EE324 Control Systems Lab

Problem Sheet 9

Prakhar Diwan, Roll no. 180100083

1. Problem 1

Given the open-loop system transfer function G(s):

$$G(s) = \frac{10}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

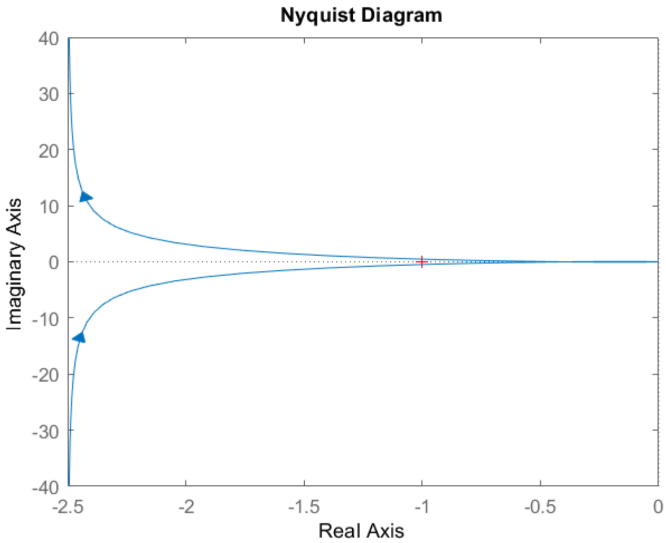

Figure 1: Nyquist Plot for G(s)

We obtain gain margin as $7.9588\ dB$ and phase margin as 22.5359°.

1.1. Part (a)

$$C(s) = \frac{s+3}{s+1}$$

$$C(s)G(s) = \frac{10}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}\left(\frac{s+3}{s+1}\right)$$

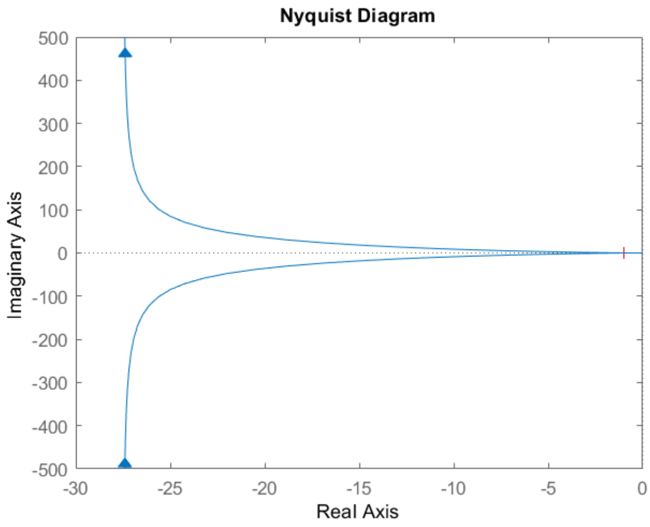

Figure 2: $GM = 2.0762 \; dB \; PM = 4.0248 \; ^\circ$

We obtain gain margin as $2.0762 \; dB$ and phase margin as 4.0248°.

1.2. Part (b)

$$G(s) = \frac{10}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)} \frac{s+1}{s+3}$$

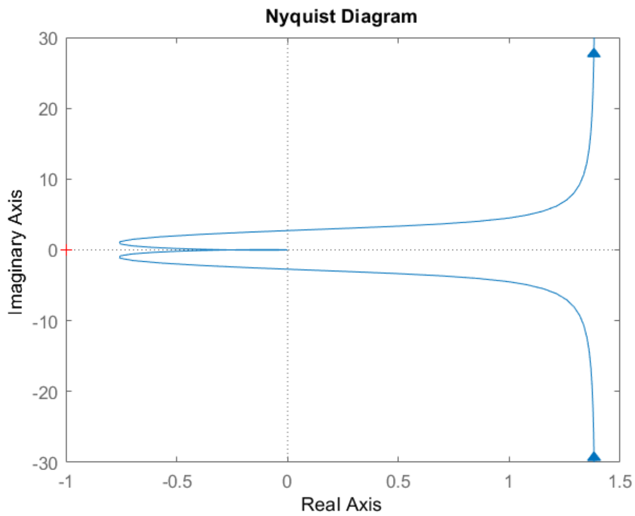

Figure 3: $GM = 11.7598 \; dB \; PM = 43.1732 \; ^\circ$

We obtain gain margin as $11.7598 \; dB$ and phase margin as 43.1732°.
The Lag compensator reduces both the phase and gain margins of $C(s)G(s)$ as compared to $G(s)$, while the lead compensator increases both of them.

