<u>Introduction to Financial Engineering</u>
Markowitz Portfolio Optimization

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Problem Statement:

1. Choose any 10 risky assets from the market. These could be stocks, bonds, ETFs, or any other investable assets with readily available price data.
2. Gather the closing prices for your chosen assets over the past 3 months.
3. Calculate the simple/log returns for each asset over the chosen period.
4. Apply Markowitz's mean-variance optimization to construct the efficient frontier.
5. Choose two points on the efficient frontier representing two different risk tolerance levels.
For each point, calculate the corresponding weights for each asset to construct a portfolio that maximizes expected return for that given level of risk.

Dataset Collection:

1. Apple (AAPL)
2. Microsoft (MSFT)
3. Alphabet Inc. (Formerly Google) (GOOGL)
4. Amazon (AMZN)
5. Cisco (CSCO)
6. Tesla (TSLA)
7. Nvidia (NVDA)
8. JP Morgan (JPM)
9. Goldman Sachs (GS)
10. Netflix (NFLX)

Some Important Terms Used :

Portfolio : A combination of different assets held in a specific proportion to achieve a desired risk-return profile. It is determined by optimizing the weights of individual assets in the portfolio, considering their expected returns, risk, and correlations.
Return of Portfolio : The return of a portfolio represents the gain or loss on an investment over a specific period. It's typically calculated as the weighted average of the returns of individual assets in the portfolio.

$$\boxed{R_p = \sum_i w_i \times R_i}$$

Risk of Portfolio : The risk of a portfolio measures the uncertainty or variability of returns associated with the portfolio. It's often quantified by metrics such as standard deviation or variance.

$$\boxed{\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i \times w_j \times \sigma_i \times \sigma_j \times \rho_{ij}}}$$

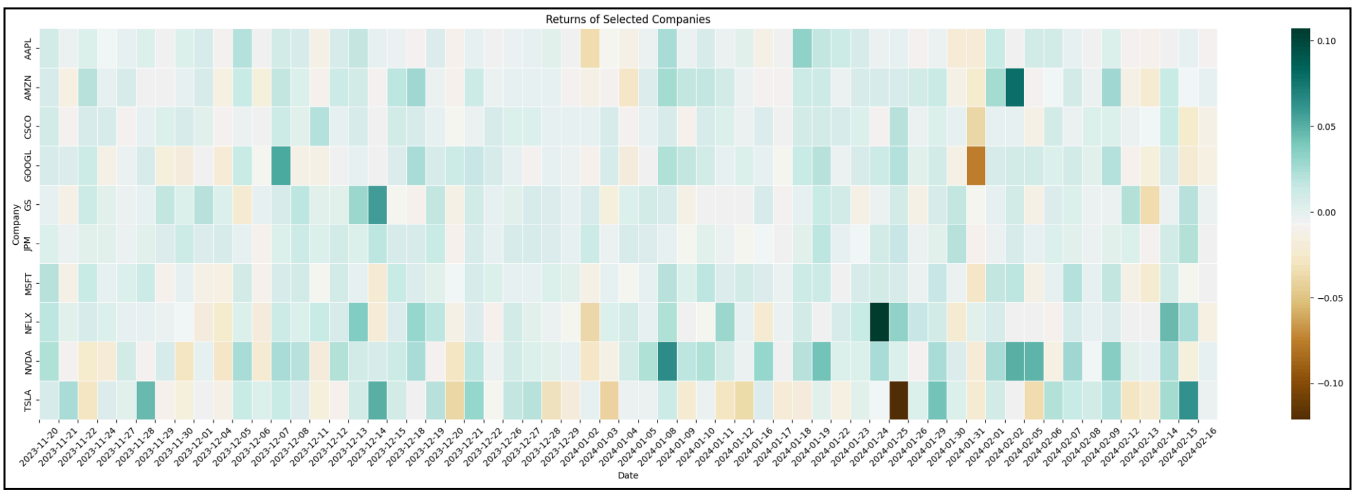
Efficient Frontier : A set of portfolios that offer the maximum expected return for a given level of risk or the minimum risk for a given level of return. It is represented in a scatter plot where the x-axis is portfolio risk and the y-axis is portfolio return.

