# Parameter Constraints

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## 1 Introduction

I have reproduced the plots given in the original paper Cao et al (2203.10825 ) for Non-Flat $LCDM$ and XCDM models. For each model I used emcee to compute the Markov chains and use them to calculate 1D marginalized distributions corresponding to each free parameter. Here I first present my results for LCDM Model and then XCDM Model.

## 2 Non-FLat LCDM Model

### 2.1 Data

Since the orginal paper Cao et al (2203.10825) cotains the combined results for H(z)+BAO data given in table 1 and 2 of the paper so I have also done the analysis for H(z)+BAO dataset so as to compare my results.

### 2.2 Method

First I wrote the code for $\chi^2$ function and then minimized it using `scipy.minimize`. It returns the minimized value of function $\chi^2_{min}$ and the best fit value of free parameters $\Omega_{ko}, \Omega_b h^2, \Omega_c h^2, H_o$ which are the same as used in the paper. From these free parameters we can also calculate the best fit value of derived parameter $\Omega_m = \frac{\Omega_b h^2 + \Omega_c h^2 + \Omega_\nu h^2}{h^2}$ using the value of neutrino density parameter, $\Omega_\nu h^2 = 0.06/93.14$ given in the paper. Then I ran the emcee code to obtain Markov chains for these free parameters and the derive parameter $\Omega_m$. While running the mcmc simulation for all the parameters (except $H_o$) I used the same priors as in the paper. For $H_o$ the paper uses the priors from 0 to $\infty$ while I used the priors (as suggested in Ryan's thesis) from 50 to 85. These set of priors were used to save time while running the MCMC simulations. I ran the simulations for 5000 steps and for 200 walkers.

### 2.3 Comparison of Results

In the following table, I have made a comparison of results of my work with that of orginal paper.

**Note:** The paper contains only parameter values and $\chi^2_{min}$ for the H(z)+BAO analysis but not the plots. So I've included the plots which contain analysis of H(z)+BAO+SN data (in red coloured contours in figure 1).

| Attribute | Cao et al | Using emcee and scipy |
|---|---|---|
| $\Omega_b h^2$ | 0.026 | 0.026 |
| $\Omega_c h^2$ | 0.1098 | 0.1097 |
| $\Omega_m$ | 0.292 | 0.2917 |
| $\Omega_k$ | 0.048 | 0.0477 |
| $H_o$ | 68.35 | 68.37 |
| $\chi^2_{min}$ | 25.30 | 25.28 |
| AIC | 33.30 | 33.28 |
| BIC | 40.43 | 40.42 |

Table 1: Unmarginalized Best Fitting Parameter Values for BAO+H(z)data

| Attribute | Cao et al | Using emcee and scipy |
|---|---|---|
| Plots | Figure 1 | Figure 2 |
| $\Omega_b h^2$ | $0.0266^{+0.0039}_{-0.0045}$ | $0.0275^{+0.0035}_{-0.0057}$ |
| $\Omega_c h^2$ | $0.1088 \pm 0.0166$ | $0.107 \pm 0.017$ |
| $\Omega_m$ | $0.291 \pm 0.023$ | $0.289 \pm 0.022$ |
| $\Omega_k$ | $0.059^{+0.081}_{-0.091}$ | $0.064^{+0.079}_{-0.090}$ |
| $H_o$ | $68.37 \pm 2.10$ | $68.4 \pm 2.0$ |

Table 2: One-dimensional marginalized posterior mean values and uncertainties ($\pm 1\sigma$ error bars or $2\sigma$ limits) of the parameters

