Report

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Global force planner

- *Modified Ge cui potential fields*
- *Gaussian hills for continuous ranges*
- *Tuning of hill parameters*
- *Velocity planning (speed)*

Local force planner

- *Minguez space transformations*
- *Obstacles in cars frame*
- *C-Space*
- *Dynamics*
- *EKT space*
- *Local force planner*

Hybrid force and controllers

- *Method of hybrid force tuning*
- *Controller for Ackermann kinematic model for car like robot*

Global force planner

Modified Ge Cui Potential fields

$$F_{attractive} = Ge\ cui\ attractive\ force * C_{VAR_{ATTRACTION}}$$

$$F_{repulsive} = C_{VAR_{NRO}} * (Frep1 + Frep2) + C_{VAR_{NRO_{PERP}}} * Frep2 + C_{VAR_{REVERSE}} * Frep2$$

$$\mathbf{F}_{rep1} = \frac{-\eta}{(\rho_s(\mathbf{p}, \mathbf{p}_{obs}) - \rho_m(v_{RO}))^2} \left(1 + \frac{v_{RO}}{a_{\max}}\right) \mathbf{n}_{RO}$$

$$\mathbf{F}_{rep2} = \frac{\eta v_{RO} v_{RO\perp}}{\rho_s(\mathbf{p}, \mathbf{p}_{obs}) a_{\max} (\rho_s(\mathbf{p}, \mathbf{p}_{obs}) - \rho_m(v_{RO}))^2} \mathbf{n}_{RO\perp}$$

**removed vro_perp term in Frep2*

Refer ge cui 2002 paper for more details

$$C\_VAR_{NRO} = \boldsymbol{k} * \boldsymbol{e}^{-(\boldsymbol{D_{OBS}} - \boldsymbol{x2})^2/\boldsymbol{\beta}^2}$$

$$C\_VAR_{NRO_{PERP}} = \boldsymbol{k} * \boldsymbol{e}^{-(\boldsymbol{D_{OBS}} - \boldsymbol{x1})^2/\boldsymbol{\beta}^2}$$

$$C_{VAR_{ATTRACTION}} = \mathbf{1}/\left(\mathbf{1} + \boldsymbol{e}^{-(\boldsymbol{D_{OBS}} - \boldsymbol{K1})}\right)$$

$$C_{VAR_{REVERSE}} = \boldsymbol{k} * \mathbf{1}/\left(\mathbf{1} + \mathbf{0.1} * \boldsymbol{e}^{-(\boldsymbol{D_{OBS}} - \boldsymbol{x3})}\right)$$

$$\boldsymbol{F\_NET\_Global} =$$

$$\left\{ \begin{array}{ll} \boldsymbol{IF\ vro > 0} & : \boldsymbol{F_{ATTRACTIVE}} + \boldsymbol{F_{REPULSIVE}} \\ \boldsymbol{ELSE} & : \boldsymbol{F_{ATTRACTIVE}} \end{array} \right\}$$

Parameters:

