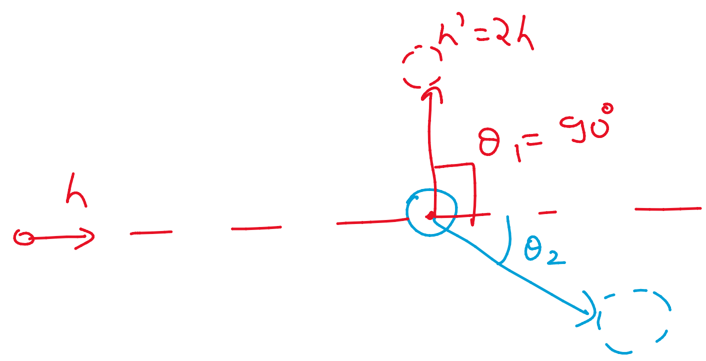
Compton-Tutorial 1 Solutions

11 December 2021 14:27

(1)

1. A photon of energy $h\nu$ is scattered through 90° by an electron initially at rest. The scattered photon has a wavelength twice that of the incident photon. Find the frequency of the incident photon and the recoil angle of the electron.



Energy = E

momentum = P

$$(2\lambda - \lambda) = \frac{h}{m_e c}(1 - \cos\theta_1)$$

$$\lambda = \frac{h}{m_e c}$$

$$\boxed{\nu = \frac{c}{\lambda} = \frac{m_e c^2}{h}}$$

$$\frac{h}{\lambda} = P\cos\theta_2$$

$$\frac{h}{2\lambda} = p\sin\theta_2$$

$$\tan\theta_2 = 2 \quad \Rightarrow \boxed{\theta_2 = \tan^{-1}(2)}$$

(2)

2. Find the energy of the incident x-ray if the maximum kinetic energy of the Compton electron is $m_0c^2/2.5$.

Energy of Compton $e^{\ominus}$

$$E^2 = p^2c^2 - m_0^2c^4$$

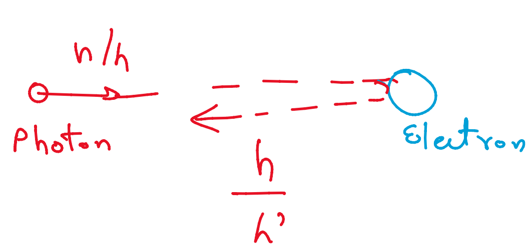
E is manimisied when p is <u>manimised</u>.

momentum of $e^{\ominus}$ is manimised when photon looses man. momentum,

that happens for recoil angle $\theta = 180°$ for photon.

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos(180°)) = \frac{2h}{m_0 c}$$

$$\lambda' = \frac{2h}{m_0 c} + \lambda$$



momentum of electron $= P_{e^-} = \frac{h}{\lambda} + \frac{h}{\lambda'} = \left(\frac{h}{\lambda} + \frac{h}{\frac{2h}{m_oc} + \lambda}\right) =$

$KE_{max} = \frac{m_oc^2}{2.5}$ $\quad E_{max} = m_oc^2 + \frac{m_oc^2}{2.5} = \frac{7}{5} m_oc^2$ $\quad m_o^2c^2\left[\frac{v^2 + c^2 - v^2}{1 - v^2/c^2}\right]$

Applying energy conservation

$$\frac{hc}{\lambda} + m_oc^2 = \frac{hc}{\lambda'} + \frac{7}{5} m_oc^2$$

$$\frac{hc}{\lambda} - \frac{hc}{\frac{2h}{m_oc} + \lambda} = \frac{2}{5} m_oc^2$$

$\left\{ \lambda, \lambda' \to \text{wavelength of photon} \quad \nu, \nu' \to \text{corresponding frequencies} \right\}$

Let $\frac{h}{m_oc} = \lambda_o$

$$\frac{\lambda_o}{\lambda} - \frac{\lambda_o}{2\lambda_o + \lambda} = \frac{2}{5}$$

$$\lambda_o \left[\frac{2\lambda_o}{\lambda(2\lambda_o + \lambda)}\right] = \frac{2}{5}$$

$$\Rightarrow 5\lambda_o^2 = 2\lambda_o\lambda + \lambda^2$$

$$\Rightarrow \lambda^2 + 2\lambda\lambda_o - 5\lambda_o^2 = 0$$

$$\Rightarrow \lambda = \frac{-2\lambda \pm \sqrt{24\lambda_o^2}}{2} = (\sqrt{6} - 1)\lambda_o = (\sqrt{6} - 1)\frac{h}{m_oc}$$

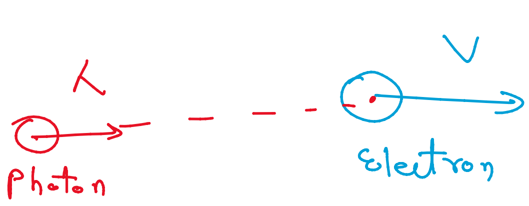
$$\Rightarrow E_p = \frac{hc}{\lambda} = \frac{m_oc^2}{(\sqrt{6} - 1)} = \frac{0.511 \text{ MeV}}{(\sqrt{6} - 1)} = 0.352 \text{ MeV}$$

(3)

3. Show that a free electron cannot absorb a photon so that a photoelectron requires bound electron. However, the electron can be free in Compton Effect. Why?

Sol^n Absorbing the photon means that no new photon is released & all the energy & momentum of photon gets transferred to e^-.