## 2.4 Plots

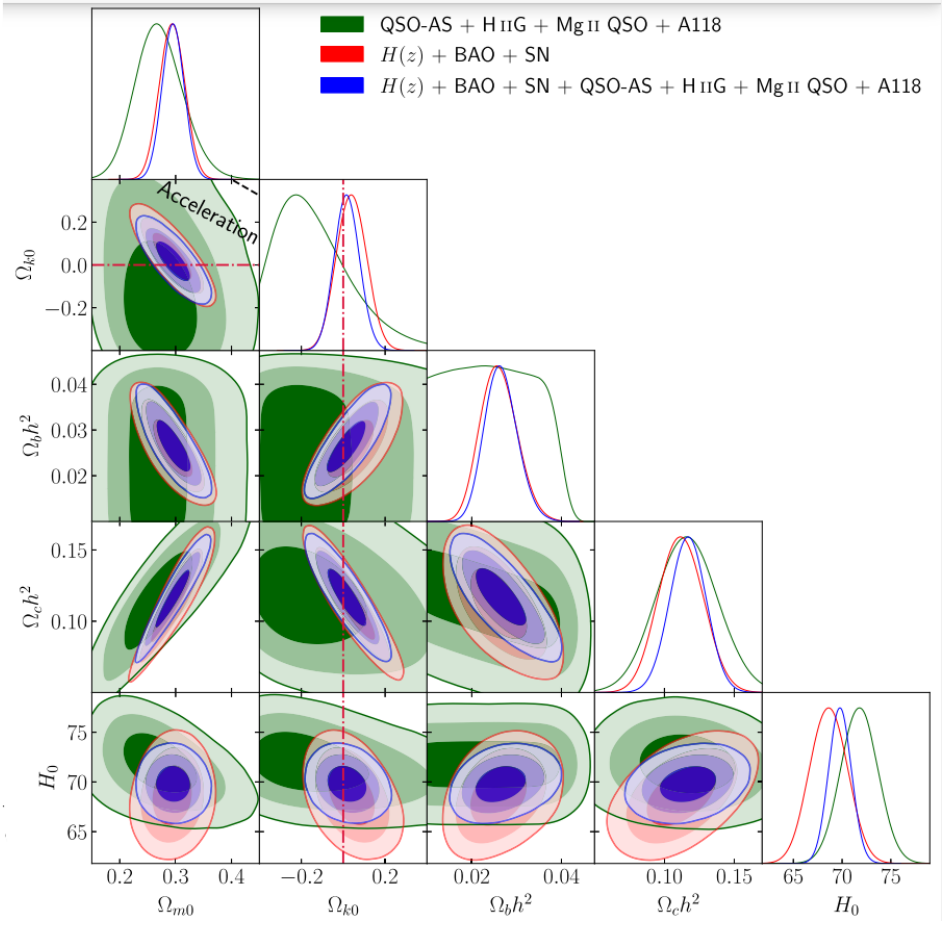


Figure 1: Original Plots in Cao et al

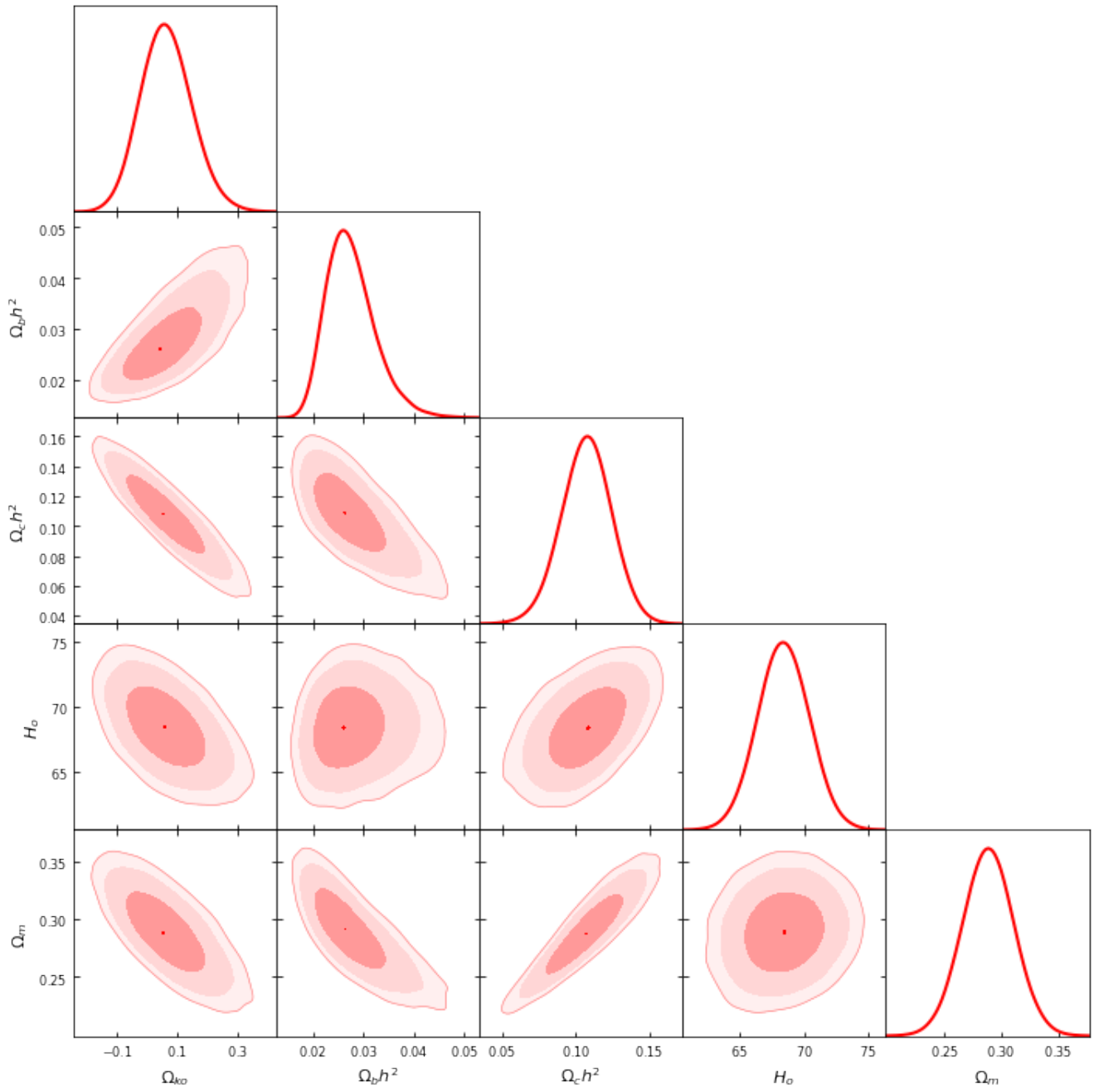


Figure 2: Plots generated using emcee and scipy

# 3 Non-Flat XCDM Model

## 3.1 Data

Same as used above

## 3.2 Method

The whole methodology remains same as used for Non-Flat LCDM Model. Except here the number of free parameters have increased from four to five $(\Omega_{ko}, \Omega_b h^2, \Omega_c h^2, H_o, \omega_x)$. Since the number of parameters were increased, longer chains had to used as autocorrelation time of MCMC analysis increased. Here I've used chain length of 8000 for 200 walkers.

### 3.3 Comparison of Results

In the following table, I have made a comparison of results of my work with that of orginal paper.

**Note:** The paper contains only parameter values and $\chi^2_{min}$ for the H(z)+BAO analysis but not the plots. So I've included the plots which contain analysis of H(z)+BAO+SN data (in red coloured contours in figure 3).

| Attribute | Cao et al | Using emcee and scipy |