Short Selling :

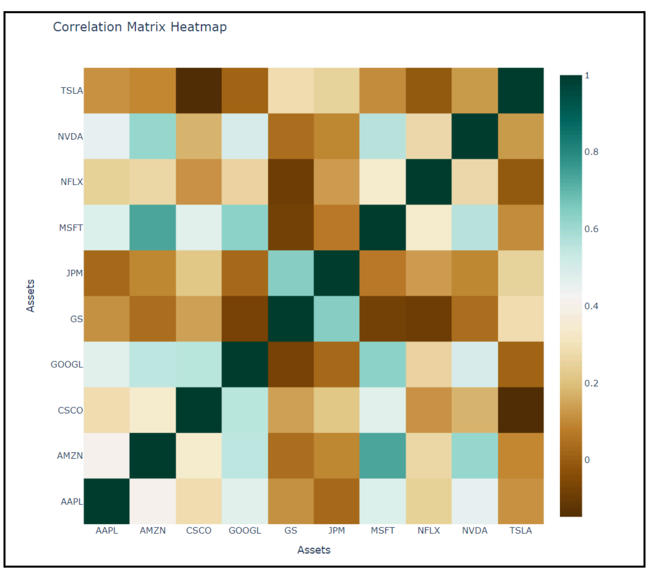
Short selling occurs when an investor borrows a security and sells it on the open market, planning to buy it back later for less money.

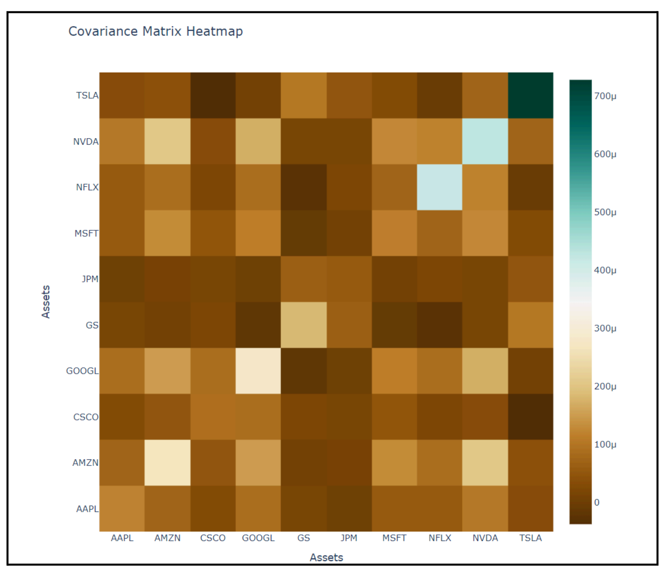
Mathematically, weights assigned in the portfolio can be negative when short selling is allowed.

Visualizing Returns



Correlation and Covariance Matrix for returns





Return of the Assets		Risk of the assets	
AAPL	-0.000735	AAPL	0.010957
AMZN	0.002605	AMZN	0.016290
CSCO	0.000233	CSCO	0.009520
GOOGL	0.000653	GOOGL	0.016680
GS	0.002302	GS	0.013533
JPM	0.002721	JPM	0.007611
MSFT	0.001224	MSFT	0.010745
NFLX	0.003666	NFLX	0.020474
NVDA	0.006313	NVDA	0.020761
TSLA	-0.002364	TSLA	0.026984