eta1_global and eta2_global is for global repulsive force

*** highlighted parts are the changes made*

Beta- width of hills

x1 – peak of hill 1

x2 -peak of hill 2

x3 -peak of hill 3

k – height of hills

*k1- **effective distance below which sigmoid decreases fastest to 0***

Gaussian hill parameters and tuning

Tuning of parameter's

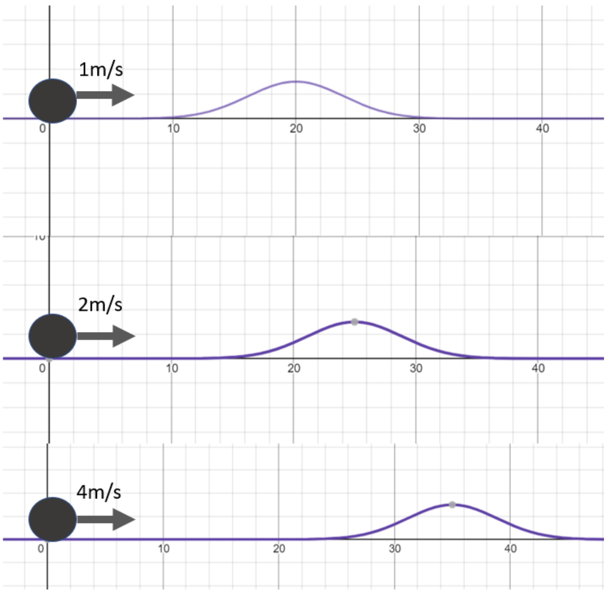
1) *Tuning of x1 x2 and x3*

$$\boldsymbol{if vro \le |velocity_{car}| : x1 = k1; x2 = k2; x3 = k3}$$

$$\boldsymbol{else\ if\ vro > |velocity_{car}| : x1 = k1 + 0.9 * vro\ ; x2 = k2 + 0.9 * vro\ ; x3 = k3 + 0.9 * vro;}$$

$$\boldsymbol{if\ x1\ \ge k3\ ||\ x2\ \ge k4\ ||\ x3 \ge k5 :\ x1 = k3;\ x2 = k4; x3 = k5;}$$

- *varying hill peaks based on relative velocity of car and obstacles*



Velocity planning

$$\boldsymbol{if cos\left(differ_{heading}\right) < 0}$$

{

$$|\boldsymbol{velocity_{car}}| = |\boldsymbol{velocity_{car}}| - \boldsymbol{a_{max}} * \boldsymbol{t_{simulator}}$$

}

Else

{

$$|\boldsymbol{velocity_{car}}| = |\boldsymbol{velocity_{car}}| + \boldsymbol{a_{plan}} * \boldsymbol{t_{simulator}}$$

}

Differ_heading is the angle between heading of car and f_net (local +global)

$$\boldsymbol{a_{plan}} = \boldsymbol{a_{max}} * \boldsymbol{tanh}\frac{\left(0.1*\left((alpha_{param}*param_{dist}+beta_{param}*param_{var})\right)\right)}{\left(alpha_{param}+beta_{param}\right)}$$

$$\boldsymbol{param_{dist}} = \boldsymbol{L_{MAX}} * \left(\boldsymbol{1} - \frac{\boldsymbol{VAR}}{\boldsymbol{50}}\right)$$

$$\boldsymbol{param_{var}} = \boldsymbol{L_{MAX}} * \left(\frac{\boldsymbol{MIN_{D_{OBS}}}}{\boldsymbol{d_{safe}}} - \boldsymbol{1}\right)$$

VAR → variance of forces acting on car

$$\sum_{i=1}^{n} \frac{|fi|}{\sum|fi|} * ((atan2\left(f_{net(2)}, f_{net(1)}\right) - atan2(fi(2), fi(1)))^2$$

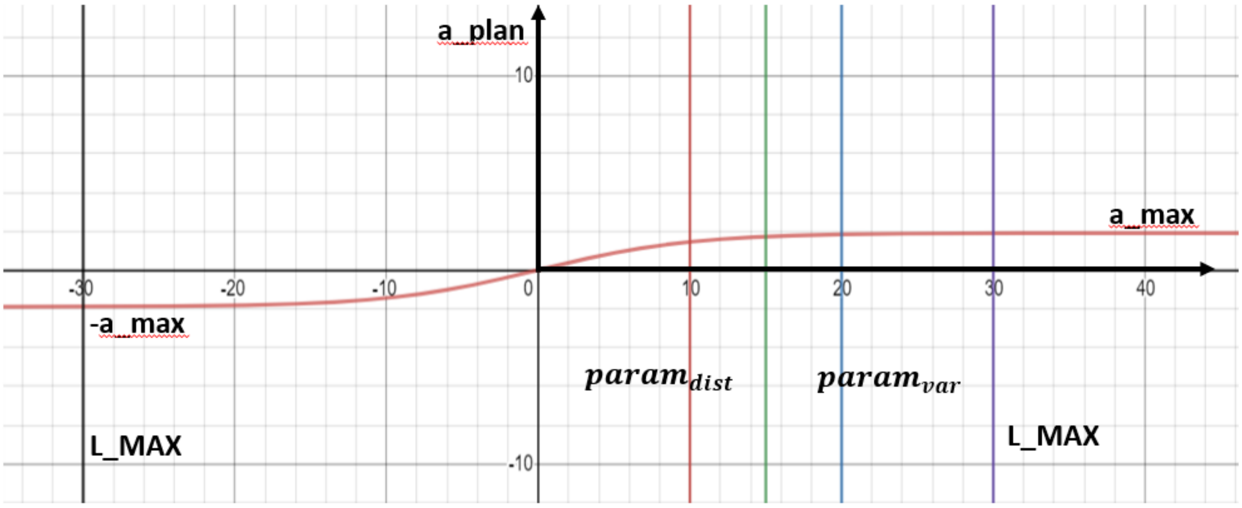
$\boldsymbol{MIN_{D_{OBS}} is\ the\ minimum\ distance\ to\ obstacle}$

