Microscopic Traffic Flow Model

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Traffic flow is the study of interactions between travellers (like cars, trucks, buses, cyclist, pedestrians) and infrastructure (like highways, traffic stop, turns) with the purpose of understanding and developing an optimal network for efficient movement of traffic and minimal traffic congestion problems.

Traffic behaves in a complex and non linear way, depending upon interactions of large number of vehicles. Traffic flow theory refers to steam variables of speed, flow and concentration.

Terms:

Speed: It is the distance covered per unit time. To get average speed of vehicles, there are two types of speed average -

- Time mean speed Instantaneous speed of different vehicles is measured at a point on path. Then average of that speed is taken.

- Space mean speed A snapshot of track is taken for getting speed of all vehicles at same time. Average of those speed is taken to get space mean speed.

Density: It is defined as the number of vehicles per unit length of the roadway. In traffic flow critical density and jam density are the most important density which describe a lot about a system.

Flow: It is the number of vehicles passing a reference point per unit of time (vehicles per hour).

Methods of Analysis:

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*wikipedia,

1. Microscopic Scale Most basic level. Every vehicle is considered as individual. Equations (ODE- Ordinary Differential Equation) for each individual can be written and their nature can be deduced.

2. Macroscopic Scale Advanced level where system as a whole is considered. It is considered useful to employ a system of partial differential equations, which gives quantities of interest; e.g., the density of vehicles or their mean velocity.

3. Mesoscopic Scale Intermediate between microscopic and macroscopic scale. Uses method of statistical mechanics.

Microscopic Scale:

In this level, individual vehicles are observed and data analysis is done in detailed level.

Our Model: Based on "Follow the leader Theory"

- In our model we will consider a single lane and therefore no overtaking is allowed.

- For driver behaviour and action we are using the 'follow the leader' theory.

- We are ignoring traffic and road infrastructure like traffic lights, crossing, turns so as to keep our model simple and understandable.

- We will also assume that no vehicles crash into each other and thus each vehicle maintains atleast some distance even in worst case scenerio.

Subject Vehicle (n)   Leading Vehicle (n-1)
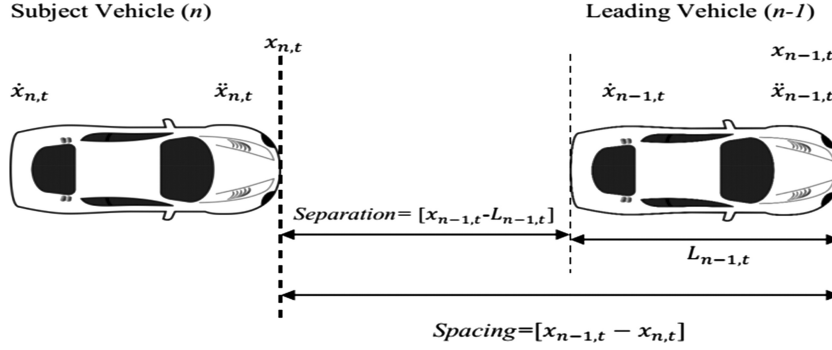
$$x_{n-1}(t) = x_n + d + L_{n-1} \quad \text{L represent the lenght of } n-1_{th} \text{ vehicle} \qquad (1)$$

For n and n+1 vehicle.

$$x_n(t) - x_{n+1}(t) = d + L \quad \text{L represent the lenght of nth vehicle} \qquad (2)$$

d is the separation between vehicles.

Leaders Model says : Response = Stimulus * Response time.

Stimulus for driver can be any or combination of these events

1. acceleration of vehicles

2. distance between vehicles

In our model we incooperate the acceleration of vehicles as stimulus for response that will then act as stimulus for preceding vehicles and so on.

$$\triangle x_{n+1}^t = \triangle x_{safe} + \tau v_{n+1}^t \qquad (3)$$

$\triangle x_{n+1}$ denotes the gap between $n+1^{th}$ vehicle and n^{th} vehicle. $\triangle x_{safe}$ denotes the least distance or gap that should be present even when the presiding vehicle is faster than successive vehicle. v_{n+1}^t denotes the speed of $n+1^{th}$ vehicle. τ is the sensitivity according to which $n+1^{th}$ vehicle adjust its speed.

Therefore now we have:

$$x_n^t - x_{n+1}^t = \triangle x_{safe} + \tau v_{n+1}^t \qquad (4)$$

Differentiating the above equation with respect to time, we get

$$v_n^t - v_{n+1}^t = \tau a_{n+1}^t \qquad (5)$$

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$$a_{n+1}^t = \frac{1}{\tau}[v_n^t - v_{n+1}^t] \tag{6}$$

Thus our final equation is

$$\frac{d^2 x_{n+1}^t}{dt^2} = \frac{1}{\tau}[\frac{dx_n^t}{dt} - \frac{dx_{n+1}^t}{dt}] \tag{7}$$

Observations :

- Resultant equation is an ODE (ordinary differential equation) with order 2.

- $\frac{1}{\tau}$ is sensitivity constant

- If $\frac{dx_n}{dt} > \frac{dx_{n+1}}{dt}$ Leader is faster. Follower would accelerate.
  Therefore $\frac{1}{\tau} > 0$

- If $\frac{dx_n}{dt} < \frac{dx_{n+1}}{dt}$ Leader is slower. Follower would deaccelerate.
  Therefore$\frac{1}{\tau} > 0$

- This ODE is difficult to solve because acceleration of each vehicle is dependent on vehicle successive to it. Thus we need to solve n ODE to get acceleration and velocity of n+1 vehicle.

