EE324 Control Systems Lab

Problem Sheet 4

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1. Problem 1

1.1. Part (a)

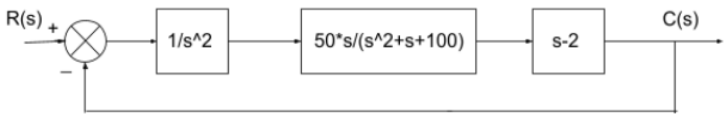


Figure 1: System A

The three blocks are in series, so simply a product of three transfer functions leads to equivalent transfer function along forward path. Using standard equation for feedback we get the transfer function of the system.
Using Scilab, we obtain:

```
    -100 + 50s
--------------------
                2   3
-100 + 150s + s + s
```

Figure 2: Input-Output Transfer Function obtained for System A

Scilab Code

```
s = poly(0, 's');
S_1 = 1/(s^2);
S_2 = (50*s)/(s^2+s+100);
S_3 = s-2;
G = S_1*S_2*S_3;// combination of 3 blocks
T = G/(1+G); // Upon solving
```

1.2. Part (b)

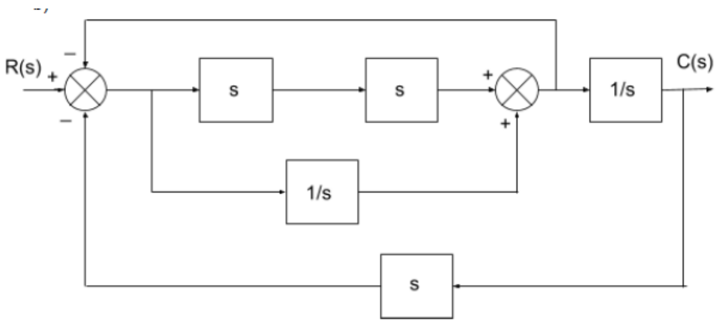


Figure 3: System B

Upon simplifiying the block diagram and writing the equations we get:

$$sC(s) = (R(s) - 2sC(s))\left(s^2 + 1/s\right)$$

$$\Longrightarrow C(s)\left(2s^3 + s + 2\right) = R(s)\left(s^2 + 1/s\right)$$

Using Scilab, we obtain:

$$\frac{1 + s^3}{2s + s^2 + 2s^4}$$

Figure 4: Input-Output Transfer Function for Part b

Scilab Code

```
s = poly(0, 's');
S_1 = s^2 + (1/s);
S_2 = 2*s^3 + s + 2;
T = S_1/S_2; // Upon solving
```

1.3. Part (c)

Upon simplifiying the block diagram and writing the equations we get:

$$2s(R(s)-C(s)) \; - \; 5C(s) \; + \left(3s^2/(s+1)\right)(R(s)-C(s)) \; = (s+1)C(s)$$

which finally leads to:

$$\implies C(s)\left(3s+6+\left(3s^2/(s+1)\right)\right) \; = R(s)\left(\left(3s^2/(s+1)\right)+2s\right)$$

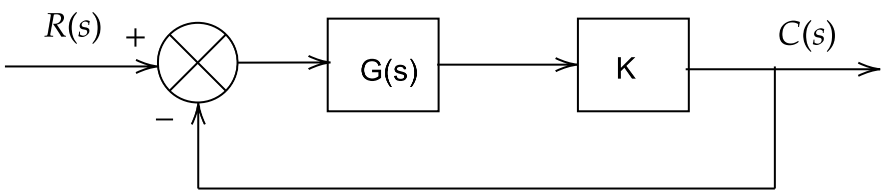
Using Scilab, we obtain:

```
                          2
0.3333333s + 0.8333333s
------------------------
                   2
    1 + 1.5s + s
```

Scilab Code

```
s = poly(0, 's');
S_1 = 2*s + ((3*s^2)/(s+1));
S_2 = ((3*s^2)/(s+1)) + 6 + 3*s;
T = S_1/S_2; // Upon solving
```