$\boldsymbol{d_{safe}}$ is the safe distance from obstacle

$\boldsymbol{L_{MAX}}$ is a constant that is defined for tanh functioning

$\boldsymbol{alpha_{param}}$ is the weight given to $\boldsymbol{param_{dist}}$

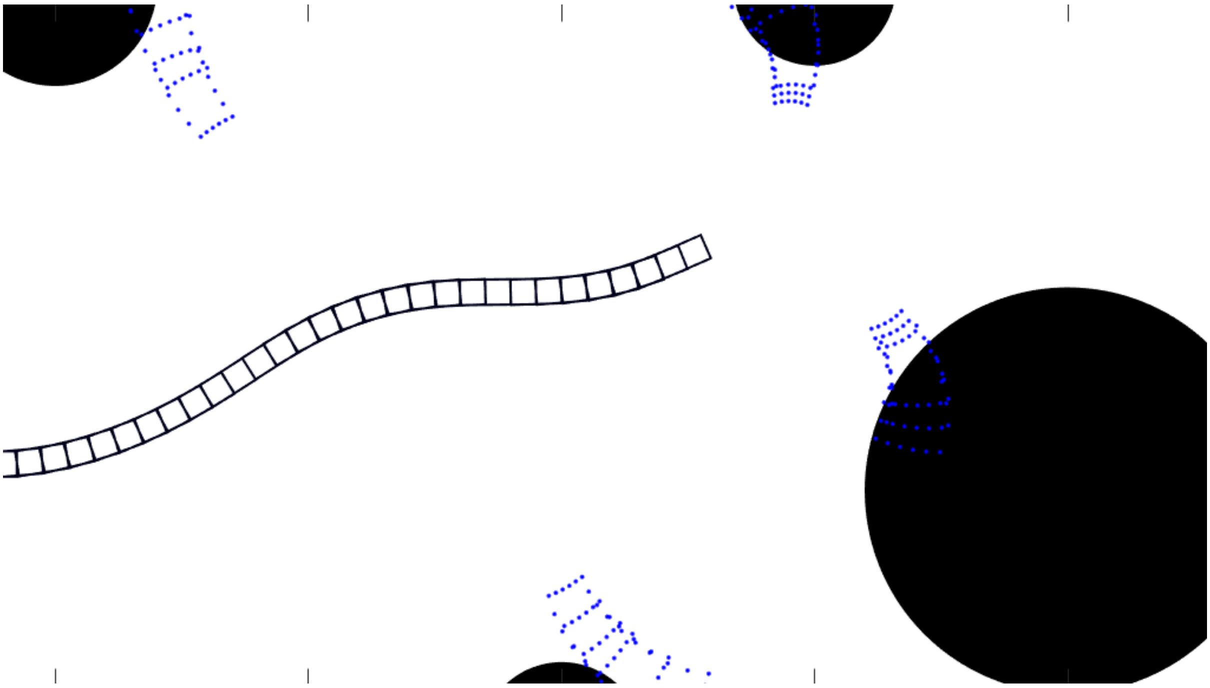
$\boldsymbol{beta_{param}}$ is the weight given to $\boldsymbol{param_{var}}$


a_plan
10
a_max
-30
-20
-10
0
10
20
30
40
-a_max
$param_{dist}$
$param_{var}$
L_MAX
L_MAX
-10