MATLAB Code

```
clear;
close all;
clc;

syms s;
s=tf('s');
G = 10/(s*(s/5 +1)*(s/20 + 1));

clag = (s+3)/(s+1);
clead = (s+1)/(s+3);

figure;nyquist(G);
[Gm,Pm,~,~] = margin(G);
fprintf("GM = %2.3fdeg \n", 20*log10(Gm));
fprintf("PM = %2.3fdeg \n", Pm);

figure;nyquist(G*clag);
[Gm,Pm,~,~] = margin(G*clag);
fprintf("GM for Lag compensated system = %2.3fdB \n", 20*log10(Gm));
fprintf("PM for Lag compensated system = %2.3fdeg \n", Pm);
figure;nyquist(G*clead);
[Gm,Pm,~,~] = margin(G*clead);
fprintf("GM for Lead compensated system = %2.3fdB \n", 20*log10(Gm));
fprintf("PM for Lead compensated system = %2.3fdeg \n", Pm);
```

2. Problem 2

The transfer function of a standard notch filter is:

$$H(s) = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_z s}{Q} + \omega_z^2}$$

$$H(j\omega) = \frac{\omega_z^2 - \omega^2}{\omega_z^2 - \omega^2 + j\frac{\omega_z \omega}{Q}}$$

We see that we have gain of 1 for very large and very small ω and gain around $\frac{2Q\Delta\omega}{\omega_z}$ for small deviation $\Delta\omega$ around ω_z.
Choosing the following parameter $\omega_z = 100\pi$ (for 50Hz specification), and varying Q with values of 0.05, 0.1 and 0.5 we get the following plots:

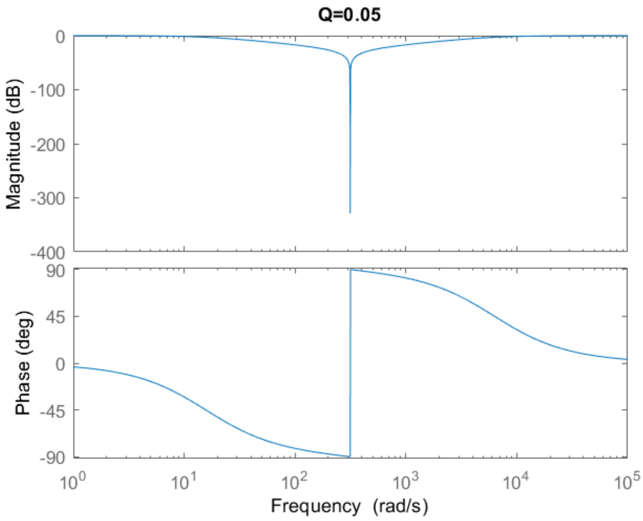

Figure 4: $Q = 0.05$

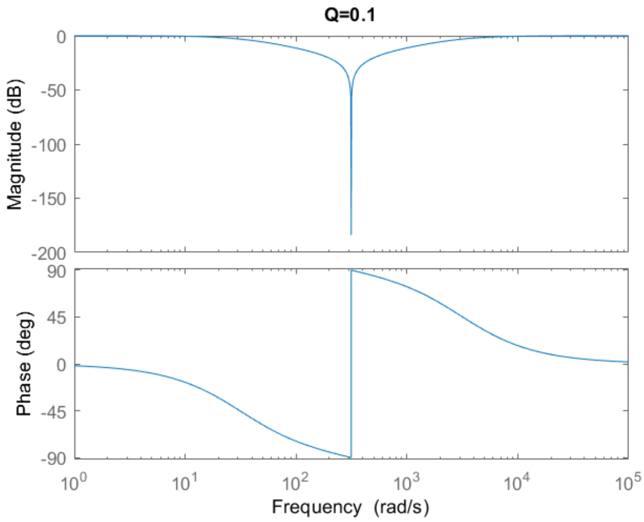

Figure 5: $Q = 0.1$

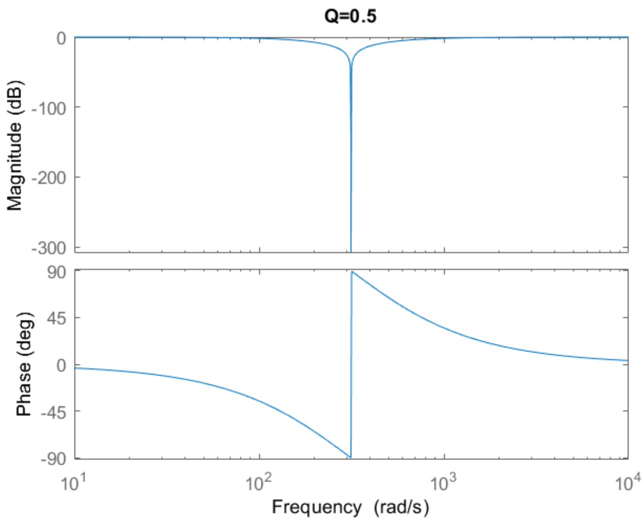

Figure 6: $Q = 0.5$

We see that upon varying Q we are able to modify the steepness of the magnitude plot for the notch filter and have proven the same by above plots. As Q increases, the steepness increases.

MATLAB Code

```
clear;
close all;
clc;

syms s;
s=tf('s');
```

```
omega_z = 2*pi*50;
q = 0.05;
G1 = (s^2 + omega_z^2)/(s^2 + omega_z*s/q + omega_z^2);
figure;bode(G1);title('Q=0.05');

q=0.1;
G2 = (s^2 + omega_z^2)/(s^2 + omega_z*s/q + omega_z^2);

figure;bode(G2);title('Q=0.1');

q=0.5;
G3 = (s^2 + omega_z^2)/(s^2 + omega_z*s/q + omega_z^2);

figure;bode(G3);title('Q=0.5');
```

3. Problem 3

Given the system transfer function C(s):

$$C(s) = \frac{100}{s+30}, \; G(s) = e^{-sT}$$

$$C(j\omega)G(j\omega) = \frac{100\, e^{-j\omega T}}{j\omega + 30}$$

$$\angle C(j\omega)G(j\omega) = \angle C(j\omega) - \omega T$$

As $|e^{-sT}| = 1$, the magnitude plot remains unaffected, and the the phase plot is linearly shifted. For verge of instability (or marginal stability), we require $\angle \omega_{gcf} = (2k+1)\pi$, upon equating we get

$$\angle C(j\omega_{gcf}) - \omega_{gcf} T = (2k+1)\pi$$

$$T = \frac{\angle C(j\omega_{gcf}) - (2k+1)\pi}{\omega_{gcf}}$$

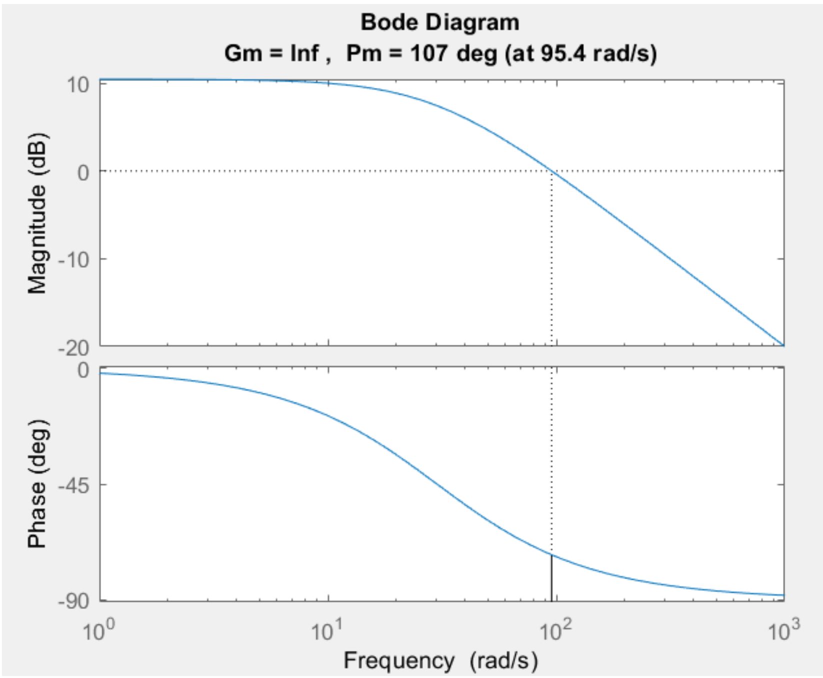

Figure 7: Bode Plot for C(s)