|---|---|---|
| $\Omega_b h^2$ | 0.0289 | 0.0295 |
| $\Omega_c h^2$ | 0.0985 | 0.0981 |
| $\Omega_m$ | 0.296 | 0.296 |
| $\Omega_k$ | -0.053 | -0.055 |
| $\omega_x$ | -0.730 | -0.728 |
| $H_o$ | 65.76 | 65.83 |
| $\chi^2_{min}$ | 22.13 | 22.12 |
| AIC | 32.13 | 32.12 |
| BIC | 41.05 | 41.04 |

Table 3: Unmarginalized Best Fitting Parameter Values for BAO+H(z)data

| Attribute | Cao et al | Using emcee and scipy |
|---|---|---|
| Plots | Figure 3 | Figure 4 |
| $\Omega_b h^2$ | $0.0294^{+0.0047}_{-0.0050}$ | $0.0344^{+0.0040}_{-0.011}$ |
| $\Omega_c h^2$ | $0.0980^{+0.0186}_{-0.0187}$ | $0.088^{+0.029}_{-0.018}$ |
| $\Omega_m$ | $0.292 \pm 0.025$ | $0.283 \pm 0.029$ |
| $\Omega_k$ | $-0.027 \pm 0.109$ | $-0.0282 \pm 0.11$ |
| $\omega_x$ | $-0.770^{+0.149}_{-0.098}$ | $-0.73^{+0.17}_{-0.11}$ |
| $H_o$ | $66.13^{+2.35}_{-2.36}$ | $65.8^{+2.2}_{-2.5}$ |