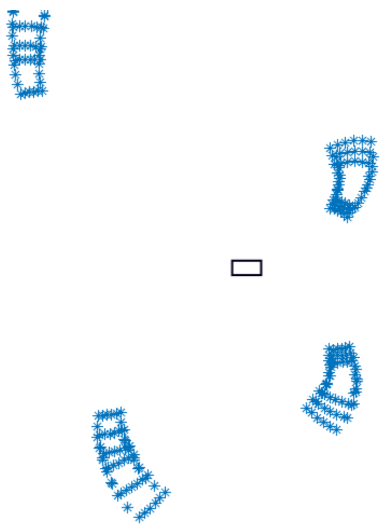
Local Planner

<u>With reference to Minguez's work</u>

➔ Transforming obstacles in cars frame and selecting local obstacle points



- *Selecting obstacle points inside* v_{radius} ($\boldsymbol{within\ a\ particluar\ radius\ depending\ on\ velcoity}_{car}$)
- *Transforming points to cars frame of reference from global frame of reference using rotation matrices*



C space with dynamic boundary of selected obstacles in cars frames

➔ C – Space

given possible robot trajectories find possible collision space

(xf,yf) ➔ obstacle point
(xi,yi) ➔ point on robot boundary

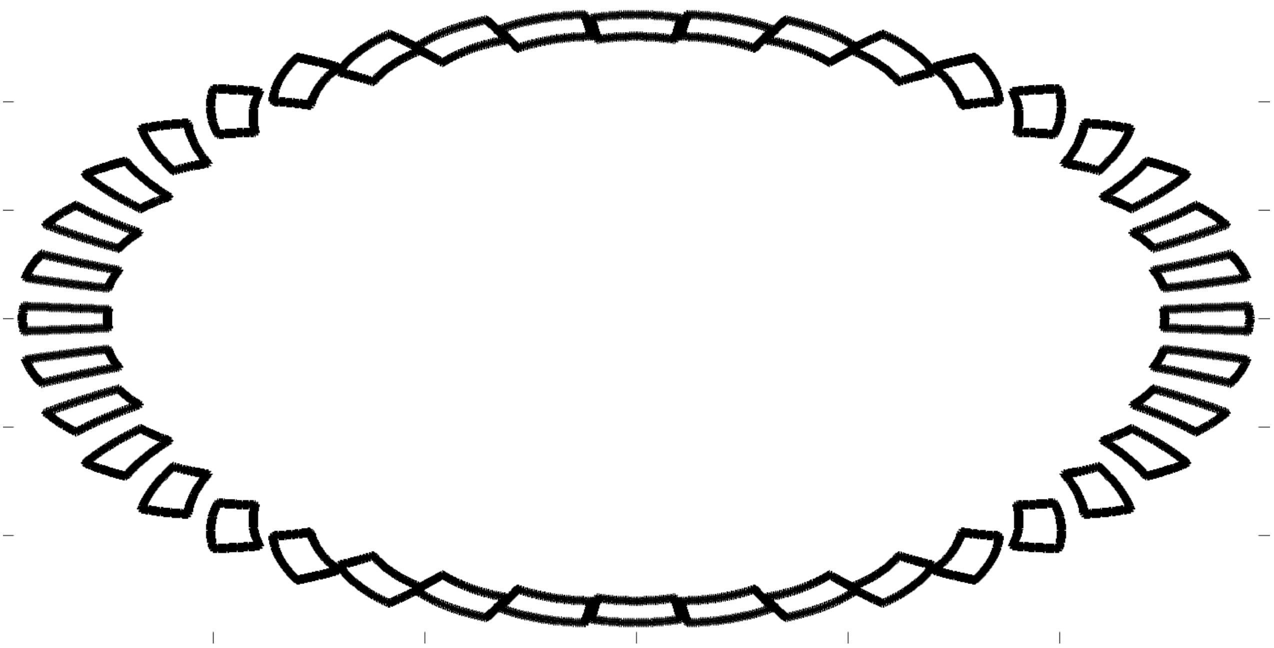
Parameterizing the robot shape as a rectangle