2. Problem 2



The transfer of the system is given by:

$$H(s, K) = \frac{KG(s)}{1 + KG(s)}$$

$$where,\ G(s) = \frac{10}{s(s+2)(s+4)}$$

2.1. Part (a)

For K = 5, we obtain the closed-loop transfer function as: (K value can be changed as required)

$$\frac{50}{50 + 8s + 6s^2 + s^3}$$

Scilab Code

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
K = 5; //Proportionality gain
C_tf = (G*K)/(1+(G*K));   // Closed loop transfer function
C_tf = syslin('c',C_tf);
disp(C_tf);
```

2.2. Part (b)

For plotting the loccii of the closed-loop poles we use evans() function from Scilab, with maximum gain value (K) as 100. The following plot is obtained:

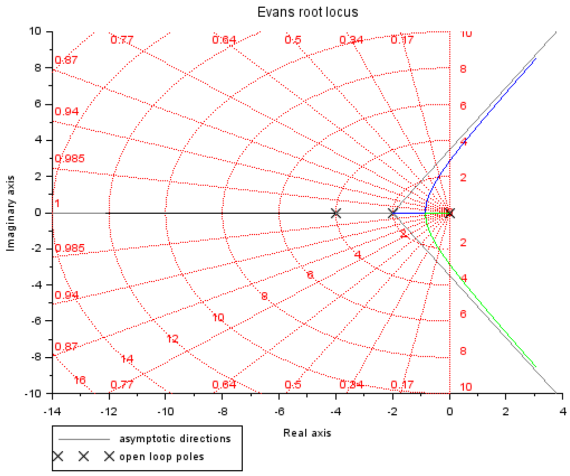


Figure 5: Loci of Closed-loop poles

Scilab Code

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
G = syslin('c',G);
// evans finds root-locii of 1+K*G(s)
evans(G, 100);  // constructing root locus
sgrid('red');
```

2.3. Part (c)

The code from part b was used here as well, so we obtain that the value of $K < 4.8$ are stable, as all poles in LHP. The plot with the critical point marked is shown below:

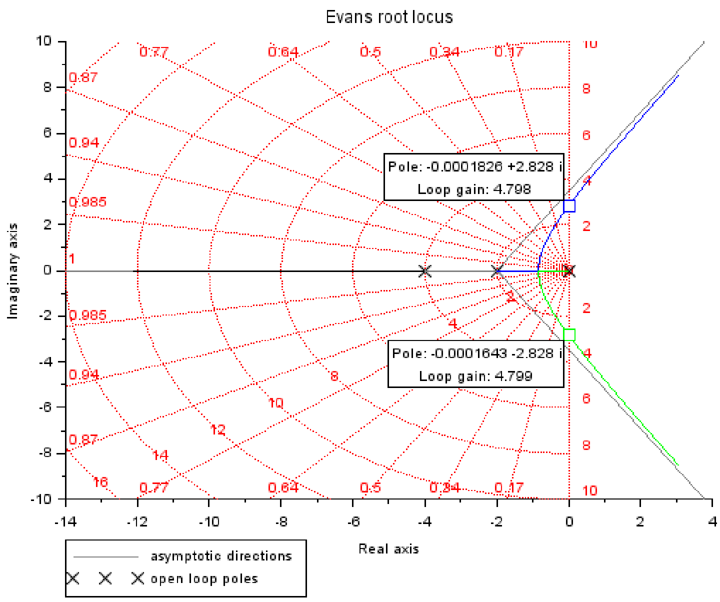


Figure 6: Critical points marked

2.4. Part (d)

To confirm our answer, we display the number of sign changes for $K_c - 0.1$, K_c, $K_c + 0.1$. As expected, we see no sign change in $H(s, K_c - 1)$ and a sign change in $H(s, K_c)$ and $H(s, K_c + 0.1)$. The routh tables obtained for each of them are shown below:

```
r  =

  1.           8.
  6.           47.
  0.1666667    0.
  47.          0.
```

Figure 7: Routh Table for $K_c - 0.1$

```
r  =

  1.           8.
  6.           48.
 -8.882D-15    0.
  48.          0.
```

Figure 8: Routh Table for K_c

```
r  =

  1.           8.
  6.           49.
 -0.1666667    0.
  49.          0.
```

Figure 9: Routh Table for $K_c + 0.1$

These confirm the results obtained in Part c.