Table 4: One-dimensional marginalized posterior mean values and uncertainties ($\pm 1\sigma$ error bars or $2\sigma$ limits) of the parameters

### 3.4 Plots

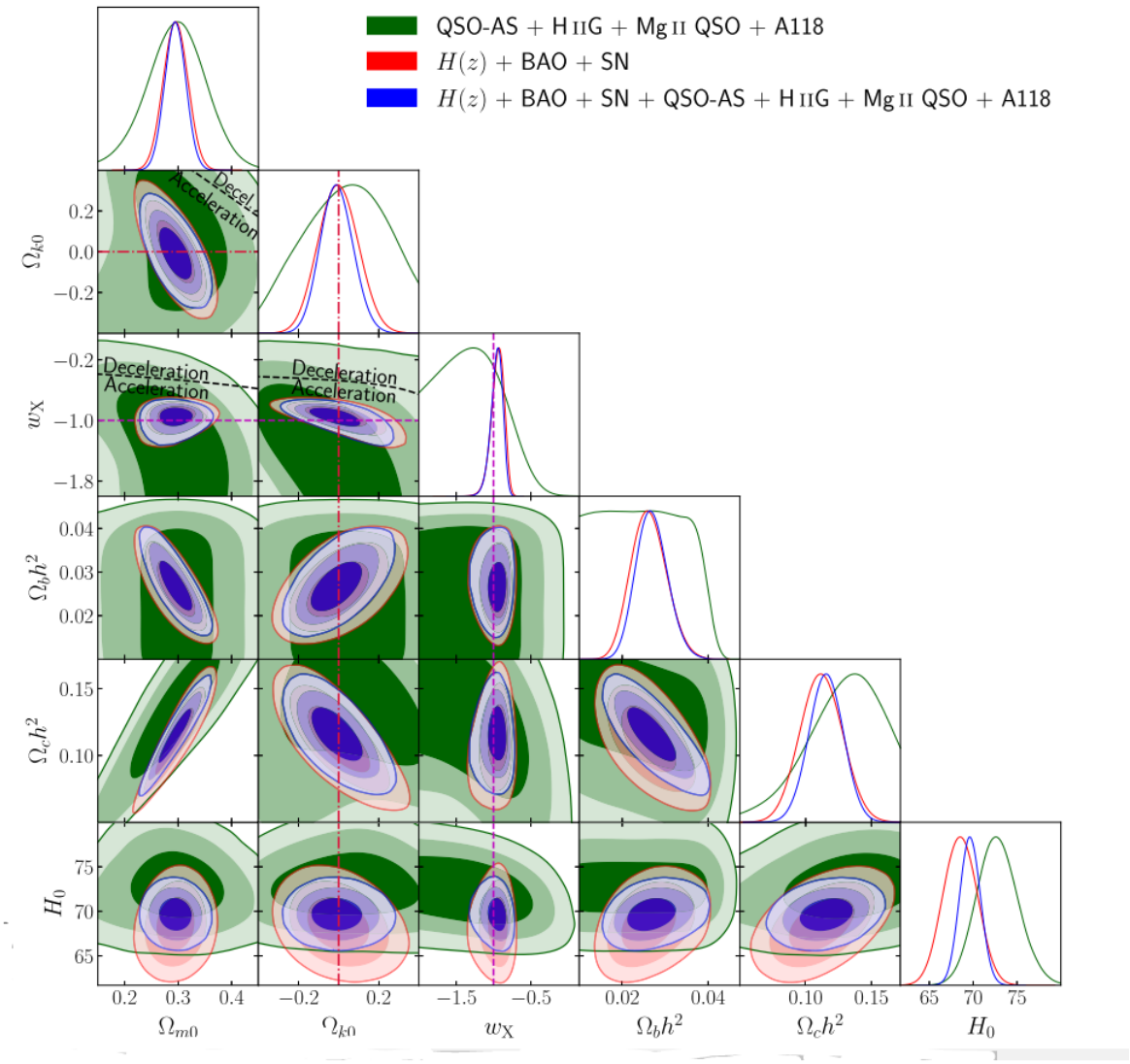