$$\begin{cases} x_i(\lambda) = x_1 + (x_2 - x_1)\cdot\lambda \\ y_i(\lambda). = y_1 + (y_2 - y_1)\cdot\lambda \end{cases}$$

Calculate c space for each point on robot boundary with the obstacle point

$$x_s = \frac{(x_f + x_i)\cdot\left[\left(y_f^2 - y_i^2\right) + \left(x_f^2 - x_i^2\right)\right]\cdot\left[(y_f - y_i)^2 + (x_f - x_i)^2\right]}{(y_f - y_i)^4 + 2\left(x_f^2 + x_i^2\right)(y_f - y_i)^2 + \left(x_f^2 - x_i^2\right)^2}$$

$$y_s = \frac{(y_f - y_i)\cdot\left[\left(y_f^2 - y_i^2\right) + \left(x_f^2 - x_i^2\right)\right]\cdot\left[(y_f - y_i)^2 + (x_f - x_i)^2\right]}{(y_f - y_i)^4 + 2\left(x_f^2 + x_i^2\right)(y_f - y_i)^2 + \left(x_f^2 - x_i^2\right)^2}$$



➔ Dynamics

region contains the configurations reached with a control after a time interval that can't be cancelled by max deceleration before collision with c-space

For detailed steps refer paper

1) Translational dynamics

$$\mathbf{p_1^v} = \begin{cases} (\text{sign}(x) \cdot L^v_{max}, 0), & \text{if } y = 0 \\ (r \sin \frac{\text{sign}(x) \cdot L^v_{max}}{r}, \\ \quad r(1 - \cos \frac{\text{sign}(x) \cdot L^v_{max}}{r})), & \text{otherwise} \end{cases}$$

$$L^v_{max} = L - L^v_{brake} \tag{8}$$

where $L^v_{max} = vT$, and $L^v_{brake} = \frac{v^2}{2a_v}$. Expanding and solving:

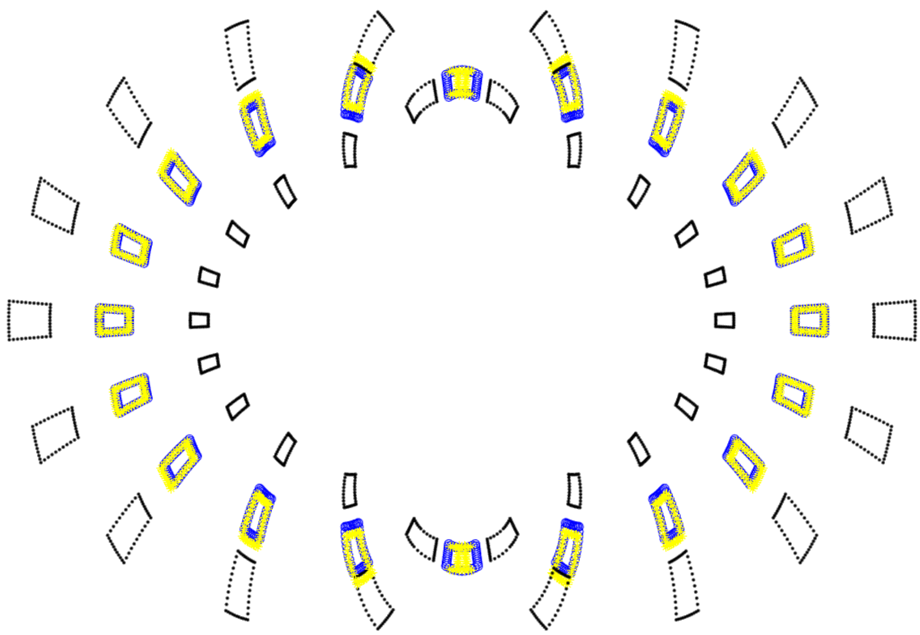
$$L^v_{max} = a_v T^2 (\sqrt{1 + \frac{2L}{a_v T^2}} - 1) \tag{9}$$