$$\omega_{gcf} = 95.4$$

$$\angle C(j\omega_{gcf}) = -1.2669$$

$$T = \frac{-1.2669 + \pi}{95.4}$$

$$= 0.01965$$

$$\therefore G(s) = e^{-0.01965s}$$

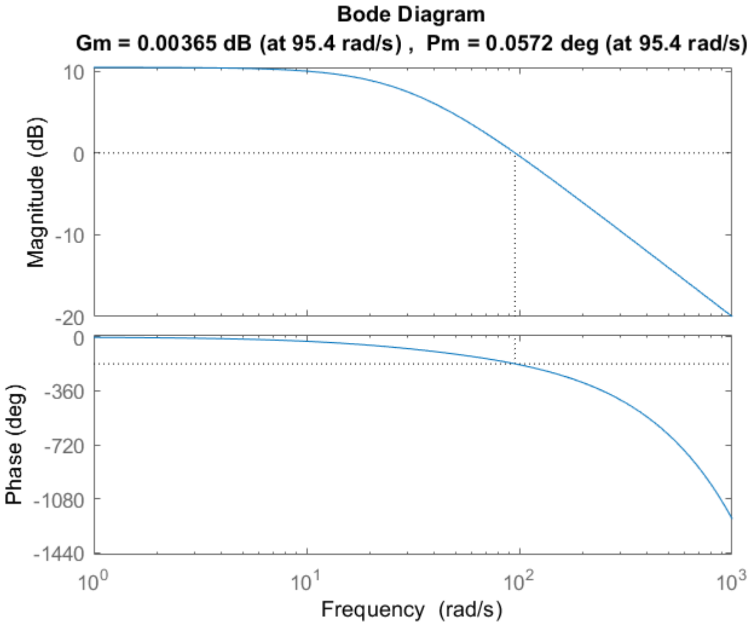

Figure 8: Bode plot for required destabilized system

Without the delay unit, we had infinite gain margin and phase margin of $\pi - 1.2669\ rads$ = 107 degrees. But with this delay unit we have $\omega_{pcf} = \omega_{gcf}$ hence 0 gain and phase margin.

MATLAB Code

```
clear;
close all;
clc;

syms s;
s=tf('s');
C = 100/(s+30);
%figure;margin(C);
%[Gm,Pm,Wcg,Wcp] = margin(C);

G = exp(-0.01965*s);
figure;margin(G*C);
```

4. Problem 4

Given the open-loop system transfer function:

$$G(s) = \frac{1}{(s^3 + 3s^2 + 2s)}$$

The gain margin is mentioned in linear units when not specified as dB.