Markowitz Portfolio Optimization

Case 1 : Short Selling is allowed

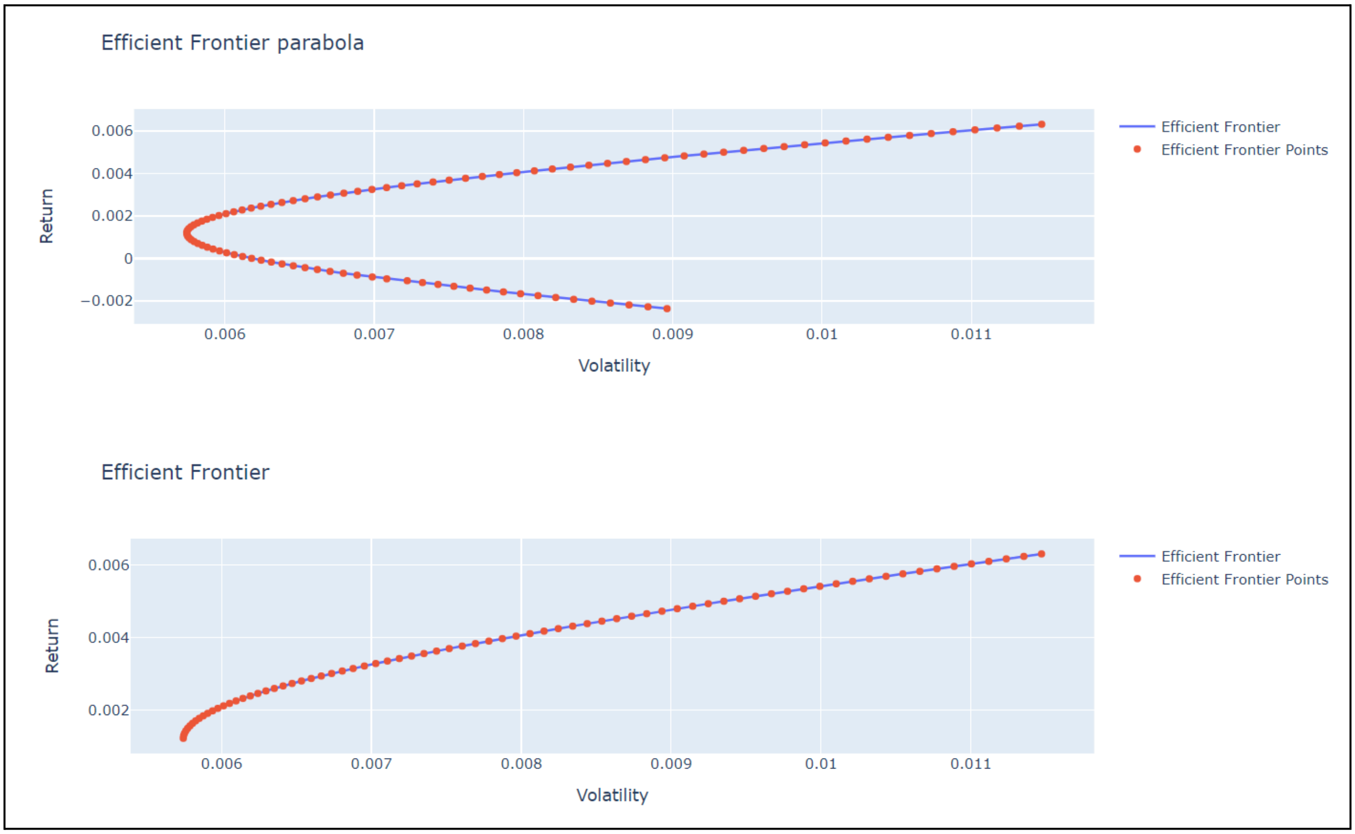
When target return is not specified :

$$\begin{array}{ll} \text{minimize} & \frac{1}{2} w^T \Sigma w \\ \text{subject to} & \mathrm{e}^T w = 1 \ . \end{array}$$

Results for selected Assets :

```
Optimal Weights:
AAPL : 19.72
MSFT : -3.42
GOOGL : 23.1
AMZN : -7.92
CSCO : -5.81
TSLA : 54.91
NVDA : 19.7
JPM : 0.23
GS : -1.97
NFLX : 1.45
Optimal Risk: 0.57 %
Optimal Return: 0.12 %
```

Efficient Frontier :



For a given target return :

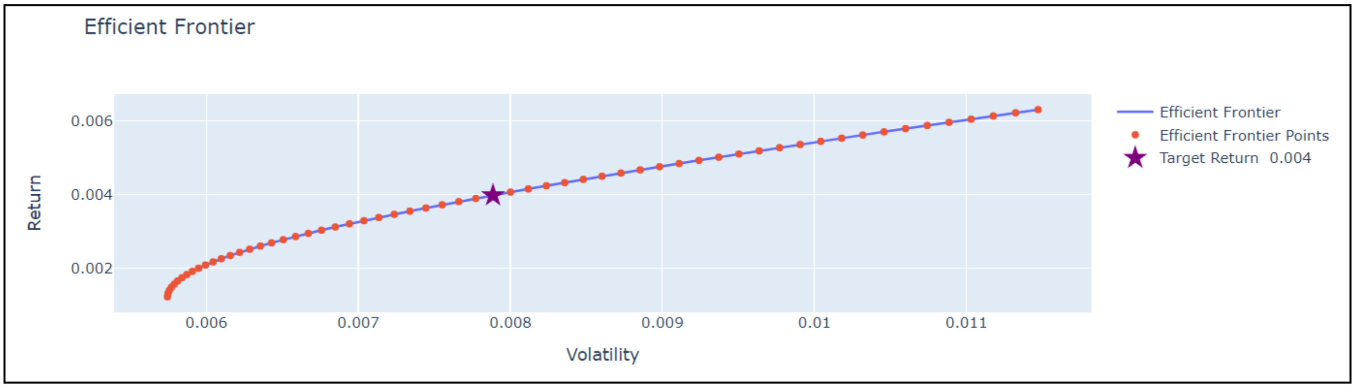
Minimize: $w^T \Sigma w$

Subject to: $\sum_i w_i = 1, w^T \mu = R_{\text{target}}$

Results for selected Assets:

```
Markowitz Optimization for Target Return 0.004 Results:
Optimal Weights:
AAPL : -7.73
MSFT : -3.98
GOOGL : 2.82
AMZN : -9.92
CSCO : -1.52
TSLA : 84.75
NVDA : 20.23
JPM : 4.14
GS : 19.59
NFLX : -8.37
Optimal Risk: 0.79 %
Optimal Return: 0.4 %
```