2) Rotational dynamics

$$\mathbf{p_1^\omega} = \begin{cases} (\infty, 0), & \text{if } y = 0 \\ (r \sin(\text{sign}(y) \cdot \theta^\omega_{max}), \\ \quad r(1 - \cos(\text{sign}(y) \cdot \theta^\omega_{max}))), & \text{otherwise} \end{cases} \tag{10}$$

$$\theta^\omega_{max} = sign(\theta) \cdot a_\omega T^2 (\sqrt{1 + \frac{2|\theta|}{a_\omega T^2}} - 1)$$

Selecting the regions farther from c space as dynamics boundary



➔ EKT transformation (considering non holonomic kinematics) -ego-kinematic transformation

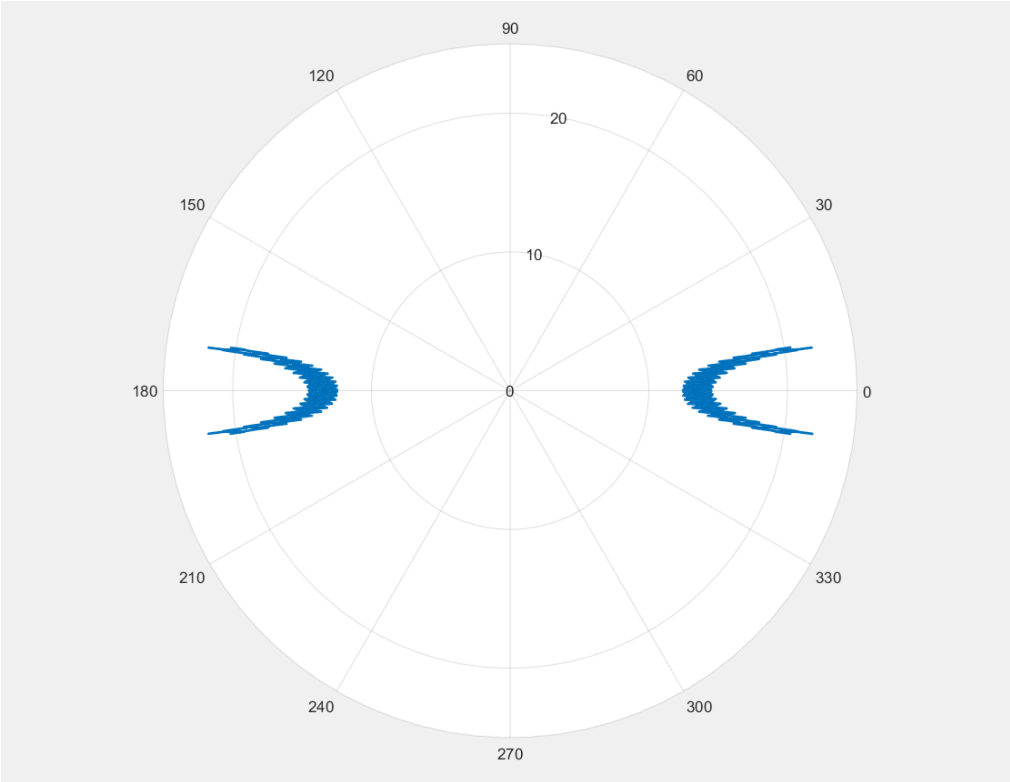
$$(x, y) \rightarrow (L, \alpha)$$

$$L = \begin{cases} |x|, y = 0 \\ |r \cdot \theta|, y \neq 0 \end{cases}$$
$$= \begin{cases} |x|, & y = 0 \\ \left| \frac{x^2 + y^2}{2y} \operatorname{atan2}(2xy, x^2 - y^2) \right|, & y \neq 0 \end{cases}$$

