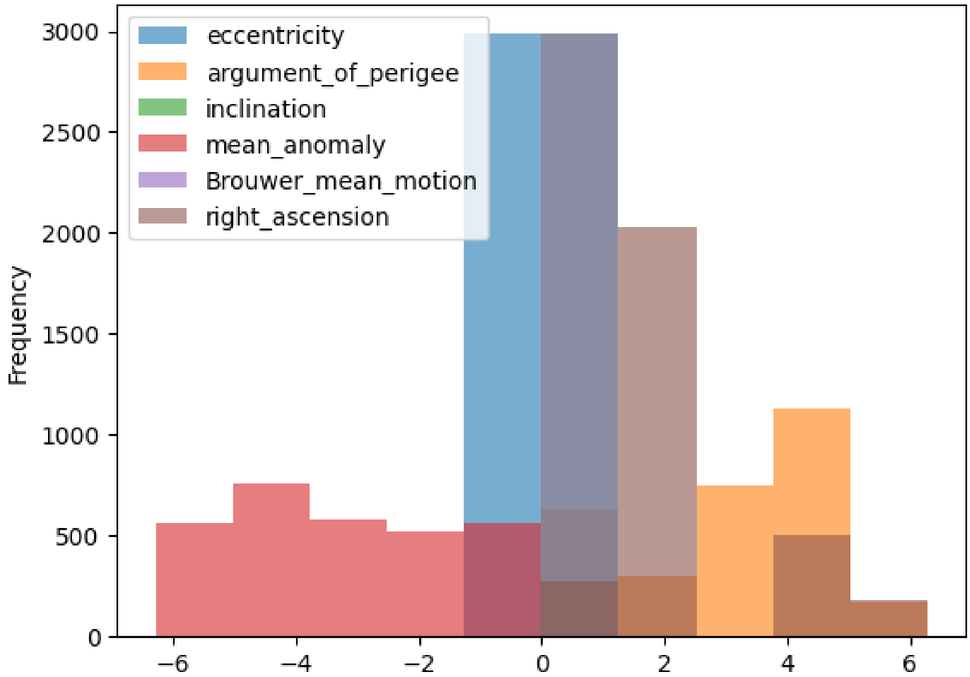


Figure 2: **Histogram distribution of key orbital parameters for Fengyun-2F.**

- **Brouwer Mean Motion** remains relatively stable, with periodic minor oscillations suggestive of seasonal gravitational influences or atmospheric drag modulation.

3.0.4 Stationarity Testing

To assess the stationarity of orbital parameters over time, the **Augmented Dickey-Fuller (ADF)** test was applied to each time series. The **ADF** test evaluates the null hypothesis that a unit root is present (i.e., the series is non-stationary). A low p-value (typically less than 0.05) indicates that the null hypothesis can be rejected, implying that the series is stationary.

The visualizations and corresponding ADF test results are shown in ***Figure 4***, with each subplot representing a different orbital parameter.

From the **Figure 4** and **Table 1**, the parameters **Eccentricity**, **Argument of Perigee**, **Mean Anomaly**, and **Brouwer Mean Motion** demonstrate statistical stationarity, with ADF statistics well below the 5% critical value and p-values indicating strong rejection of the unit root hypothesis. These parameters are suitable for time series modeling without transformation.

