ASSIGNMENT-1
Probability & Random Variables

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Question

Two dice are thrown at the same time. Determine the probabiity that the difference of the numbers on the two dice is 2.

Solution

Let X and Y be the random variables representing the numbers on the first and the second dice, respectively. As each dice has 6 equally likely outcomes, we have:

$$P(X = i), i = (1, 2, 3, 4, 5, 6)$$

$$P(Y = j), j = (1, 2, 3, 4, 5, 6)$$

Let $D = |X - Y|$ be the absolute difference between the numbers on the two dice,where $D = 1, 2, 3, 4, 5, 6$ etc and CDF of D is:

$$F(X) = \left(\frac{6}{36}\right), D = 0$$

$$\left(\frac{16}{36}\right), D \geq 1$$

$$\left(\frac{24}{36}\right), D \geq 2$$

$$\left(\frac{30}{36}\right), D \geq 3$$

$$\left(\frac{34}{36}\right), D \geq 4$$

$$\left(\frac{36}{36}\right), D \geq 5$$

And we want to find $P(E = 2)$, which is the probability that the absolute difference is exactly 2.

We are going to use this formula;

$$P(E = 2) = \sum P(X = i)P(y = j)$$

,Where the sum is taken over all possible pairs(i,j) of numbers on the 1st and 2nd dice.

Those Pairs are:

$$(1, 3), (2, 4), (3, 5), (4, 6), (6, 4), (5, 3), (4, 2), (3, 1)$$

$\therefore P(E = 2) = P(X = 1)P(Y = 3) + P(X = 2)P(Y = 4) + P(X = 3)P(Y = 5) + P(X = 4)P(Y = 6)$

$+ P(X = 6)P(Y = 4) + P(X = 5)P(Y = 3) + P(X = 4)P(Y = 2) + P(X = 3)P(Y = 1)$

Thus,

$$P(E = 2) = \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)$$

$$\therefore P(E = 2) = \left(\frac{8}{36}\right) = \left(\frac{2}{9}\right)$$

, the required Probability.