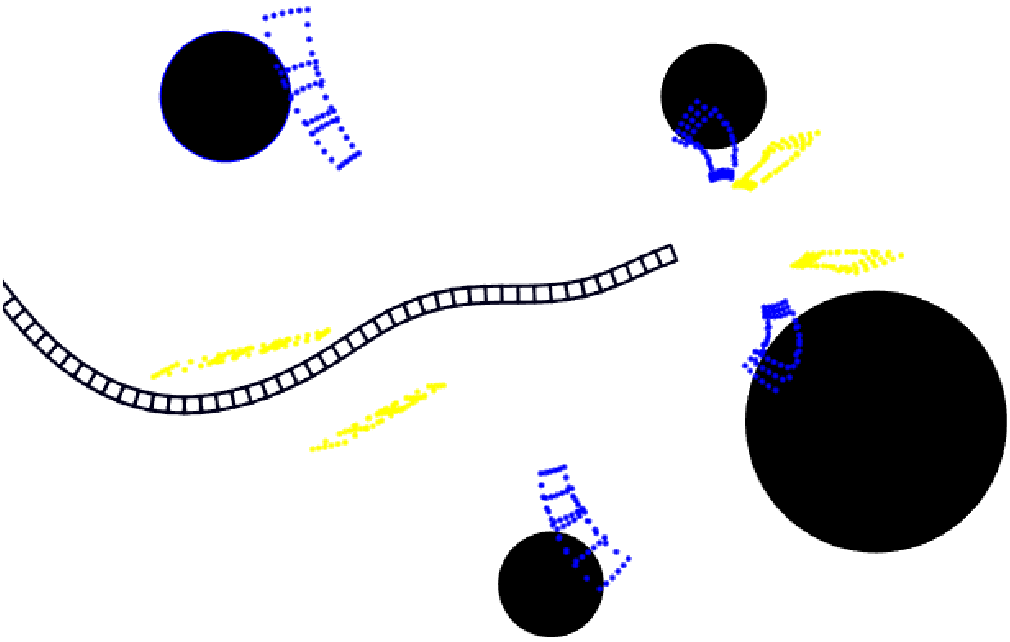
$$\alpha' = \arctan\left(\frac{2y}{x^2 + y^2}\right)$$

The second coordinate is then:

$$\alpha = \begin{cases} \alpha', & x \geq 0 \\ \operatorname{sign}(y) \cdot \pi - \alpha', & x < 0 \end{cases}$$



ekt space in polar plot



Ekt with c-space and dynamics in cartesian plot

➔ Local force planner

Using modified khatibs potential fields as local planner ($\boldsymbol{l}, \boldsymbol{\alpha}$ from ekt)

$$\boldsymbol{if\ l \leq \rho}\ \{\ \boldsymbol{f_{local_{rep}}} = \boldsymbol{eta1_{local}} * \boldsymbol{u}(-\boldsymbol{l}+\boldsymbol{\epsilon}) * \left(\frac{\boldsymbol{1}}{\boldsymbol{l}} - \frac{\boldsymbol{1}}{\boldsymbol{\mu}}\right) * \left(\frac{\boldsymbol{1}}{\boldsymbol{l}^2}\right) * (-\boldsymbol{nro}) +$$

$$\boldsymbol{eta2_{local}} * \boldsymbol{u}(\boldsymbol{l}-\boldsymbol{\epsilon})\ \boldsymbol{*}* \left(\frac{1}{l} - \frac{1}{\mu}\right) * \left(\frac{1}{l^2}\right) * (\boldsymbol{nro\_perp})\}$$

$$\boldsymbol{else}\ \{\boldsymbol{f_{local_{rep}}} = [\boldsymbol{0};\boldsymbol{0}]\}$$