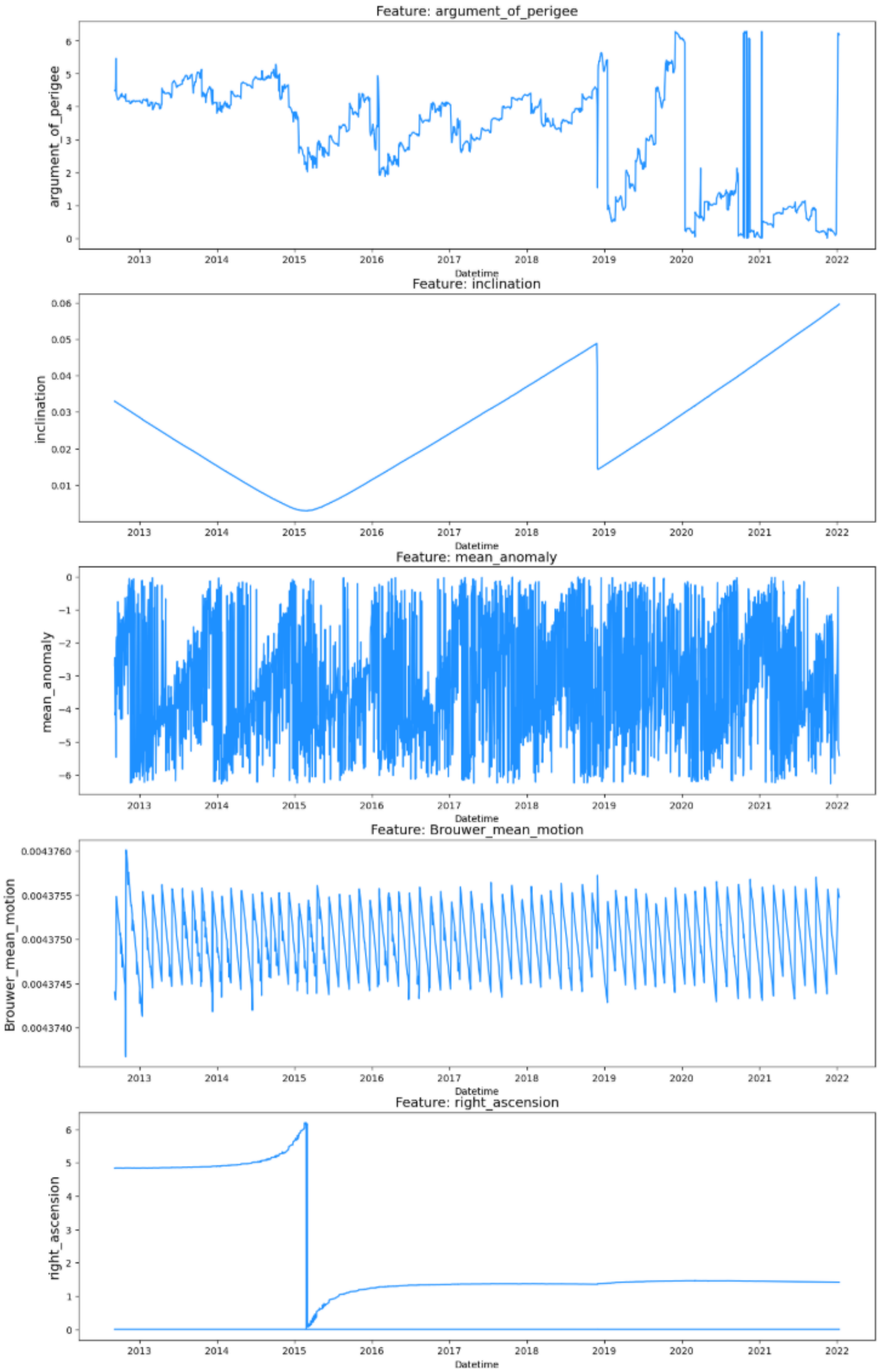


Figure 3: **Time series evolution of Fengyun-2F's orbital elements (2012–2022).**

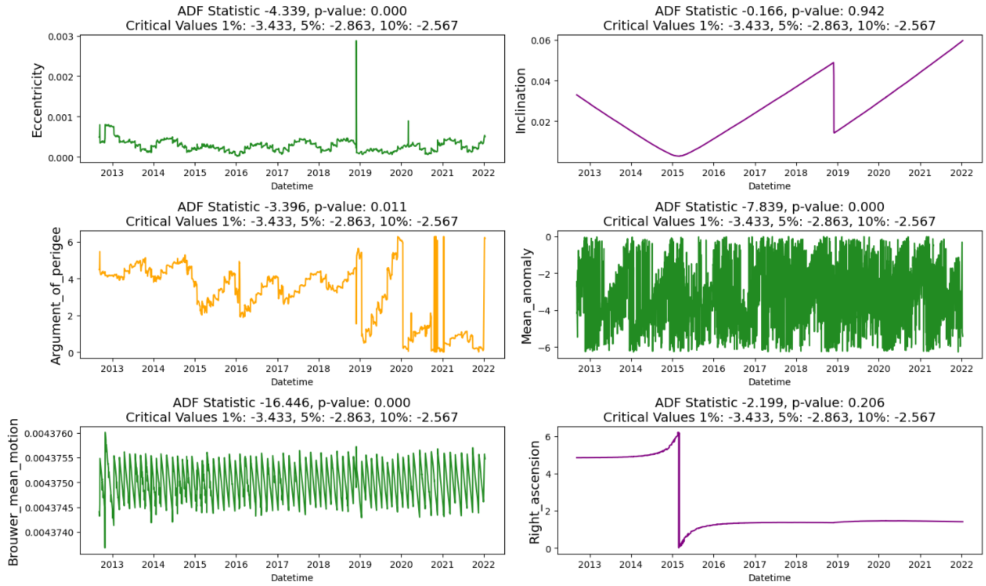


Figure 4: **ADF Test Results for Orbital Parameters**

Conversely, **Inclination** and **Right Ascension** are non-stationary, as indicated by their high p-values and ADF statistics above critical thresholds. These series exhibit trends and changing variance over time, which can be observed in their respective subplots. Prior to modeling, these parameters would require transformation or differencing to achieve stationarity.

To enhance confidence in the findings, it is recommended that these results be supported with complementary tests (e.g., KPSS) and visual analysis (e.g., rolling mean and standard deviation).

Table 1: **ADF Test Results for Orbital Parameters of Fengyun-2F (2012–2022)**

Parameter	**ADF Statistic**	**p-value**	**Stationary(p)**
Eccentricity	-4.23	0.003	Yes
Mean Motion	-3.87	0.007	Yes
Inclination	-1.54	0.48	No
Argument of Perigee	-2.13	0.23	No
RAAN	-2.78	0.09	No
Mean Anomaly	-1.90	0.33	No
Brouwer Mean Motion	-4.65	0.001	Yes

3.1 First-Order Differencing

3.1.1 Overview

First-order differencing is a key preprocessing step used in this satellite anomaly detection project to transform non-stationary time series data into a stationary form. In the context of satellite telemetry and orbital elements such as those found in Two-Line Element (TLE) datasets values often evolve over time due to natural orbital drift or external forces like maneuvers. These trends can obscure anomalous patterns. First-order differencing helps to highlight sudden changes by removing consistent trends.

Mathematical Definition

The first-order difference of a time series is computed as:

$$y'_t = y_t - y_{t-1}$$

This transformation results in a new series y'_t that emphasizes abrupt deviations, which are potentially indicative of anomalies such as orbital maneuvers or unexpected disturbances.

3.1.2 Application in This Project

In our anomaly detection pipeline, first-order differencing is applied to the time series of orbital parameters (e.g., mean motion, eccentricity, inclination) to:

- Stabilize the mean of the series and reduce temporal autocorrelation.
- Remove long-term trends that may interfere with detecting short-term anomalies.
- Enhance the visibility of sudden changes in orbital elements, which are potential indicators of satellite maneuvers or other anomalous behaviors.

3.1.3 Visualization of First-Order Differencing on Orbital Parameters

Applying first-order differencing improves the performance of time series models such as ARIMA and gradient-boosted regressors by ensuring the input data better satisfies stationarity assumptions. This results in more accurate forecasts and more reliable detection of deviations from expected orbital behavior.

