[redacted]

1. What is the coordinate vector of X if $X = (-15, 35, 2)$ and if the basis used is $B = (u_1, u_2, u_3)$ where $u_1 = (1, 3, 2)$, $u_2 = (-2, 4, -1)$, $u_3 = (2, -2, 0)$?

2. Let $f(t) = 5t^3 + 7t^2 - 8t + 9$. Is the set $\{f, f', f'', f'''\}$ a basis for P_3 ?

3. Let $A = \begin{bmatrix} 1 & 2 & 3 & 2 & 3 \\ 3 & 6 & 9 & 2 & 3 \\ 2 & 4 & 6 & 2 & 3 \\ 1 & 2 & 3 & 2 & 3 \end{bmatrix}$. Find the bases for each of the spaces Null(A), Col(A) and Row(A).

4. Find the rank of $A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$.

[redacted]

[redacted]

5. Consider the linear system:

$$\begin{aligned} x_1 + 2x_2 + 4x_3 + 4x_4 &= 7 \\ x_2 + x_3 + 2x_4 &= 3 \\ x_1 + 0x_2 + 2x_3 + 0x_4 &= 1 \end{aligned}$$

(a) Let A be the coefficient matrix of the associated homogeneous system. Find the reduced form of A.
(b) Determine whether the system is consistent and, if so, find the general solution.

6. Find the reduced row echelon form for each of the following matrices which are assumed to be over R:

$$A_1 = \begin{bmatrix} 2 & 1 & -1 & 1 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

7. Find the inverse of the following real matrix.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

[redacted]

[redacted]

8. Find the matrix of the following transformations:

   (a) $F(x_1, x_2, x_3)^T = (2x_1 - 3x_3, x_1 + x_2 - x_3, x_1, x_2 - x_3)^T$.

   (b) $G(x_1, x_2, x_3, x_4)^T = (x_1 - x_2 + x_3 + x_4, x_2 + 2x_3 - 3x_4)^T$.

9. Diagonalize the following matrix:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & -2 \end{bmatrix}.$$

10. Let $W \subset R^4$ be the span of $(1,0,1,1)^T$, $(-1,1,0,0)^T$, and $(1,0,1,-1)^T$. Find an orthonormal basis of W. Expand $(0,0,0,1)^T$ and $(1,0,0,0)^T$ in terms of this basis.

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2 points

Let P_n denote the $n \times n$ matrix whose entries are all ones except the zeros directly below the main diagonal.
Then determinant of P_n is

$$2$$

.....

$$-1$$

1

....

1

..

0

...

2

Suppose that a system of linear equations has a 3×5 coefficient matrix with three pivot columns. The system is

Neither consistent nor inconsistent

.....

Inconsistent

....

Consistent and unique solution

..

Consistent and infinitely many solutions

...

Let A be a 3×3 upper triangular matrix with real entries. If $a_{11} = 1$, $a_{22} = 2$, $a_{33} = 3$, determine α, β and γ such that $A^{-1} = \alpha A^2 + \beta A + \gamma I$.

$$\alpha = \frac{1}{6},\ \beta = -1,\ \gamma = \frac{11}{6}$$

..

$$\alpha = -1,\ \beta = \frac{2}{19},\ \gamma = \frac{3}{19}$$

.....

5

$$\alpha = -1,\ \beta = \frac{2}{9},\ \gamma = \frac{3}{11}$$

...

$$\alpha = \frac{11}{6},\ \beta = -1,\ \gamma = \frac{11}{3}$$

....

The product of two orthogonal matrices is orthogonal and that the inverse of an orthogonal matrix is

Skew-Symmetric

....

Orthogonal

...

Symmetric

◯ ..

Hermition

◯ .....

Which of the following matrices are similar:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

C is similar to both A and B

.....

A is similar to B

..

Consider the vector $v = (1, 2, 3, 4) \in R^4$. A basis of the subspace of R^4 consisting of all vectors perpendicular to v is

$$(-2, 1, 0, 0), (-3, 0, 1, 0), (-4, 0, 0, 1)$$

..

$$(1, 0, 0, 0), (0, 2, 0, 0), (0, 0, 3, 4)$$

...

Determine which of the following functions is linear transformation

$h: R^3 \to R^3$ such that

$$h\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} xy \\ z \\ 2y \end{bmatrix}$$

- ....

$g: R \to R$ such that $g(x) = 5x + 3$

- ...

$T : R \to R$ such that $T(x) = 9x^2 + 3$

○ .....

$f : R^3 \to R$ such that f maps each vector $\overline{x} \in R^3$ to

its dot product with the vector $\overline{v} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

○ ..

Suppose that A is a square matrix such that $B^T AB$ is diagonal for some orthogonal matrix B. Then

A is skew Hermition

.....

A is necessarily symmetric

..

$U = \text{span}\,\{(1,2,3,4),(5,6,7,8)\}$. Then $U^{\perp}$ is

$$(1,-2,1,0),(2,-3,0,1)$$

...

$$(1,-2,2,0),(5,6,7,8)$$

.....

$$(1, 2, 3, 4), (2, -4, 0, 1)$$

....

$$(1, -2, 2, 0), (2, -4, 0, 0)$$

..

Suppose that A is a square matrix such that $B^T AB$ is diagonal for some orthogonal matrix B. Then

A is skew Hermition

.....

A is necessarily symmetric

..

A is skew-symmetric

...

A is Hermition

....