$\boldsymbol{if}$ *{cos* $(\boldsymbol{\alpha})$ *<=-visibility* $\boldsymbol{f_{local_{rep}}} = [\boldsymbol{0};\boldsymbol{0}]\}$

***** *Giving perp force direction that has closest cosine similarity with heading direction*

$$\boldsymbol{nro} = [\boldsymbol{cos}(\boldsymbol{alpha})\ \boldsymbol{sin}(\boldsymbol{alpha})]^{\boldsymbol{T}}$$

Parameters

$\boldsymbol{\rho}, \boldsymbol{\mu}, \boldsymbol{\epsilon}, \boldsymbol{eta1_{local}}, \boldsymbol{eta2_{local}}$

➔ *Transforming local force in global frame*

Hybrid Tuning

1) *Tuning of forces*

$$f_{net} = param_{local} * \frac{f_{net_{local}}}{||f_{net_{local}}||} + param_{global} * \frac{f_{net_{global}}}{||f_{net_{global}}||}$$

$$param_{local} = mean_{local} + modulation_{local}$$

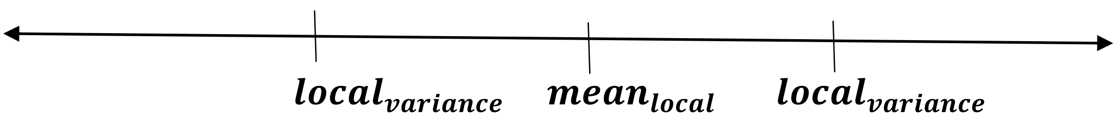
$$modulation_{local} = 2 * local_{variance} * \frac{differ_{local}}{\pi} - local_{variance}$$

$$differ_{local} = \left| atan2\left(\frac{\sin(differ)}{\cos(differ)}\right) \right| \in [0, \pi]$$

$$differ = cos^{-1}\left(\frac{f_{net_{global}}}{||f_{net_{global}}||} . \frac{f_{net_{local}}}{||f_{net_{local}}||}\right)$$

Parameters:

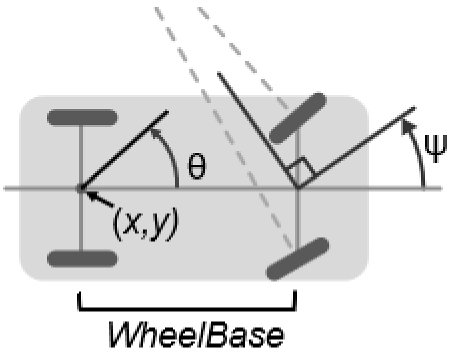
$mean_{local}$, $local_{variance}$



$$param_{global} = 1 - param_{local}$$

Controller

Kinematic model of car – Ackermann kinematics



State: $[\mathbf{x}, \mathbf{y}, \boldsymbol{\theta}, \boldsymbol{\psi}]^{\mathbf{T}}$ **input :** $[\mathbf{v}, \dot{\boldsymbol{\psi}}]^{T}$

$$\dot{x} = v * cos(\theta + \psi)$$

$$\dot{y} = v * sin(\theta + \psi)$$

$$\dot{\theta} = \frac{v}{l} sin(\psi)$$

$$\dot{\psi} = \sigma$$

Heading controller (tracking net force) – controlling car like unicycle model

1) $\gamma = atan2 \frac{f_{net_y}}{f_{net_x}}$
2) $\omega = PID(\gamma - \theta)$
3) $\sigma = c * [\frac{\omega l}{v} - sin(\psi)]$

Minimum parameters to tune

1) Eta1_global and Eta2_global: safety versus oscillations and big effect [1-10]
   To be tuned with respect to attractive force parameters

2) Gaussian parameter: k – coupled with eta1_global and eta2_global, x1, x2, x3 base values tuning and width of hills tuning

3) Khatibs parameters for tuning

4) Velocity planning - $alpha_{param}$ and $beta_{param}$ for giving weights to variance term and d_obs term

5) Hybrid: $mean_{local}$, $local_{variance}$