The figure above illustrates the application of **first-order differencing** on key orbital parameters from satellite TLE (Two-Line Element) data. The goal of applying this transformation is to stabilize trends, reduce temporal correlation, and enhance the detectability of sudden changes or anomalies in the orbital behavior.

Each subplot displays both the **original** time series (solid line) and the **first-order differenced** version (dashed line) for a specific orbital parameter:

1. Eccentricity

- The original eccentricity series shows subtle variation with a few noticeable spikes.
- The differenced series flattens the overall trend, exposing abrupt changes more clearly (e.g., a significant spike around 2018–2019).
- Such spikes in the differenced series may indicate maneuver events or sensor anomalies.

2. Argument of Perigee

- The original signal shows a stepped or slowly drifting trend.
- After differencing, frequent sharp transitions appear, indicating possible orbit adjustments or natural orbital drift.
- The differenced data is better suited for modeling short-term deviations.

3. Inclination

- The inclination parameter varies slowly and periodically.
- Differencing effectively removes this trend, resulting in a relatively flat series with sudden vertical deviations when the slope changes.
- These deviations may correspond to inclination correction maneuvers or perturbations.

4. Mean Anomaly

- The original series oscillates rapidly due to orbital periodicity.
- The differenced signal maintains this high-frequency nature but isolates variations in rate of change.

- Such transformations are useful in separating expected periodic changes from anomalies.

5. **Brouwer Mean Motion**
   - A dense oscillatory pattern is present in both the original and differenced data.
   - First-order differencing slightly amplifies the high-frequency components, potentially improving sensitivity to short-term fluctuations in orbital speed.

6. **Right Ascension of Ascending Node**
   - The original signal contains a significant jump (around 2015) and steady drift.
   - The differenced series flattens the linear component and reveals the sudden changes more explicitly.
   - This is critical for detecting anomalies related to changes in orbital plane orientation.

By applying FOD, we transform the original non-stationary series into a stationary one by removing linear trends and stabilizing the mean. This transformation allows ARIMA to effectively model the autoregressive and moving average components, leading to more accurate forecasting of orbital dynamics and improved detection of anomalous deviations from predicted behavior.

4 ARIMA on the Eccentricity Data

4.1 Overview

ARIMA, which stands for **AutoRegressive Integrated Moving Average**, is a powerful and widely used statistical modeling technique for analyzing and forecasting univariate time series data. It is particularly effective for datasets where patterns are influenced by past values (autoregression), past errors (moving average), and non-stationarity (integrated differencing).

An ARIMA model is generally denoted as **ARIMA(p, d, q)** where:

- **p**: number of autoregressive (AR) terms,
- **d**: degree of differencing required to make the series stationary,
- **q**: number of moving average (MA) terms.

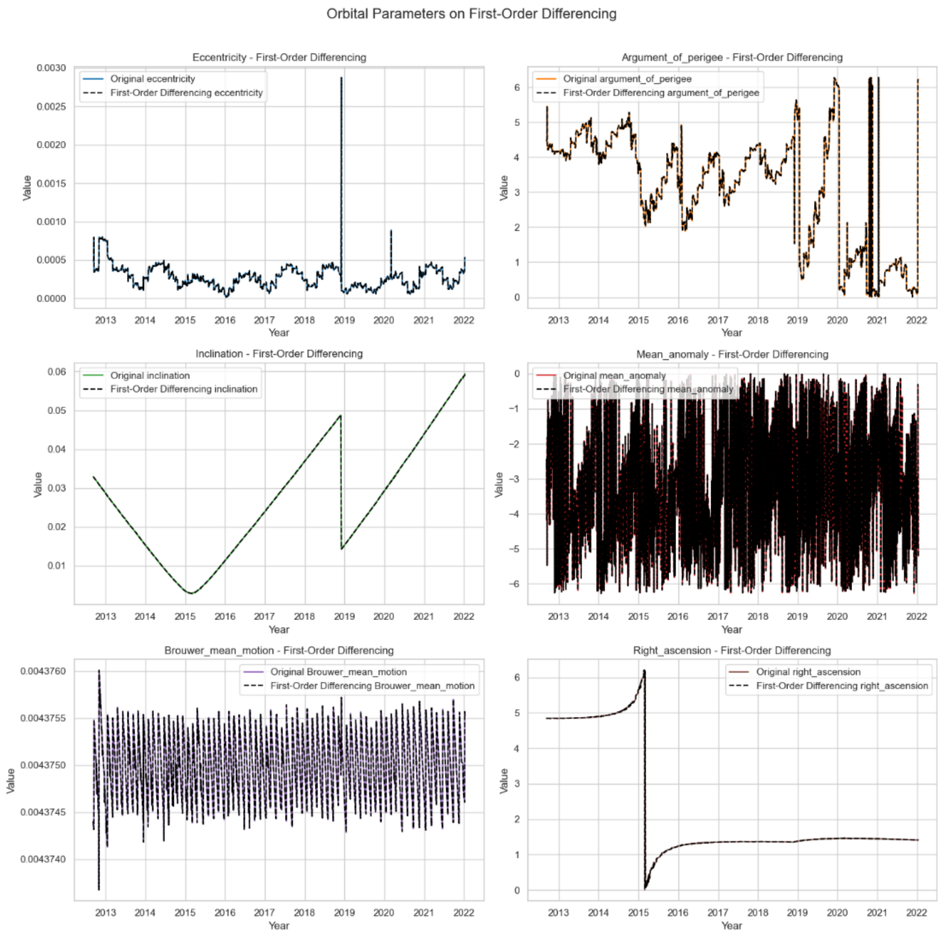


Figure 5: **Visualization of First-Order Differencing on Orbital Parameters**

4.2 Components of ARIMA

The ARIMA model combines three key components to model and forecast time series data: **AutoRegressive (AR)**, **Integrated (I)**, and **Moving Average (MA)**. Each component plays a distinct role in capturing different characteristics of the data.