Speed density Relationship -

$$\frac{dx_{n+1}^t}{dt^t} = \int \frac{d^2 x_{n+1}^t}{dt^2} \tag{8}$$

$$\frac{d^2 x_{n+1}^t}{dt^2} = \frac{1}{\tau}[\frac{dx_n^t}{dt} - \frac{dx_{n+1}^t}{dt}] \tag{9}$$

Using (8) and (9) we get

$$\frac{dx_{n+1}^t}{dt} = \int \frac{1}{\tau}[\frac{dx_n^t}{dt} - \frac{dx_{n+1}^t}{dt}] \tag{10}$$

Integrating the above equation with respect to t:

$$\frac{dx_{n+1}^t}{dt} = \frac{1}{\tau}[x_n^t - x_{n+1}^t] + c_{n+1} \tag{11}$$

$$v_{n+1}^t = \frac{1}{\tau}[x_n^t - x_{n+1}^t] + c_{n+1} \tag{12}$$

Speed of $n+1_{th}$ is dependent on space ahead of it.

Also:

$$v_{avg} = \frac{\sum_1^N v_n}{N} \text{ where N is the total number of vehicles} \tag{13}$$

Therefore (12) can be written as:

$$v_1^t + v_2^t + v_3^t + \cdots + v_{n+1}^t = \frac{1}{\tau}[(x_0^t - x_1^t) + (x_1^t - x_2^t) + \cdots + (x_n^t + x_{n+1}^t)] + c' \tag{14}$$

$$\frac{v_1^t + v_2^t + v_3^t + \cdots + v_{n+1}^t}{N} = \frac{\frac{1}{\tau}[(x_0^t - x_1^t) + (x_1^t - x_2^t) + \cdots + (x_n^t + x_{n+1}^t)]}{N} + c'' \tag{15}$$

$$v_{avg} = \frac{1}{\tau}\frac{[x_0^t - x_{n+1}^t]}{N} + c'' \tag{16}$$

As space between 0_{th} vehicle and n_{th} represent the path lenght , thus $\frac{[x_0^t - x_{n+1}^t]}{N} = L$. L is the path length or road segment we have taken is consideration.

Also $\frac{N}{L} = \rho$ where ρ is the linear density of path or road. Linear density is defined as number of vehicles per unit length.

$$v_{avg} = \frac{1}{\tau\rho} + c'' \tag{17}$$

To get value of c" , we set condiction that at ρ_{max} , $v_{avg} = 0$
Therefore speed-velocity relationship equation:

$$v_{avg} = \frac{1}{\tau}[\frac{1}{\rho} - \frac{1}{\rho_{max}}] \tag{18}$$

Observation:

- $\frac{dv_{max}}{d\rho} = \frac{1}{\tau}(-\frac{1}{\rho^2})$

- $\rho = \rho_{max}$ then $v_{avg}^t = 0$
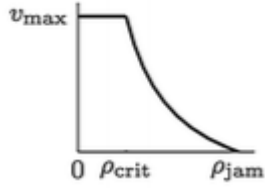
- $\rho = 0$, $v_{avg}^t = \infty$

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The third observation gives room for improvement in speed-velocity equation as when $\rho = 0$, $v_{avg} = v_{max}$

Finalized speed-velocity equation:

$$v_{avg} = \frac{1}{\tau}[\frac{1}{\rho} - \frac{1}{\rho_{max}}] \text{ when } \rho > \rho_{critical} \tag{19}$$

$$v_{avg} = v_{max} \text{ when } \rho < \rho_{critical} \tag{20}$$

Graph of avg velocity vs density :

Flow density relationship -
q (Flow) $= \rho \times v$
Therefore
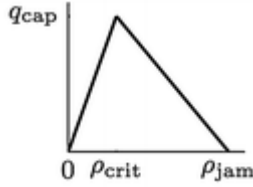
$$q = \frac{1}{\tau}[1 - \frac{\rho}{\rho_{max}}] \text{ when } \rho > \rho_{critical} \tag{21}$$

$$q = \rho \times v_{max} \text{ when } \rho < \rho_{critical} \tag{22}$$

At contant v_{avg}
Graph of density vs flow

Result:

- We solved a very simple model based on a real world complex network of traffic flow and arrived at an ODE.

- That ODE, if solved for whole model will give property of each vehicle like their instanteous speed and acceleration at any time t.

- Using ODE we found that speed density relationship and flow density relationship that better depicts of what happens in real world.

- Even though our model is far too simple, still we get idea of how density of vehicles affect the number of vehicles passing per unit time (flow).

Model Limitation:-

- Our model takes certain assumptions that deviate from real world like we assumed that sensitivity constant for each driver is same while in real world more experienced drivers have better sensitivity that less experienced ones.

- Our model is a very simplisitic approach for a very complex real world problem. We did not take account for various infrastructure and surrounding that affect speed and density (like traffic light, crossroads, turns, u turns etc)

- In our leader theory we took acceleration as the only stimulus that effect driver while in real world its not the only stimulus.

References:

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