Efficient Frontier :



Case 2 : Short Selling is not allowed (w >=0) :

When target return is not specified:

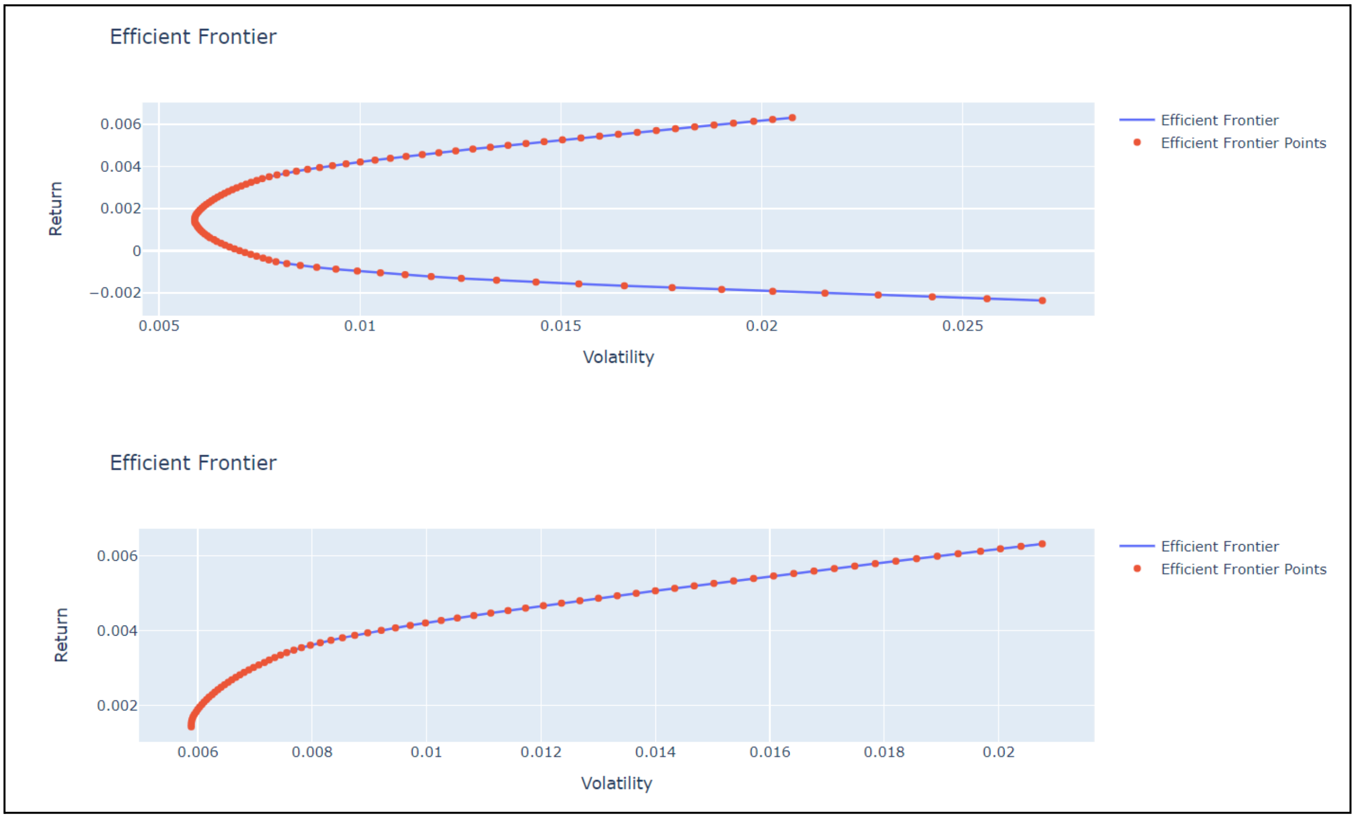
Minimize: $w^T \Sigma w$

Subject to: $\sum_i w_i = 1, w^T \mu = R_{\text{target}}, w_i \geq 0$ for all i

Optimal Weights for selected assets :

```
Markowitz Optimization Results:
Optimal Weights:
AAPL : 15.86
MSFT : 0.0
GOOGL : 21.88
AMZN : 0.0
CSCO : 0.0
TSLA : 50.96
NVDA : 9.18
JPM : 0.81
GS : 0.0
NFLX : 1.31
Optimal Risk: 0.59 %
Optimal Return: 0.14 %
```

