Analog Modulation and Demodulation

EE 340: Prelab Reading Material for Experiment 2

August 22, 2019

Modulation is the process of manipulation of a carrier wave to add message signal to it. The carrier is generally a high frequency periodic signal, whereas the message signal is typically a lower frequency signal occupying a finite non-zero bandwidth. Generally, the message signals are referred to as baseband signals and are lowpass in nature. On the other hand, the modulated signals are passband in nature, which makes it easier to transmit them over communication channels, such as the wireless channel. The process of *demodulation* is the inverse of the modulation process, which basically involves recovery of the message signal from the received modulated carrier. At a very basic level, the analog modulation techniques can be classified as amplitude modulation (AM), phase modulation (PM) and frequency modulation (FM). In Experiment 2, we will study modulation and demodulation techniques for AM and FM signals, which are commonly used for transmitting analog signals.

1 Amplitude Modulation

In amplitude modulation (AM), the amplitude of the carrier wave, $c(t) = \cos(2\pi f_c t)$, is varied with the amplitude of the message signal $x(t)$ to obtain the desired modulated signal $s(t)$. In Experiment 2, we will study the following two AM schemes, which are commonly used for transmission of analog message signals:

1. DSB-FC (double side-band with full carrier) In DSB-FC, the carrier is multiplied by the message signal to obtain the desired modulated signal, but after adding an offset to the signal before multiplication, so that the envelope of the signal never crosses zero. Therefore, we can write

$$s(t) = [1 + m.x(t)] \cos(2\pi f_c t)$$

where m is called the modulation index, which is chosen to ensure that $|m.x(t)| < 1$ always, so that the envelope never crosses zero (as shown in Fig.1 for the DSB-FC signal)

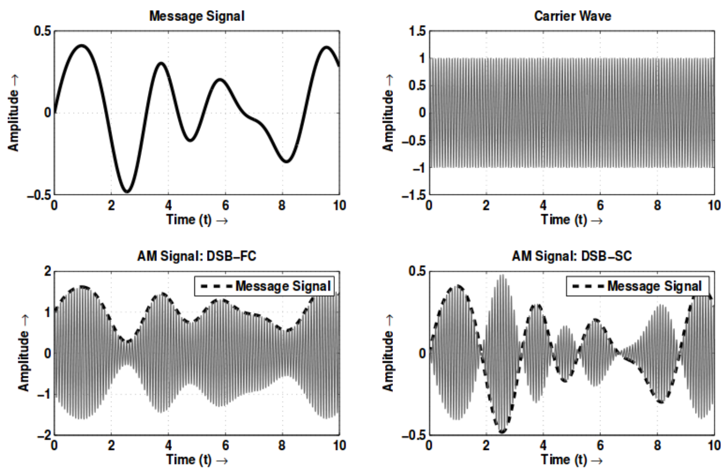

Figure 1: DSB-FC and DSB-SC AM signals in time domain

2. DSB-SC (double side-band suppressed carrier): In DSB-SC, the carrier is directly multiplied by the message signal to obtain the desired modulated signal. For DSB-SC, we can write the modulated signal as

$$s(t) = x(t)\cos(2\pi f_c t)$$

The time domain and frequency domain representations of the DSB-FC and DSB-SC AM signals are shown in Figures 1 and 2, respectively. Transmission of DSB-SC signals is more power efficient as no power is spent in transmission of the carrier. However, it is easier to demodulate a DSB-FC signal as it requires a simple envelope detector and does not require precise knowledge of the carrier frequency.

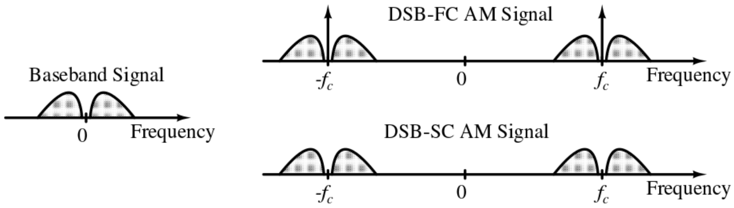

Figure 2: Baseband message signal and its corresponding DSB-FC and DSB-SC AM signals in frequency domain

1.1 Demodulation of AM signals

As can be observed from Fig. 1, the message signal can be demodulated from the DSB-FC signal by simply using an envelope detector, i.e. by passing the signal through a rectifier (or taking its absolute value), and then passing the resultant waveform through a low-pass filter. However, this approach cannot be used for demodulating the DSB-SC signals as the zero crossings in the message signal result in sign ambiguity because the envelope detector output is always positive. For demodulating a DSB-SC signal, the modulated signal is first multiplied by the carrier signal $\cos(2\pi f_c t)$. The resultant waveform is passed through a low pass filter (LPF) to remove the $2f_c$ frequency component from it to obtain the message signal as the demodulated output:

$$x(t).\cos(2\pi f_c t) \times \cos(2\pi f_c t) = \frac{x(t)}{2} + \frac{1}{2}\cos(4\pi f_c t) \xrightarrow{LPF} \frac{x(t)}{2}$$

For this operation to work successfully, the carrier frequency f_c has to be estimated precisely at the receiver, which is not very straight-forward.