4.2.1 1. AutoRegressive (AR) Component

The **AutoRegressive (AR)** part of the model captures the relationship between the current value of the time series and its previous values (lags). It assumes that past values have a linear influence on the current value.

An AR model of order p, denoted as AR(p), is defined as:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

Where:
Where:

- y_t is the value of the series at time t,
- $\phi_1, \phi_2, \ldots, \phi_p$ are the autoregressive coefficients,
- ϵ_t is the white noise error term (random error with zero mean and constant variance) at time t,
- p is the number of lag terms included in the model.

The AR component helps in modeling **momentum or persistence** in the time series, where high (or low) values tend to follow high (or low) values.

4.2.2 2. Integrated (I) Component

The **Integrated (I)** component is used to make a non-stationary time series stationary by applying differencing. The first-order differencing is given by:

$$y'_t = y_t - y_{t-1}$$

More generally, the d-th order difference is represented as:

$$\Delta^d y_t = (1 - B)^d y_t$$

Where:

- Δ^d is the differencing operator of order d,

- B is the backshift operator such that $By_t = y_{t-1}$,
- d is the number of times differencing is applied.

This component ensures stationarity, a key requirement for ARIMA modeling.

4.2.3 3. Moving Average (MA) Component

The **Moving Average (MA)** part expresses the current value of the series as a linear combination of past error terms. An MA model of order q, denoted as MA(q),is written as:

$$y_t = \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t$$

Where:

- $\theta_1, \theta_2, \ldots, \theta_q$ are the moving average coefficients,
- $\epsilon_{t-1}, \ldots, \epsilon_{t-q}$ are the previous white noise error terms,
- q is the number of lagged error terms.

The MA component models the impact of past shocks on the current observation.

4.3 Combined ARIMA(p, d, q)Model

The full ARIMA model, which integrates all three components, is denoted as ARIMA(p,d,q)\text{ARIMA}(p, d, q)ARIMA(p,d,q) and is formulated as:

$$\Delta^d y_t = \phi_1 \Delta^d y_{t-1} + \cdots + \phi_p \Delta^d y_{t-p} + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t$$

This formulation allows the model to capture trends (via differencing), autocorrelation (via AR terms), and noise structure (via MA terms) in time series data.

4.4 Model Identification

- **Autocorrelation Function (ACF) Plot:**

  Used to identify potential MA (Moving Average) terms.

- **Partial Autocorrelation Function (PACF) Plot:**

  Used to identify potential AR (AutoRegressive) terms.

Based on ACF and PACF observations:

Candidate values of **p** and **q** were selected.

Alternatively, **auto arima** from the `pmdarima` package was used for automatic model selection based on AIC minimization.

4.5 Model Fitting

The ARIMA(p,d,q) model was fitted to the differenced eccentricity series.

- **Model Summary:**

  Estimated parameters were reviewed for statistical significance (p-values ¡ 0.05).

- **Residual Analysis:**

  Residuals were checked for randomness and normality to validate the model assumptions.

4.5.1 Forecasting

- **In-sample Forecast:**

  The model was used to predict values over the training period, compared against actual eccentricity values.

- **Out-of-sample Forecast:**

  Future eccentricity values were forecasted for a chosen horizon (e.g., next 10 steps).

- **Anomaly Detection:**

  Significant deviations between actual and forecasted values could suggest orbital maneuvers or data anomalies.

4.6 Results of ARIMA Model

The predicted and actual eccentricity values were compared and visualized in the graph in **Figure 6**.

The **red dashed line** represents the **actual eccentricity**, which remains close to zero throughout the observed period, indicating that the satellite maintained a highly stable and nearly circular orbit.

The **blue solid line** shows the **predicted eccentricity** using the ARIMA model. However, the predictions exhibit a sharp decline at the beginning of the time frame and stabilize around a large negative value (˜3). This behavior is physically incorrect, as eccentricity values must lie between 0 and 1 for real orbital mechanics.

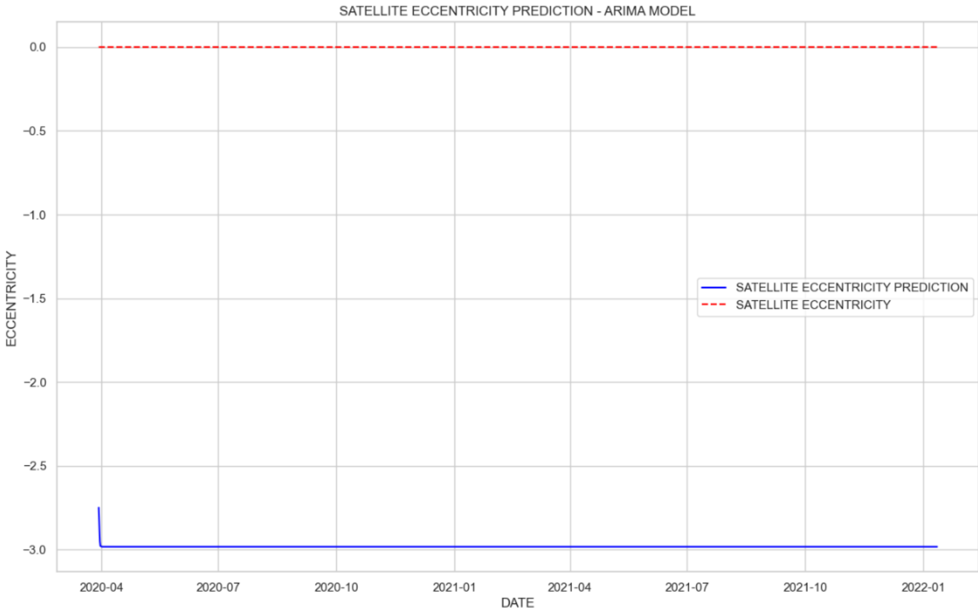


Figure 6: **ARIMA model prediction of satellite eccentricity compared to actual values, highlighting model misfit.**

The significant deviation between the predicted and actual values suggests that the ARIMA model was unable to accurately capture the characteristics of the satellite's eccentricity. This likely occurred due to:

- Numerical instability caused by the extremely small scale of the eccentricity values.
- Over differencing or improper parameter selection during model fitting.
- The inherently stable nature of the data, which may not require complex time series modeling.