Efficient Frontier :



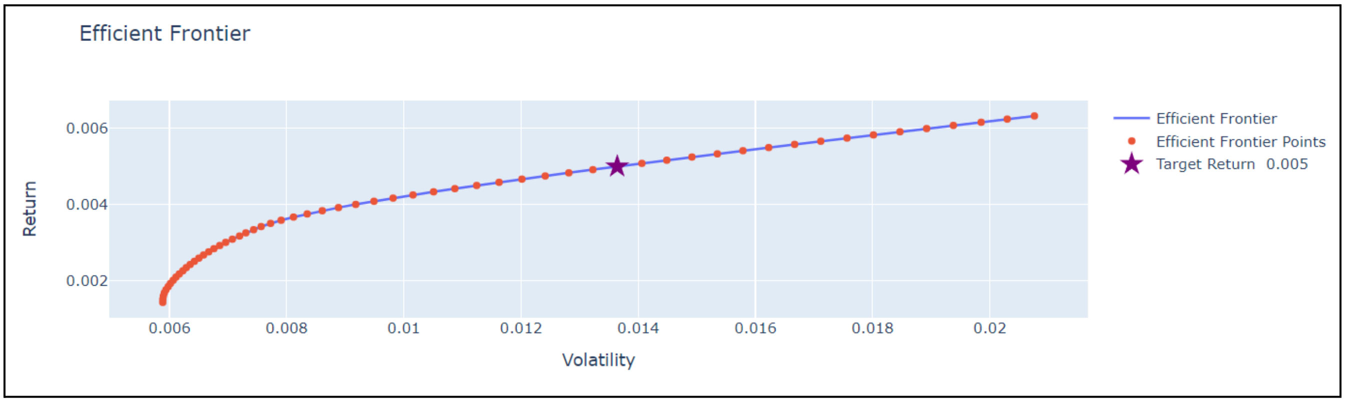
For a given target return :

$$
\begin{array}{ll}
\text{minimize} & \frac{1}{2} w^T \Sigma w \\
\text{subject to} & \mathrm{m}^T w \geq \mu_b, \ \text{and} \ \mathrm{e}^T w = 1 \\
& w \geq 0
\end{array}
$$

Markowitz Optimization Results :

```
Markowitz Optimization for Target Return 0.005 Results:
Optimal Weights:
AAPL : 0.0
MSFT : 0.0
GOOGL : 0.0
AMZN : 0.0
CSCO : 0.0
TSLA : 33.92
NVDA : 0.0
JPM : 3.57
GS : 62.51
NFLX : 0.0
Optimal Risk: 1.37 %
Optimal Return: 0.5 %
```

Efficient Frontier :



Selecting Two points on Efficient Frontier

Case 1 : Short Selling allowed :

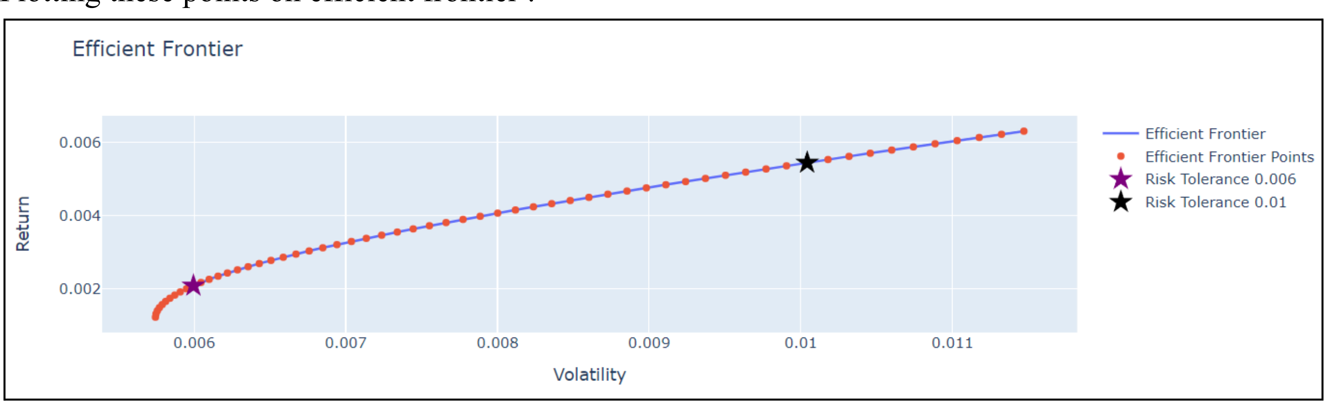
Risk Tolerance 1 : 0.006 (0.6 %)