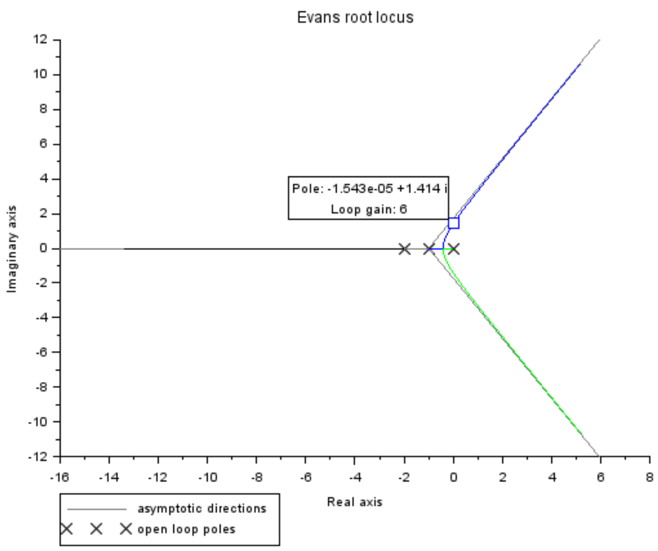

Figure 9: Root Locus of G(s): critical point marked

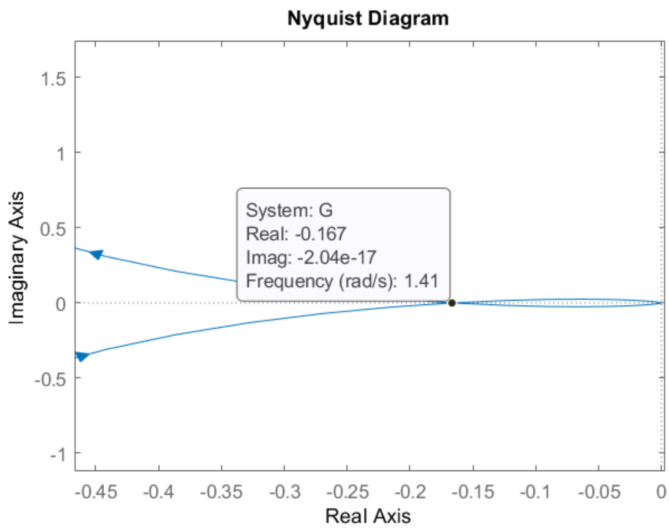

Figure 10: Nyquist Plot of G(s): -p+j0 point marked

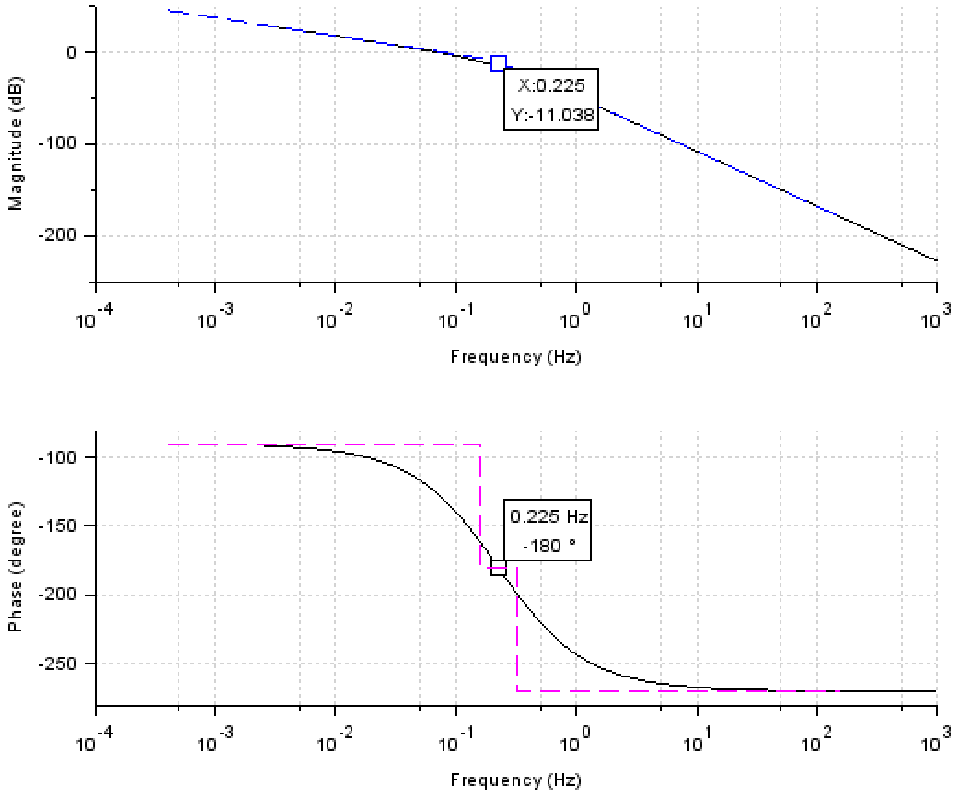

Figure 11: Asymptotic Bode Plot

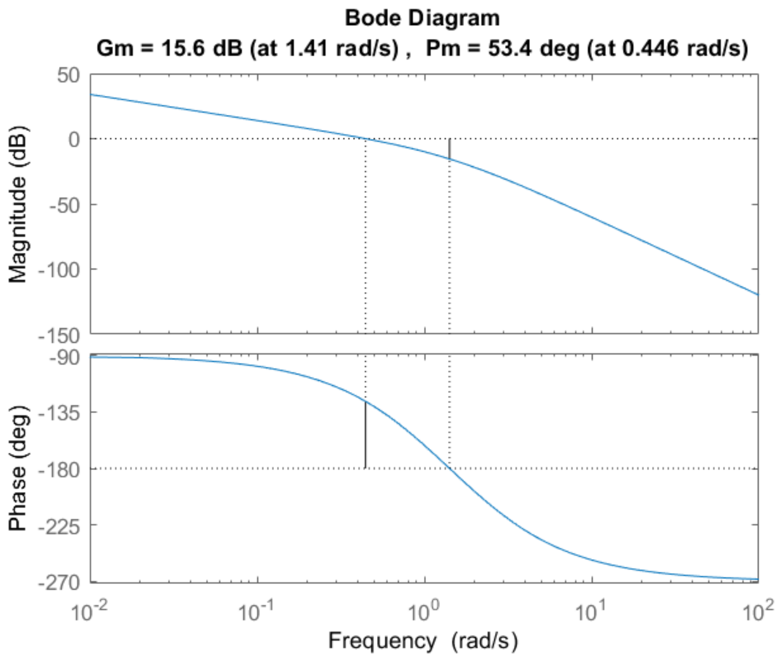

Figure 12: Bode Plot

Method Used	Calculation	K (gain margin magnitude)(in lin units)
Root locus	$GM = K_{crit} = 6$	6.000
Nyquist plot	$GM = \frac{1}{\lvert p \rvert} = \frac{1}{0.167} = 5.988$	5.988
Asymptotic Bode plot	$GM = 10^{11.038/20} = 3.564$	3.564
Bode plot	$GM = 10^{15.6/20} = 6.025$	6.025

MATLAB Code

```
clear;
close all;
clc;

syms s;
s=tf('s');
G=1/(s^3+3*s^2+2*s);

figure(1);nyquist(G);
figure(2);margin(G);
```