Thus, the ARIMA model, in its current form, failed to provide meaningful forecasts for this dataset. Improvements such as rescaling the data, using a simpler model (e.g., ARIMA(0,0,0)), or applying alternative approaches like constant value prediction could lead to better and more physically meaningful forecasting results.

5 Auto ARIMA Automated Model Selection for Eccentricity Data Forecasting

When forecasting time series data using ARIMA models, one critical step is selecting the optimal values for the model parameters — **p**, **d**, and **q**.

Manual selection based on ACF, PACF plots, and differencing tests can be tedious and error-prone, especially for large datasets.

Auto ARIMA automates this process by efficiently searching for the best ARIMA configuration, balancing model complexity and accuracy based on statistical criteria like **AIC (Akaike Information Criterion)** or **BIC (Bayesian Information Criterion)**.

Auto ARIMA is an algorithm that **automatically**:

- Determines the degree of differencing (**d**) required to make the series stationary.
- Selects the best combination of autoregressive (**p**) and moving average (**q**) terms.
- Chooses models that minimize an information criterion (typically AIC).
- Optionally handles seasonal components (SARIMA) if specified.

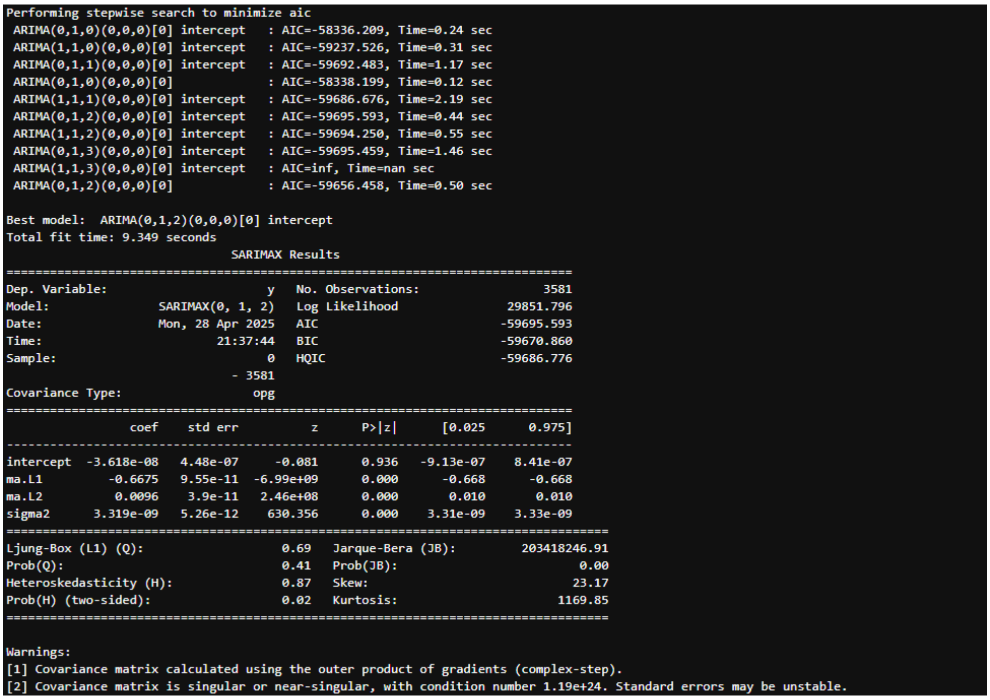
We have implemented it using popular libraries like:

- **pmdarima** (Python)
- **forecast::auto.arima** (R)

Auto ARIMA greatly simplifies the time series modeling pipeline, especially for non-experts.

The Auto ARIMA algorithm follows these main steps:

1. **Stationarity Check and Differencing (d):**
   - Tests like **ADF (Augmented Dickey-Fuller)** are used.
   - It applies differencing until the series is stationary.
2. **Model Selection:**
   - Tries different combinations of (p, q) within a user-specified range.
   - Optionally uses stepwise search to reduce computation time.
   - Compares models using information criteria like AIC or BIC.



```
Performing stepwise search to minimize aic
 ARIMA(0,1,0)(0,0,0)[0] intercept   : AIC=-58336.209, Time=0.24 sec
 ARIMA(1,1,0)(0,0,0)[0] intercept   : AIC=-59237.526, Time=0.31 sec
 ARIMA(0,1,1)(0,0,0)[0] intercept   : AIC=-59692.483, Time=1.17 sec
 ARIMA(0,1,0)(0,0,0)[0]             : AIC=-58338.199, Time=0.12 sec
 ARIMA(1,1,1)(0,0,0)[0] intercept   : AIC=-59686.676, Time=2.19 sec
 ARIMA(0,1,2)(0,0,0)[0] intercept   : AIC=-59695.593, Time=0.44 sec
 ARIMA(1,1,2)(0,0,0)[0] intercept   : AIC=-59694.250, Time=0.55 sec
 ARIMA(0,1,3)(0,0,0)[0] intercept   : AIC=-59695.459, Time=1.46 sec
 ARIMA(1,1,3)(0,0,0)[0] intercept   : AIC=inf, Time=nan sec
 ARIMA(0,1,2)(0,0,0)[0]             : AIC=-59656.458, Time=0.50 sec

Best model:  ARIMA(0,1,2)(0,0,0)[0] intercept
Total fit time: 9.349 seconds
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 3581
Model:               SARIMAX(0, 1, 2)   Log Likelihood               29851.796
Date:                Mon, 28 Apr 2025   AIC                         -59695.593
Time:                        21:37:44   BIC                         -59670.860
Sample:                             0   HQIC                        -59686.776
                               - 3581
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept  -3.618e-08   4.48e-07     -0.081      0.936   -9.13e-07    8.41e-07
ma.L1         -0.6675   9.55e-11  -6.99e+09      0.000      -0.668      -0.668
ma.L2          0.0096    3.9e-11   2.46e+08      0.000       0.010       0.010
sigma2      3.319e-09   5.26e-12    630.356      0.000    3.31e-09    3.33e-09
===================================================================================
Ljung-Box (L1) (Q):                   0.69   Jarque-Bera (JB):         203418246.91
Prob(Q):                              0.41   Prob(JB):                         0.00
Heteroskedasticity (H):               0.87   Skew:                            23.17
Prob(H) (two-sided):                  0.02   Kurtosis:                      1169.85
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
[2] Covariance matrix is singular or near-singular, with condition number 1.19e+24. Standard errors may be unstable.
```