Risk Tolerance 2 : 0.01 (1%)

```
For risk :  0.006
Expected Return :  0.21 %
Optimal Weights
AAPL  :  9.75
MSFT  :  -3.34
GOOGL  :  17.67
AMZN  :  -7.53
CSCO  :  -4.1
TSLA  :  62.85
NVDA  :  19.14
JPM  :  1.96
GS  :  4.71
NFLX  :  -1.1
```

```
For risk :  0.01
Expected Return :  0.54 %
Optimal Weights
AAPL  :  -33.29
MSFT  :  -3.89
GOOGL  :  4.43
AMZN  :  -9.46
CSCO  :  5.05
TSLA  :  88.23
NVDA  :  20.21
JPM  :  9.89
GS  :  29.84
NFLX  :  -11.02
```

Plotting these points on efficient frontier :

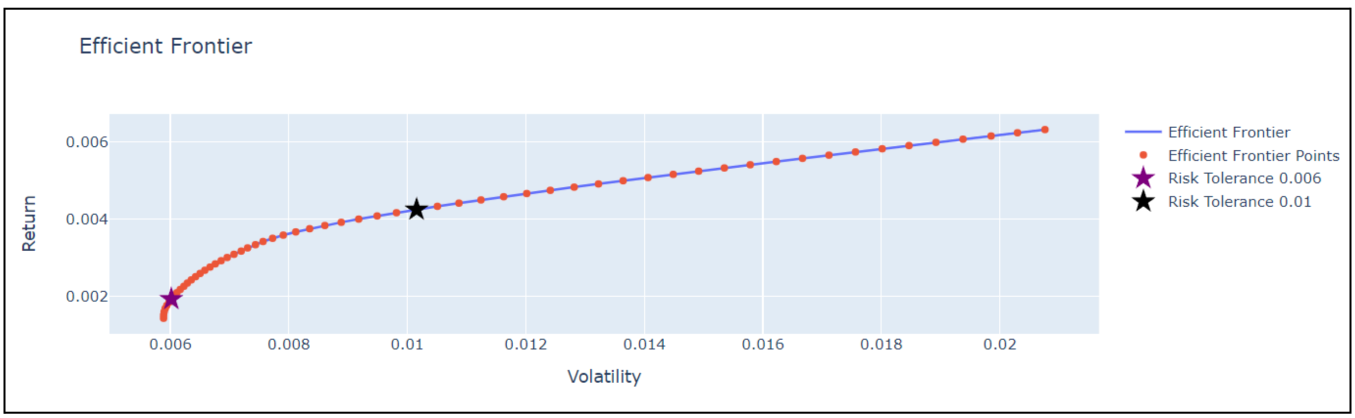


Case 2 : Short Selling not allowed
Risk Tolerance 1 : 0.006 (0.6%)
Risk Tolerance 2 : 0.01 (1%)

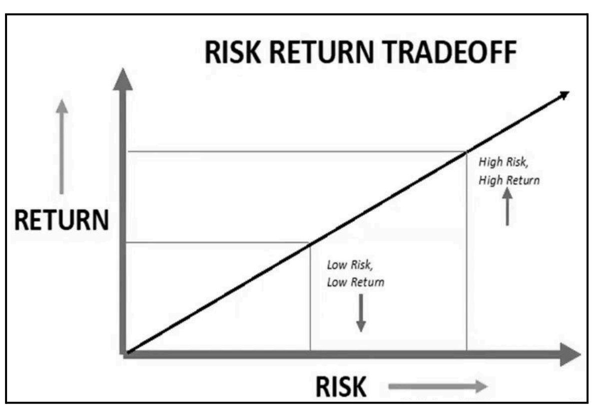
```
For risk :  0.006
Expected Return :  0.19 %
Optimal Weights
AAPL --- 10.65
MSFT --- 0.0
GOOGL --- 15.31
AMZN --- 0.0
CSCO --- 0.0
TSLA --- 59.54
NVDA --- 10.96
JPM --- 1.82
GS --- 1.73
NFLX --- 0.0
```

```
For risk :  0.01
Expected Return :  0.42 %
Optimal Weights
AAPL --- 0.0
MSFT --- 0.0
GOOGL --- 0.0
AMZN --- 0.0
CSCO --- 0.0
TSLA --- 55.22
NVDA --- 0.0
JPM --- 4.63
GS --- 40.15
NFLX --- 0.0
```

Plotting these risk tolerance on efficient frontier :



Trade-off between Risk and Return



Risk-return tradeoff states that the potential return rises with an increase in risk. Using this principle, individuals associate low levels of uncertainty with low potential returns, and high levels of uncertainty or risk with high potential returns.According to risk-return tradeoff, invested money can render higher profits only if the investor will accept a higher possibility of losses.