2 Frequency Modulation

In the case of FM, the instantaneous frequency of the carrier wave, $c(t) = \cos(2\pi f_c t)$, is varied with the amplitude of message signal $x(t)$. The instantaneous frequency of the modulated signal can be written as $f(t) = f_c + f_\Delta \cdot x(t)$, where f_Δ is the maximum frequency deviation away from f_c if $x(t)$ is normalized such that the maximum value $|x(t)| = 1$. Therefore, the phase of the FM signal is

$$\phi(t) = \int_0^t 2\pi f(\tau)d\tau = 2\pi f_c t + 2\pi f_\Delta \int_0^t x(\tau)d\tau$$

and the modulated FM signal can be written as

$$s(t) = \cos(2\pi f_c t + 2\pi f_\Delta \int_0^t x(\tau)d\tau)$$

If the message signal is represented by the sinusoid $x(t) = \sin(2\pi f_m t)$, the FM signal, ignoring the constant of integration, becomes

$$s(t) = \cos(2\pi f_c t + \frac{f_\Delta}{f_m}\cos(2\pi f_m t))$$

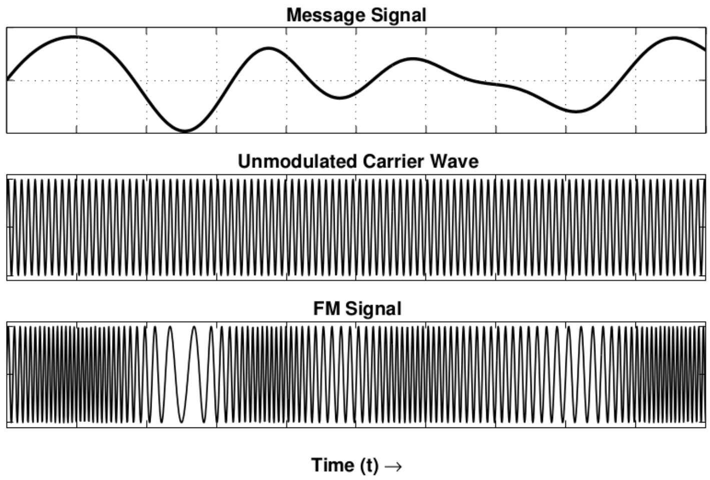

Figure 3: Time domain representation of an FM (Frequency Modulated) signal

The ratio $h = f_\Delta/f_m$ is called the modulation index of the FM signal, where f_m is the highest frequency component in $x(t)$. If $h \ll 1$, the modulation is called *Narrow-band FM (NBFM)*, while for $h \leq 1$, it is called *Wide-band FM (WBFM)*. In theory, an FM signal occupies an infinite bandwidth.

However, as per the *Carson's Rule*[1], for practical purposes, most of the power in the FM signal spectrum is restricted between the frequencies $(f_c - f_m - f_\Delta)$ and $(f_c + f_m + f_\Delta)$. The bandwidth occupied by the FM signal is $BW \approx 2(f_m + f_\Delta)$. In general, the WBFM signals have much better signal-to-noise ratio compared to the NBFM signals, but occupy much larger bandwidth.

2.1 Demodulation of the FM Signals

In the analog domain, the FM signals can be demodulated using a phase locked loop (PLL), in which a feedback loop tracks the incoming signal frequency by adjusting the control voltage applied to the VCO (voltage controlled oscillator) in the loop, as shown in Fig. 4. Therefore, this control voltage changes with the instantaneous frequency of the incoming signal (which depends on the message signal amplitude), and hence demodulates the message signal. Some other analog domain techniques for demodulating FM signal are also used, but will not be a part of this discussion. Generally, to demodulate the FM signal in a digital signal processor or in a software, first the phase of the incoming signal is extracted. The frequency information is then obtained by differentiating the phase with respect to time, to obtain the demodulated message signal. You will be using this approach for demodulating FM signals in GNU radio.