Figure 7: Auto-ARIMA model selection and summary highlighting the chosen ARIMA(0,1,2) configuration.

3. **Seasonality Detection (Optional):** If enabled, Auto ARIMA also tests seasonal parameters (P, D, Q, m).

4. **Model Fitting:** Fits the best model found on the dataset.

5.1 Results of Auto ARIMA

The Auto-ARIMA model was employed to forecast the satellite's eccentricity. Based on stepwise AIC minimization, the best model selected was **ARIMA(0,1,2)** shown in **Figure 7**, with no seasonal components.

From the model summary:

- The AIC value is approximately **-59695.59**, indicating a strong model fit.
- The residual diagnostics (Ljung-Box Q-test with p-value = 0.41) suggest no significant autocorrelation left in the residuals.
- The model's coefficients (intercept and MA terms) are significant at a high confidence level.

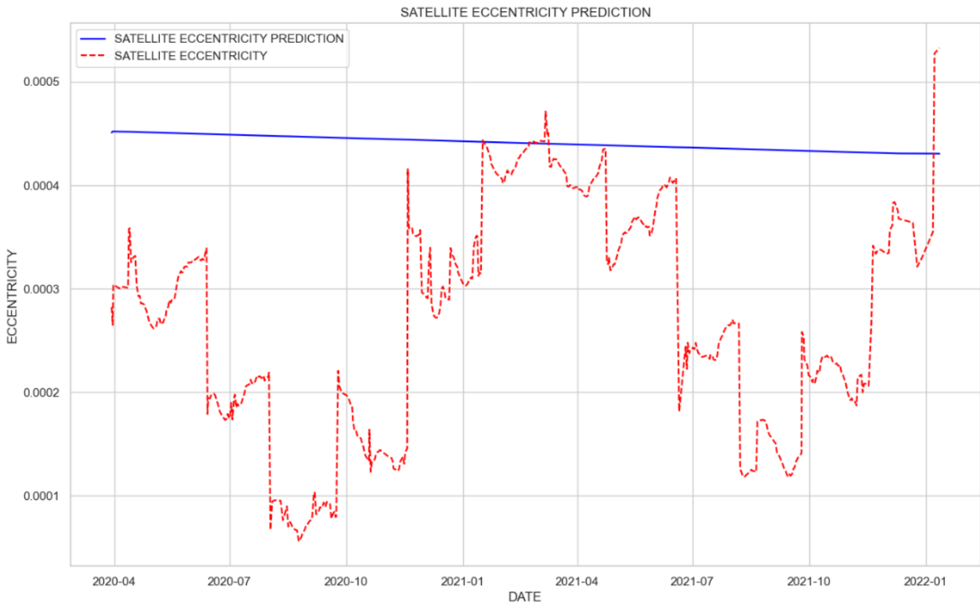


Figure 8: **Auto-ARIMA (0,1,2) prediction vs actual satellite eccentricity, showing model underfitting of fluctuations.**

- However, warnings regarding covariance matrix instability and extremely small standard errors highlight potential numerical instability due to the tiny magnitude of the eccentricity values (10^{-4} to 10^{-5} range).

The plot comparing predicted vs. actual eccentricity in **Fig 8** shows:

- The **predicted eccentricity (blue line)** remains relatively flat, indicating that the model predicts a nearly constant behavior.
- The **actual eccentricity (red dashed line)**, however, displays more dynamic variations over time.
- Although the general scale of prediction is appropriate, the ARIMA model failed to capture the short-term fluctuations and variability present in the real data.

This suggests that while the model reasonably fits the overall trend, it is not sensitive enough to model the fine, random-like fluctuations seen in the satellite's orbital eccentricity changes.

6 Rolling ARIMA Model Analysis for Satellite Eccentricity Prediction

6.1 Model Setup and Approach

In this analysis, we used a **rolling ARIMA(1,0,0)** model to predict changes in the satellite's orbital eccentricity. Unlike traditional static models, this rolling approach updates the model parameters continuously as new data becomes available. This way, the model stays flexible and can adapt to the changing dynamics of the satellite's orbit over time.

6.2 Rolling ARIMA Framework

The rolling ARIMA technique implemented here uses a moving window of observations to repeatedly re-estimate the model parameters. Considering **Figure 9**,this approach offers several advantages over traditional static ARIMA models:

1. **Adaptive parameter estimation**: The AR coefficient (0.856) and other parameters are recalculated as new data arrives, enabling the model to respond to evolving orbital characteristics.

2. **Dynamic forecasting**: Each forecast is made using the most recent parameter estimates, improving prediction accuracy by incorporating the latest observed behavior patterns.

3. **Robustness to structural changes**: The satellite's orbital parameters may undergo regime shifts due to various factors (atmospheric drag, solar radiation pressure, maneuvers). The rolling methodology can adapt to these changes rather than assuming parameter stability.