Risk-return tradeoff is the trading principle that links high risk with high reward. The appropriate risk-return tradeoff depends on a variety of factors that include an investor's risk tolerance, the investor's years to retirement, and the potential to replace lost funds.

Risk Return Trade-off at Portfolio Levels : Risk-return tradeoff also exists at the portfolio level. For example, a portfolio composed of all equities presents both higher risk and higher potential returns. Within an all-equity portfolio, risk and reward can be increased by concentrating investments in specific sectors or by taking on single positions that represent a large percentage of holdings. For investors, assessing the cumulative risk-return tradeoff of all positions can provide insight on whether a portfolio assumes enough risk to achieve long-term return objectives or if the risk levels are too high with the existing mix of holdings.

Limitations of Markowitz Portfolio Optimization

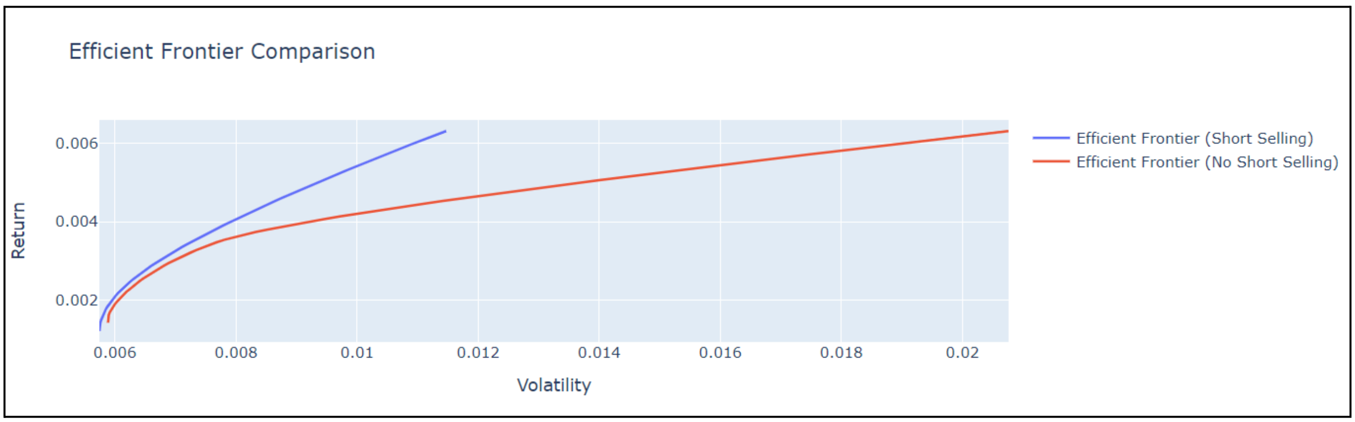
- **Sensitivity to Input Estimates:** Markowitz optimization heavily relies on estimates of expected returns, variances, and covariances of assets. These estimates can be sensitive to changes in input parameters, leading to potentially unstable and unreliable portfolio allocations.
- **Assumption of Normality:** The optimization assumes that asset returns follow a normal distribution. In reality, asset returns often exhibit non-normal distributions, such as fat tails or skewness, which can lead to inaccurate risk assessments and suboptimal portfolios.
- **Estimation Error and Data Mining:** Historical data used to estimate expected returns, variances, and covariances may not accurately reflect future market conditions. Estimation errors and data mining biases can lead to suboptimal portfolio allocations and increased exposure to unforeseen risks.
- **No Consideration of Transaction Costs:** Markowitz optimization does not explicitly consider transaction costs, such as brokerage fees, taxes, and market impact costs. High transaction costs can significantly impact the performance of optimized portfolios and may render them impractical to implement.
- **Lack of Robustness to Model Assumptions:** The optimization is sensitive to the choice of model assumptions, such as the choice of risk measure (e.g., variance), the time period used for estimation, and the specification of constraints. Small changes in these assumptions can lead to vastly different portfolio allocations.
- **No Incorporation of Behavioral Biases:** Markowitz optimization assumes that investors are rational and risk-averse, with homogeneous preferences for risk and return. In reality, investors exhibit behavioral biases, such as loss aversion and herding behavior, which can lead to suboptimal investment decisions.
- **Infeasibility of Short Selling and Borrowing Constraints:** The optimization may yield portfolios that involve short selling or borrowing, which may not be feasible or practical for all investors due to regulatory constraints or risk aversion.
- **Single-Period Framework:** Markowitz optimization is based on a single-period framework, which may not capture the dynamic nature of investment decisions over time, such as changing risk preferences and evolving market conditions.