Scilab Code for assymptotic bode plot and Root locus calculations

```
clear;
clc;

s=poly(0,'s');
G = 1/(s^3+3*s^2+2*s);
sysG = syslin('c',G);

gcf();
evans(sysG);

gcf();
bode(sysG);
bode_asymp(sysG);
```

5. Problem 5

Given transfer function:

$$G(s) = \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

i) We get the following Bode plots

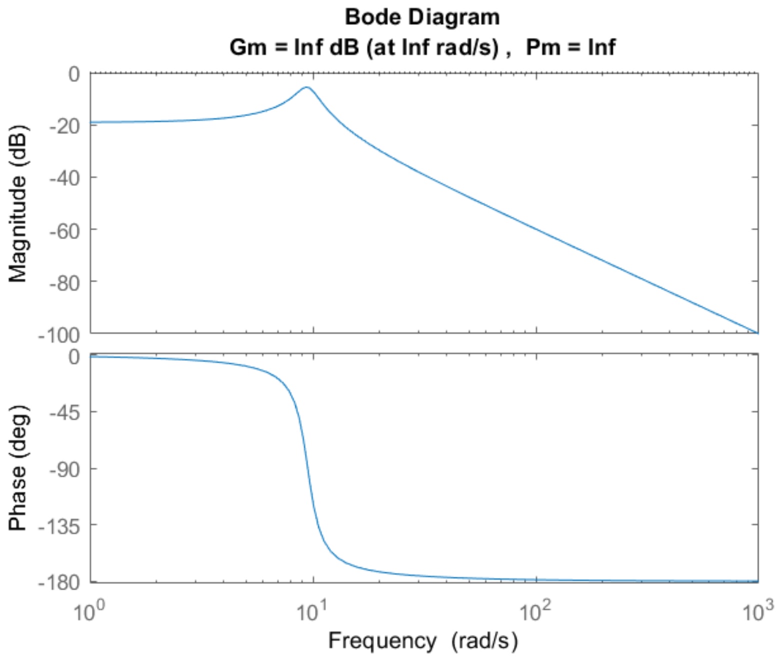

Figure 13: Bode Plot of G(s)

Here, we observe that gain margin and phase margin are both infinite in this case.

ii) As it is a 0-order system, we increase proportional gain 'K' to decrease steady state error to 0.1 for a step response as follows:

$$SSE = \frac{1}{1 + KG(0)} = 0.1$$
$$\implies KG(0) = 9$$
$$\implies K = 9/G(0) = 81.0045$$

iii) The bode plot for the modified open loop system is ashown below:

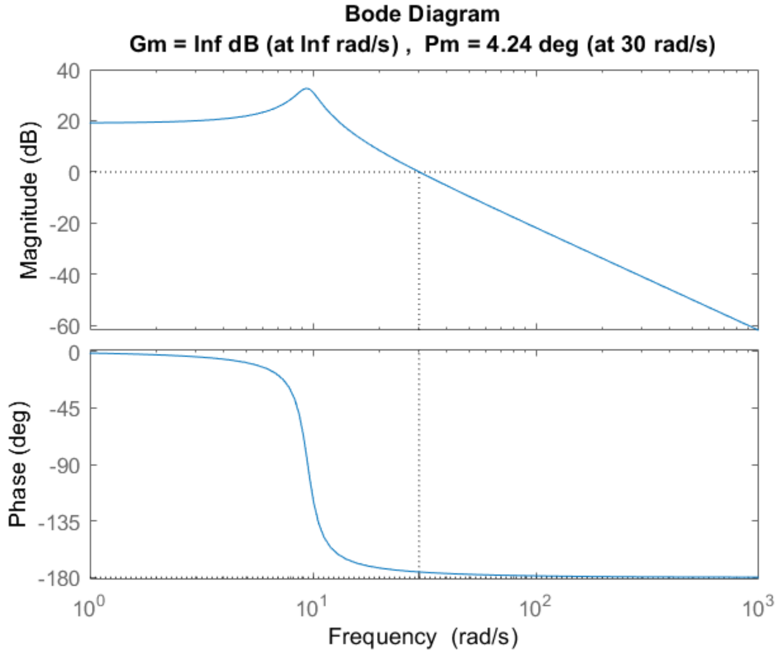

Figure 14: Bode plot for K*G(s)

Here we see that system has a phase margin of 4.24°, and gain margin still stays infinite.

iv) Now to improve the phase margin of the system, we add a zero such that the phase stops decreasing around the gain crossover frequency. We add the zero at $s = -1$.

$$G_{iv}(s) = \frac{K(10s + 2000)(s + 1)}{s^3 + 202s^2 + 490s + 18001}$$

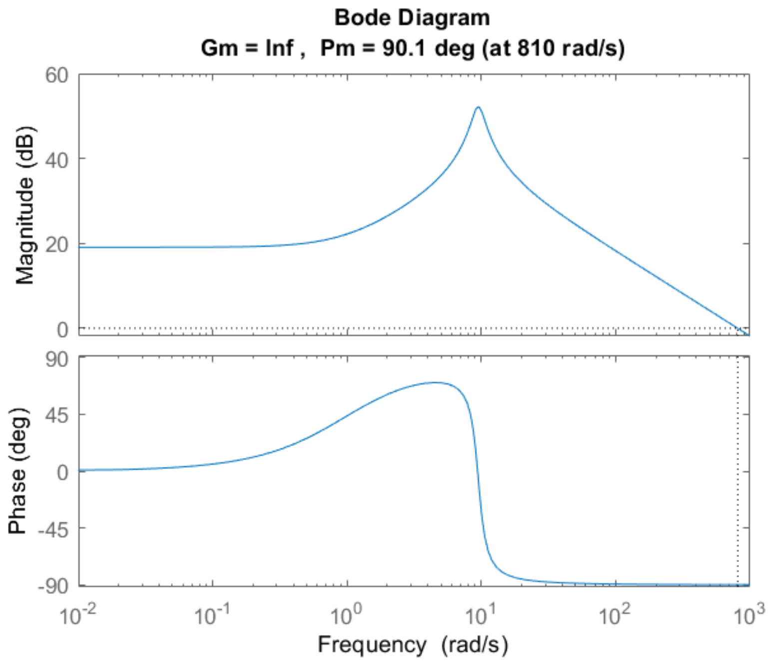

Figure 15: Bode Plot for K*G(s)*(s+1)

v) The closed loop system for $G_{iv}(s)$ is stable since we have positive gain and phase margins.

MATLAB Code

```
clear;
close all;
clc;

syms s;
s=tf('s');

G = (10*s+2000)/(s^3+202*s^2+490*s+18001);
figure;margin(G);

k=81.0045;
figure;margin(k*G);
figure;margin(k*G*(s+1));
```