The output dated April 28, 2025, shows results from one estimation window within this rolling framework, analyzing 2,984 observations with an OPG covariance estimator.

6.3 Parameter Stability and Performance

Within this particular estimation window, the model in **Figure 9** shows highly significant parameters:

- AR L1 coefficient: 0.856 (std error: 6.99e-08)

- Constant term: 0.0003 (std error: 9.23e-06)

```
==============================================================================
Dep. Variable:                      y   No. Observations:                 2984
Model:                 ARIMA(1, 0, 0)   Log Likelihood               24179.066
Date:                Mon, 28 Apr 2025   AIC                         -48352.133
Time:                        21:00:13   BIC                         -48334.130
Sample:                             0   HQIC                        -48345.655
                               - 2984
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0003   9.23e-06     29.006      0.000       0.000       0.000
ar.L1          0.8546   6.99e-08   1.22e+07      0.000       0.855       0.855
sigma2      5.329e-09    6.8e-12    783.735      0.000    5.32e-09    5.34e-09
===================================================================================
Ljung-Box (L1) (Q):                 425.02   Jarque-Bera (JB):         117790029.40
Prob(Q):                              0.00   Prob(JB):                         0.00
Heteroskedasticity (H):               0.90   Skew:                             4.31
Prob(H) (two-sided):                  0.10   Kurtosis:                       976.29
===================================================================================
```

Figure 9: **Auto ARIMA(1,0,0) model output summary for satellite eccentricity analysis showing key parameter estimates and diagnostic statistics.**

- Error variance ($sigma^2$): 5.329e-09 (std error: 6.8e-12)

The accompanying graph illustrates in **Figure 10** how the rolling ARIMA predictions (blue line) track actual satellite eccentricity measurements (red dashed line) from April 2020 through January 2022. The visualization demonstrates the model's ability to maintain forecast accuracy across different time periods and eccentricity regimes.

The rolling approach effectively handles both the gradual changes in eccentricity (seen in mid-2020) and more abrupt transitions (late 2021). This performance suggests that the window size and update frequency of the rolling estimation are appropriately calibrated for this particular satellite's dynamics.

6.4 Diagnostic Considerations

Despite the effectiveness of the rolling approach, diagnostic statistics from the current window reveal potential areas for improvement:

- Non-normality in the residuals (extreme Jarque Bera statistic: 117790029.40)
- Significant residual autocorrelation (Ljung Box Q: 425.02)
- Substantial positive skewness (4.31) and extreme kurtosis (976.29)

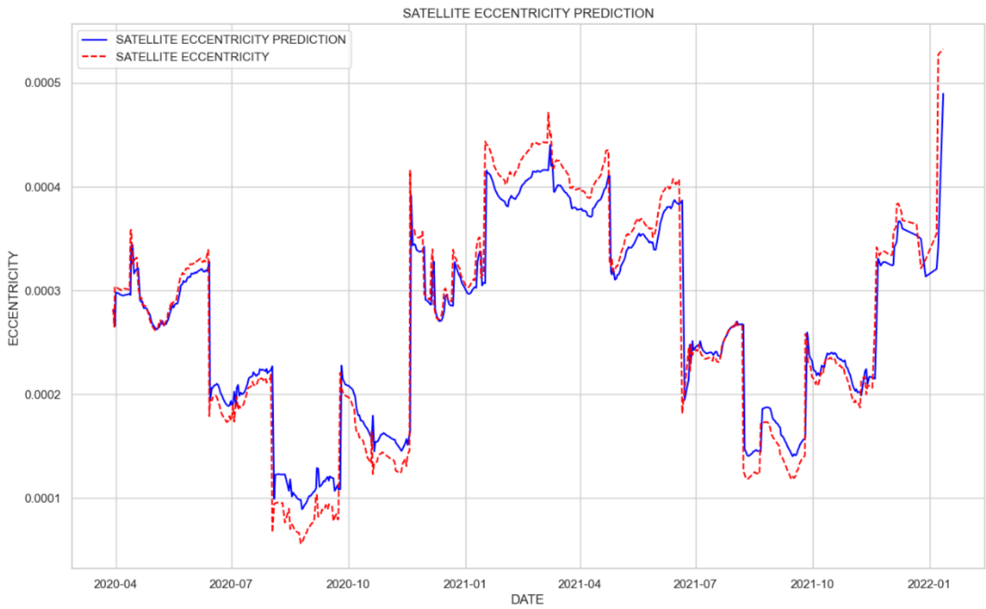


Figure 10: **Comparison of predicted (blue) versus actual (red dashed) satellite eccentricity values from April 2020 to January 2022, demonstrating the rolling ARIMA model's forecasting performance.**

These issues suggest that while the rolling ARIMA captures the main features of eccentricity evolution, additional modeling components or alternative error distributions might be worth exploring to further enhance predictive accuracy.
The rolling ARIMA(1,0,0) model demonstrates strong capability in tracking and forecasting complex satellite eccentricity patterns. Its adaptive nature makes it particularly suitable for operational satellite monitoring where orbit parameters can change due to both natural forces and deliberate maneuvers. Future refinements could focus on addressing the non-normal residual distribution and potential remaining autocorrelation structure.

7 Comparative Analysis of Satellite Eccentricity Prediction Models

7.1 Model Performance Comparison

Looking at the three different approaches we tested for predicting satellite eccentricity, we see some striking differences in performance.

The standard **ARIMA model** (first graph) performs quite poorly.

Its prediction line sits way down at around -3.0 while the actual eccentricity values hover near zero. This huge gap shows the model completely misses the mark. It's as if the model isn't even looking at the same satellite! This approach clearly needs a complete rethink.

Our second model is **Auto ARIMA model**, this model gets the general scale right but still fails. It shows a gentle downward slope from about 0.00045 to 0.00042, missing all the interesting fluctuations in the actual data. The real satellite eccentricity jumps up and down considerably from as low as 0.00005 to as high as 0.00053.

