We will use $Dir_L\ (a_1, \ldots, a_L)$ to denote the finite-dimensional Dirichlet distribution on the simplex S_L. Also,we will use $Mult_L(b_1, \ldots, b_L)$ to denote the multinomial distribution with number of trials equal to 1 and probability vector $(b_1, \ldots, b_L)$. We will form a K $\times$ V matrix β, whose t^{th} row is the t^{th} topic (how β is formed will be described shortly). Thus, β will consist of vectors $\beta_1, \ldots, \beta_K$, all lying in S_V. The LDA model is indexed by hyperparameters $\eta \in (0, \infty)$ and $\alpha \in (0, \infty)^K$. It is represented graphically in Figure 1, and described formally by the following hierarchical model:

1. $\beta_t \overset{iid}{\sim} Dir_V(\eta, \ldots, \eta), t = 1, \ldots, K.$
2. $\theta_d \overset{iid}{\sim} Dir_K(\alpha), d = 1, ..., D$, and the $\theta_d' s$ are independent of the$\beta_t' s$.
3. Given $\theta_1, \ldots, \theta_D, z_{di} \overset{iid}{\sim} Mult_K(\theta_d), i = 1, \ldots, n_d, d = 1, \ldots, D$, and the D matrices $(z_{11}, \ldots, z_{1n_1}), \ldots, (z_{D1}, \ldots, z_{Dn_D})$ are independent.
4. Given β and the $z_{di}' s$ $,the w_{di}' s$ are independently drawn from the row of β indicated by $z_{di}, i = 1, \ldots, n_d, d = 1, \ldots, D.$

Now suppose that wwe have a method for constructing a Markov chain on ψ whose invariant distribution is $\nu_{h,w}$ and which is ergodic. Two Markov chains which statisfy these criteria are discussed in later in this section. Let $h_* \in$ H be fixed but arbitrary, and let $\psi_1, \psi_2, \ldots$ be an ergodic Markov chain with invariant distribution $\nu_{h_*,w}$. For any h $\in$ H, as n $\to \infty$ we have

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\nu_h(\psi_i)}{\nu_{h_*}(\psi_i)} \overset{a.s}{\to} \int \frac{\nu_h(\psi)}{\nu_{h_*}(\psi)} d\nu_{h_*,w}(\psi) \tag{1}$$

$$= \frac{m(h)}{m(h_*)} \int \frac{l_w(\psi)\nu_h(\psi)/m(h)}{l_w(\psi)\nu_{h_*}(\psi)/m(h_*)} d\nu_{h_*,w}(\psi) \tag{2}$$

$$= \frac{m(h)}{m(h_*)} \int \frac{\nu_h(\psi)}{\nu_{h_*}(\psi)} d\nu_{h_*,w}(\psi) \tag{3}$$

$$= \frac{m(h)}{m(h_*)}. \tag{4}$$