Figure 3: Original Plots in Cao et al for XCDM Model

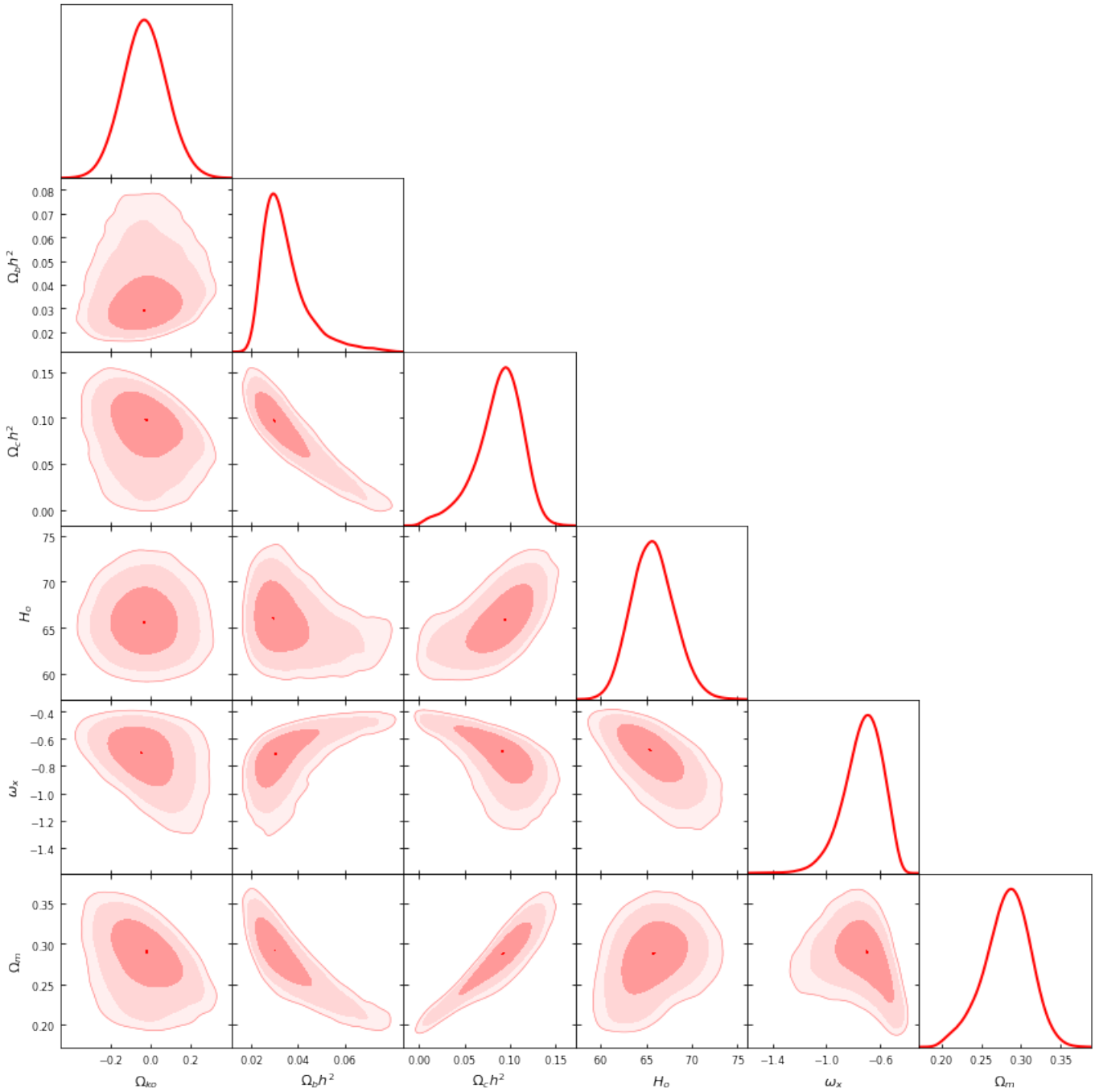


Figure 4: Plots generated using emcee and scipy for XCDM Model