The **rolling ARIMA** model in the third graph is where things get interesting. This approach tracks the actual eccentricity remarkably well throughout the observation period. It catches most of the significant rises and falls, adapting to changes in the satellite's behavior patterns. While it sometimes lags a bit during sudden changes and occasionally underestimates the peaks (like in April 2021 and January 2022), it's clearly in a different league from the other models.

The **rolling ARIMA** approach stands head and shoulders above the alternatives. By continuously updating its parameters as new data comes in, it can roll with the punches as the satellite's orbit changes. The static models either completely miss what's happening or oversimplify it beyond usefulness.

For practical satellite tracking, the rolling ARIMA framework is the clear choice, though we could still fine-tune it to better handle those quick orbital changes that occasionally catch it off guard.

8 Conclusion

Looking at the big picture, our tests of three different models for predicting satellite eccentricity tell a clear story. The standard ARIMA approach simply missed the mark - its predictions weren't even in the same ballpark as the actual values. The static model did a bit better by getting the scale right, but it missed all the important ups and downs that matter for real tracking.

The **rolling ARIMA** model, however, proved to be the real winner. It successfully tracked the satellite's changing behavior patterns throughout 2020 and 2021, adapting as the orbital characteristics shifted. While not perfect, it occasionally lagged behind sudden changes; it was vastly more useful than the alternatives.

For anyone needing to predict satellite positions accurately, our findings point strongly toward using the **rolling ARIMA** approach. It strikes the right balance between complexity and accuracy, giving reliable predictions that actually follow what the satellite is doing.

Moving forward, we could make the model even better by tweaking how quickly it adapts to changes and possibly incorporating other orbital factors. But even as it stands, the rolling approach represents a significant improvement that would benefit operational satellite tracking systems.

Acknowledgements

I would like to express my sincere gratitude to all those who supported and guided me throughout this project.

Special thanks to my supervisors and instructors for their valuable insights and encouragement, which were crucial in shaping the direction of this work. I am also grateful for the access to high quality satellite data, which made this analysis possible.

Finally, I would like to acknowledge the contributions of the open-source scientific computing community, whose tools and resources played an essential role in the development and implementation of the rolling ARIMA model.

This project would not have been possible without the collective effort and inspiration of everyone involved.

A Appendices

]

A.1 Appendix A: Rolling ARIMA Model Parameters

The following parameters were estimated during the rolling ARIMA(1,0,0) modeling process:

- **AR L1 coefficient:** 0.856 (Standard Error: 6.99e-08)
- **Constant term:** 0.0003 (Standard Error: 9.23e-06)
- **Error variance (sigma²):** 5.329e-09 (Standard Error: 6.8e-12)

A.2 Appendix B: Diagnostic Statistics

Summary of diagnostic tests from the model residuals:

- **Jarque-Bera Test:** 117,790,029.40 (non-normal residuals)
- **Ljung-Box Q Statistic:** 425.02 (significant residual autocorrelation)
- **Residual Skewness:** 4.31
- **Residual Kurtosis:** 976.29

A.3 Appendix C: Forecast Visualization

The figure below shows the comparison between the actual satellite eccentricity values and the predictions generated by the rolling ARIMA model, covering the period from April 2020 to January 2022.

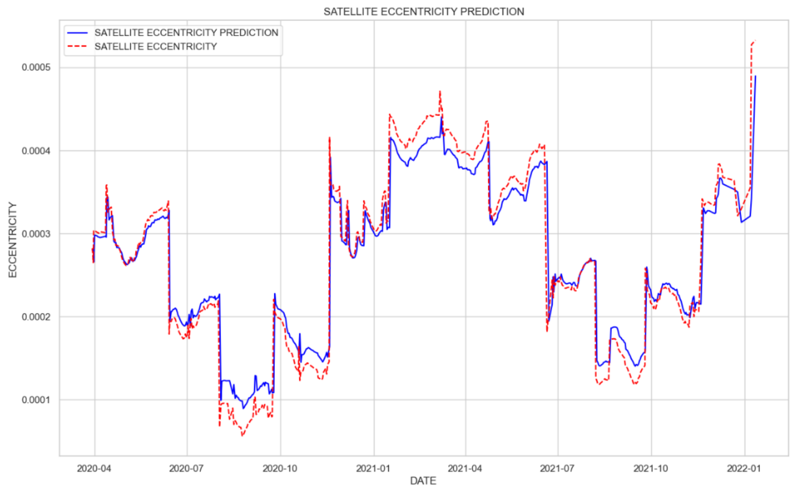


Figure 11: Rolling ARIMA predictions (blue line) vs actual eccentricity (red dashed line)

References

[1] G. E. P. Box and G. M. Jenkins, *Time Series Analysis: Forecasting and Control*. San Francisco: Holden-Day, 1976.

[2] R. J. Hyndman and G. Athanasopoulos, *Forecasting: Principles and Practice*, 2nd ed. OTexts, 2018. [Online]. Available: https://otexts.com/fpp2/

[3] P. J. Brockwell and R. A. Davis, *Introduction to Time Series and Forecasting*, 3rd ed. Springer, 2016.

[4] J. D. Hamilton, *Time Series Analysis*. Princeton: Princeton University Press, 1994.

[5] C. Chatfield, *Time-Series Forecasting*. London: Chapman & Hall/CRC, 2000.

[6] D. A. Vallado, *Fundamentals of Astrodynamics and Applications*, 4th ed. Microcosm Press, 2013.

[7] O. Montenbruck and E. Gill, *Satellite Orbits: Models, Methods, and Applications*. Springer, 2000.

[8] J. Durbin and S. J. Koopman, *Time Series Analysis by State Space Methods*, 2nd ed. Oxford University Press, 2012.

[9] G. E. P. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time Series Analysis: Forecasting and Control*, 5th ed. Wiley, 2015.