Let's suppose this is possible for now



Momentum Cons.

$$\frac{h}{\lambda} = \frac{m_o V}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \text{— (1)}$$

Energy Cons.

$$\frac{hc}{\lambda} + m_oc^2 = \frac{m_oc^2}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \text{— (2)}$$

Using (1) & (2) & $\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma$

$$m_o\gamma Vc + m_oc^2 = m_oc^2\gamma$$

$$\frac{\gamma V}{c} + 1 = \gamma$$

$$1 = v\left(1-v/c\right)$$

$$\frac{1}{v^2} = \left(1-\frac{v}{c}\right)^2$$

$$\cancel{1} - \frac{v^2}{c^2} = \cancel{1} + \frac{v^2}{c^2} - \frac{2v}{c}$$

$$\frac{v}{c} = 1$$

Not possible ($\because$ violates ①)

$$\Rightarrow \boxed{v = c} \quad \text{or} \quad \boxed{v = 0}$$

Not possible

($\because$ As a massive particle ($e^{\ominus}$ in this case) can't have velocity equal to c)

5. Find the smallest energy that a photon can have and still transfer 50% of its energy to an electron initially at rest.

Sol^n

$$\lambda' = 2\lambda \quad \left(\because \frac{hc}{\lambda'} = \frac{1}{2}\cdot\frac{hc}{\lambda}\right)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda_{max} = \frac{2h}{m_0 c} \Rightarrow \boxed{E_{min} = \frac{m_0 c^2}{2}}$$

6. *γ-rays are scattered from electrons initially at rest. Assume the it is back-scattered and its energy is much larger than the electron's rest-mass energy, $E \gg m_e c^2$.
(a) Calculate the wavelength shift
(b) Show that the energy of the scattered beam is half the rest mass energy of the electron, regardless of the energy of the incident beam
(c) Calculate the electron's recoil kinetic energy if the energy of the incident radiation is 150MeV

Sol^n Back scattered $\Rightarrow \theta = 180°$

(a) $$\Delta\lambda = \frac{2h}{m_0 c}$$

(b) $$\lambda' = \lambda + \frac{2h}{m_0 c}$$

$$\frac{hc}{\lambda'} = \frac{hc}{\lambda + \frac{2h}{m_0 c}}$$

$\lambda' \quad \frac{\quad}{m_0 c}$

$$\because \frac{hc}{\lambda} >> m_0 c^2 \Rightarrow \lambda << \frac{h}{m_0 c}$$

$$\frac{hc}{\lambda'} \simeq \frac{hc}{2h/m_0 c} = \frac{m_0 c^2}{2}$$

(c) Applying energy cons.

$$E_{e^\ominus, f} + \frac{m_0 c^2}{2} = 150\,\text{MeV} + E_{e^\ominus, i}$$

$$KE_{recoil} = \text{Recoil } KE = E_{e^\ominus, f} - m_0 c^2$$

$E_{e^\ominus, i} = m_0 c^2$ (electron at rest initially)

$$KE_{recoil} = 150\,\text{MeV} - \frac{m_0 c^2}{2}$$

$$= 150\,\text{MeV} - \frac{1}{2}(0.511\,\text{MeV}) \simeq 149.75\,\text{MeV}$$

7 . In Compton Scattering, Show that if the angle of scattering θ increases beyond a certain value θ_0, the scattered photon will never have energy larger than $2m_0c^2$, irrespective of the energy of the incident photon. Find the value of θ_0

Sol^n

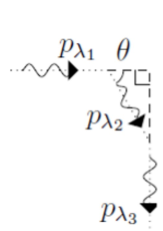
$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$$

$$E_p' = \frac{hc}{\lambda'} = \frac{hc}{\lambda + \frac{h}{m_0 c}(1-\cos\theta)} \leq 2m_0c^2 \quad \forall \lambda,\ \theta > \theta_0$$

$$\Rightarrow \frac{hc}{\frac{h}{m_0 c}(1-\cos\theta)} \leq 2m_0c^2$$

$$\Rightarrow \frac{m_0c^2}{(1-\cos\theta)} \leq 2m_0c^2 \Rightarrow \boxed{\theta_0 = 60^\circ}$$

8 8. * In a Compton scattering experiment (see figure), X-rays scattered off a free electron initially at rest at an angle $\theta(> \pi/4)$), gets re-scattered by another free electron, also initially at rest.



(a) If $\lambda_3 - \lambda_1 = 1.538 \times 10^{-12}$ m, find the value of θ.

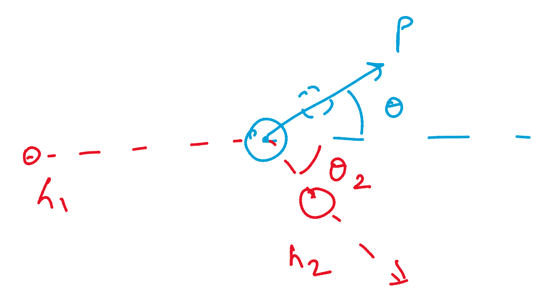
(b) If $\lambda_2 = 68 \times 10^{-12}$ m , find the angle at which the first electron recoils due to the collision.

(b) $\lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1-\cos\theta)$

$$\lambda_3 - \lambda_2 = \frac{h}{m_0 c}(1-\cos 90^\circ)$$

$$\Rightarrow \lambda_3 - \lambda_1 = \frac{h}{m_0 c}(2-\cos\theta_2)$$

(b)



$$\frac{h}{\lambda_1} = P\cos\theta + \frac{h}{\lambda_2}\cos\theta_2$$

$$P\sin\theta = \frac{h}{\lambda_2}\sin\theta_2$$

$$\tan\theta = \frac{h/\lambda_2 \sin\theta_2}{\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\cos\theta_2} = \frac{\lambda_1}{\lambda_2-\lambda_1}\tan\theta_2$$