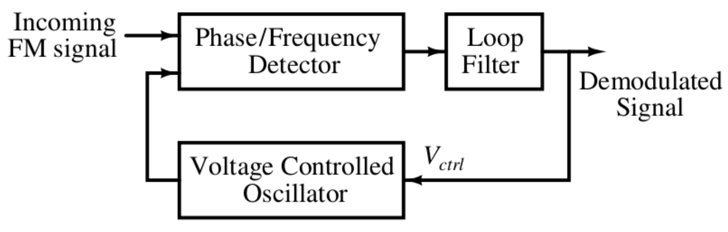

Figure 4: Use of a PLL for demodulating FM signals. The Phase/Frequency Detector (PFD) block compares the phase (represented by the time instants of the zero crossings) of the incoming signal with that of the fed-back signal. The output of the PFD followed by the Loop Filter block is a voltage signal which is proportional to the difference between the two phases

[1] J. R. Carson, Notes on the theory of modulation, *Proc. IRE* , vol. 10, no. 1, pp. 57-64, February 1922

3 Discrete Time Filters for Implementation in GNU Radio

In a computer or a digital signal processor, signals are represented as discrete-time samples. Consider an LTI system for which the n^{th} input and output samples are represented by $x[n]$ and $y[n]$, respectively. Therefore, for a causal system, one can write

$$a_0 y[n] = (a_1 y[n-1] + a_2 y[n-2] + ...) + (b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + ...) \tag{1}$$

A discrete-time signal can be represented in the *z-domain*, which is equivalent to frequency-domain representation of the discrete-time signal. In the *"z-domain"*, the delay by one sample period is represented by z^{-1}, which means $z^{-1} = e^{-j\omega T}$, where T is the sampling period. The delay by one sample period in the time-domain implies multiplication of the signal by z^{-1} in the z-domain. Therefore, the z-transform of $x[n-n_0]$ becomes $X(z)z^{-n_0}$ if $X(z)$ is the z-transform of $x[n]$. Further, if $X(z)$ and $Y(z)$ are z-transforms of $x[n]$ and $y[n]$ respectively, (1) can be rewritten in the z-domain as

$$a_0 Y(z) = Y(z)(a_1 z^{-1} + a_2 z^{-2} + ...) + X(z)(b_0 + b_1 z^{-1} + b_2 z^{-2} + ...)$$

Therefore, for this system, the transfer function in z-domain can be expressed as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ...}{a_0 - a_1 z^{-1} - a_2 z^{-2} - ...} \tag{2}$$

The above expression represents an IIR (infinite impulse response) filter, because $y[n]$ depends on the previous output samples $y[n-1]$, $y[n-2]$, $y[n-3]$, ..., in addition to the input samples, and therefore may last for an infinite duration for an impulse input. However, if a_1, a_2, a_3, ... are all zero (and assuming that the number of non-zero coefficients b_0, b_1, b_2, ... is finite), the response to an impulse inputs lasts for a finite duration only, and the corresponding filter is known as an FIR (finite impulse response) filter.
The IIR block in GNU radio software implements the above transfer function, where b_0, b_1, b_2, ... represent feed-forward taps and a_0, a_1, a_2, ... represent feedback taps. The FIR block can be used for implementing an FIR filter. Alternatively, the FIR filter can also be implemented using the IIR block and setting the filter coefficients a_1, a_2, a_3, ... $= 0$

Important Notes :

1. For implementing (1) and (2) using the IIR block, use the default setup, i.e., **Old style of taps = TRUE** in the IIR block. **If Old style of taps = FALSE is used**, the following equation is implemented:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ...}{a_0 + a_1 z^{-1} + a_2 z^{-2} + ...}$$

   which means the signs of a_1, a_2, a_3, ... have to be inverted. In this lab course, we will always be assuming that the default setup, i.e.,*Old style of taps = TRUE* is used.

2. The value of a_0 is always taken as 1 by the software, even if you specify some other value. Therefore, make sure that you have used $a_0 = 1$ in your calculations.

FM Modulator and Demodulator Implementation:
The FM modulator and demodulator implementations in GNU Radio will require use of an integrator and a differentiator, respectively. In the software, the discrete time equations $y[n] = y[n-1] + T * x[n]$ and $y[n] = (x[n] - x[n-1])/T$ can be used for implementing an integrator using the IIR block and a differentiator using the FIR block, respectively, where T is the sampling period. More details about the implementation of the modulator and demodulator blocks will be made available in the lab-sheets.