

Partial Differential Equation

A. Formation of partial differential equation by the elimination of arbitrary constants:

1. $z = ax + by + a^2 + b^2$, eliminating a and b
2. $(x - h)^2 + (y - k)^2 + z^2 = \lambda^2$, eliminating h and k
3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, eliminating a, b and c
4. $z = (x^2 + a)(y^2 + b)$, eliminating a, b

B. Derivation of partial differential equation by the elimination of arbitrary function ϕ from the equation $\phi(u, v) = 0$, where u and v are functions of x, y, z .

C. Formation of partial differential equation by the elimination of arbitrary function:

1. $\phi(x + y + z, x^2 + y^2 - z^2) = 0$
2. $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
3. $z = (x - y)\phi(x^2 + y^2)$
4. $z = x^n f\left(\frac{y}{x}\right)$
5. $z = f(x^2 - y) + g(x^2 + y)$

D. Lagrange's Method

Type-1

1. $zp = -x$
2. $y^2p - xyq = x(z - 2y)$
3. $x^2p + y^2q + z^2 = 0$

Type-2

1. $xyp + y^2q = zxy - 2x^2$
2. $py + qx = xyz^2(x^2 - y^2)$
3. $z(p - q) = z^2 + (x + y)^2$
4. $p - 2q = 3x^2 \sin(y + 2x)$

Type-3

1. $z(x + y)p + z(x - y)q = x^2 + y^2$
2. $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$
3. $(x + 2z)p + (4zx - y)q = 2x^2 + y$
4. $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$
5. $x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 2z(y^2 - x^2)$
6. $(y + zx)p - (x + yz)q + y^2 - x^2 = 0$