Scilab Code

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
K = 4.8; // K_c
Tf = (K*G)/(1+K*G);
[r,num] = routh_t(Tf.den); // r is the routh table
disp(num);          // num is the number of sign changes
```

3. Problem 3

3.1. Part (a)

$$P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$

1.	5.	1.
3.	4.	3.
3.6666667	0.	0.
4.	3.	0.
-2.75	0.	0.
3.	0.	0.

Figure 10: Routh Table

3.2. Part (b)

$$P(s) = s^5 + 6s^2 + 5s^2 + 8s + 20$$

$\frac{1}{1}$	$\frac{6}{1}$	$\frac{8}{1}$
$\frac{eps}{1}$	$\frac{5}{1}$	$\frac{20}{1}$
$\frac{-5 + 6eps}{eps}$	$\frac{-20 + 8eps}{eps}$	$\frac{0}{1}$
$\frac{-25 + 50eps - 8eps^2}{-5 + 6eps}$	$\frac{20}{1}$	$\frac{0}{1}$
$\frac{-2.274D-13 - 160eps - 64eps^2}{-25 + 50eps - 8eps^2}$	$\frac{0}{1}$	$\frac{0}{1}$
$\frac{20}{1}$	$\frac{0}{1}$	$\frac{0}{1}$

Figure 11: Routh Table

3.3. Part (c)

$$P(s) \;=\; s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4$$

1.	3.	2.
-2.	-6.	-4.
-8.	-12.	0.
-3.	-4.	0.
-1.3333333	0.	0.
-4.	0.	0.

Figure 12: Routh Table

3.4. Part (d)

$$P(s) \;= s^6 \;+\; s^5 \;-\; 6s^4 \;+\; s^2 \;+\; s \;-\; 6$$

```
1      -6   1    -6
--     ---  --   ---

1      1    1    1

1      0    1    0
--     --   --   --

1      1    1    1

-6     0    -6   0
---    --   ---  --

1      1    1    1

-24    0    0    0
----   --   --   --

 1     1    1    1

eps    -6   0    0
----   ---  --   --

 1     1    1    1

-144   0    0    0
-----  --   --   --

eps    1    1    1

864    0    0    0
-----  --   --   --

-144   1    1    1
```

Figure 13: Routh Table

Scilab Code

```
// Part a
s = poly(0, 's');
G = s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
[r,num] = routh_t(G); // r is the routh table
disp(r);          // num is the number of sign changes

//Part b
clear;
s = poly(0, 's');
G = s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
[r,num] = routh_t(G); // r is the routh table
disp(r);          // num is the number of sign changes

//Part c
clear;
s = poly(0, 's');
G = s^5 - 2*s^4 + 3*s^3 - 6*s^2 + 2*s - 4;
[r,num] = routh_t(G); // r is the routh table
disp(r);          // num is the number of sign changes

//Part d
clear;
s = poly(0, 's');
G = s^6 + s^5 - 6*s^4 + s^2 + s - 6;
[r,num] = routh_t(G); // r is the routh table
disp(r);          // num is the number of sign changes
```

4. Problem 4

4.1. Part (a)

Construct a degree 6 polynomial whose R-H table has its entire row corresponding to s^3 to be zero.

s^6	1	4	7	4
s^5	2	6	8	0
s^4	1	3	4	0
s^3	0	0	0	0

$$P(s) = s^6 + 2s^5 + 4s^4 + 6s^3 + 7s^2 + 8s + 4$$

4.2. Part (b)

Repeat Part (a) with a polynomial of degree 8 and having the entire row corresponding to s^3 to be zero.

s^8	1	7	17	19	4
s^7	1	6	13	12	0
s^6	1	4	7	4	0
s^5	2	6	8	0	0
s^4	1	3	4	0	0
s^3	0	0	0	0	0

$$P(s) = s^8 + s^7 + 7s^6 + 6s^5 + 17s^4 + 13s^3 + 19s^2 + 12s + 4$$

4.3. Part (c)

Construct a degree 6 polynomial whose R-H table has the first entry in its row corresponding to s^3 to be zero.

s^6	1	4	$\frac{15}{2}$	4
s^5	2	6	9	0
s^4	1	3	4	0
s^3	0	1	0	0

$$P(s) = s^6 + 2s^5 + 4s^4 + 6s^3 + \frac{15}{2}s^2 + 9s + 4$$