Real Word Applications of Markowitz Portfolio Optimization

- ➢ **Portfolio Construction:** Helps in constructing portfolios that maximize returns for a given level of risk or minimize risk for a given level of return, catering to the preferences and constraints of investors.
- ➢ **Risk Management:** Provides a systematic framework for managing portfolio risk by diversifying across assets with low or negative correlations, reducing overall portfolio volatility.
- ➢ **Performance Evaluation:** Enables the evaluation of portfolio performance relative to benchmarks or peer portfolios, helping investors assess the effectiveness of their investment strategies.

Conclusion

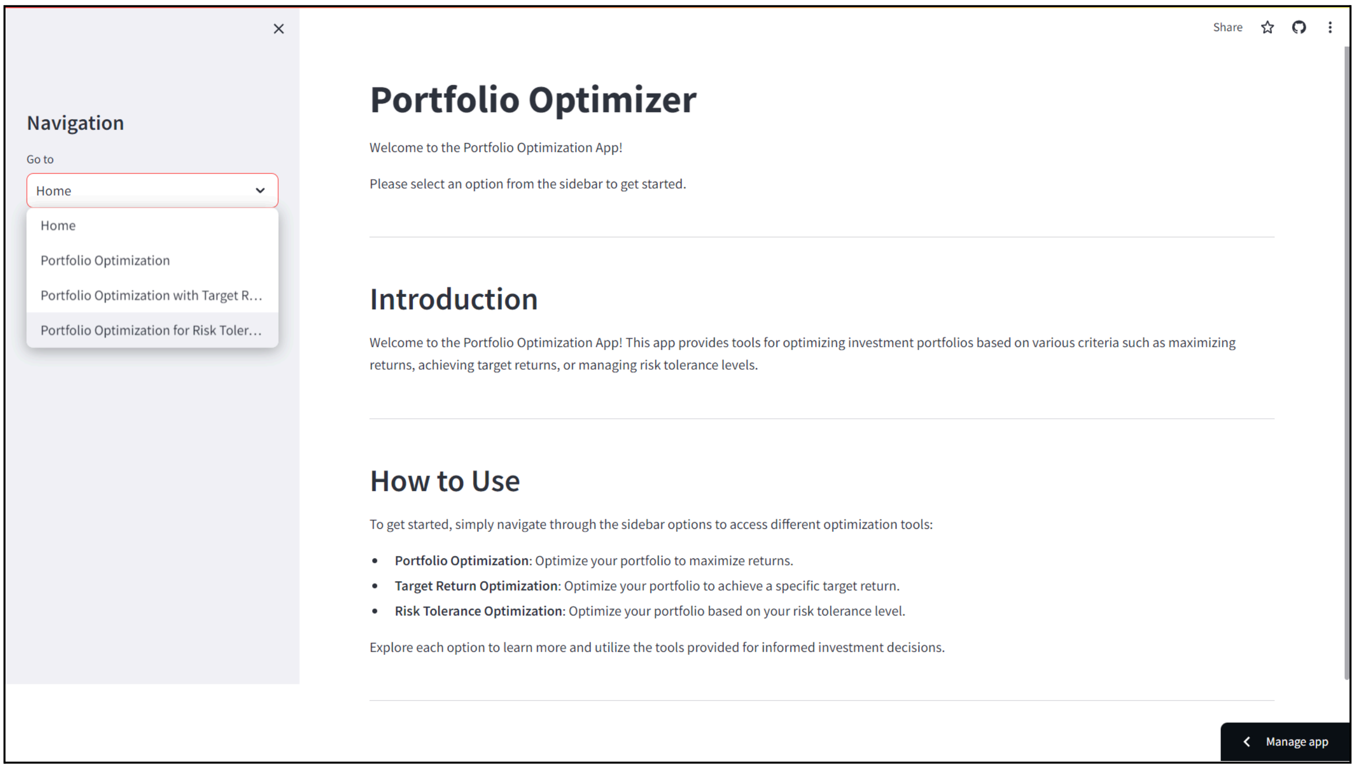
<u>Allowing short selling maximizes return for the same level of risk which is clear from the efficient frontier plot.</u>



Deploying the WebApp using Streamlit

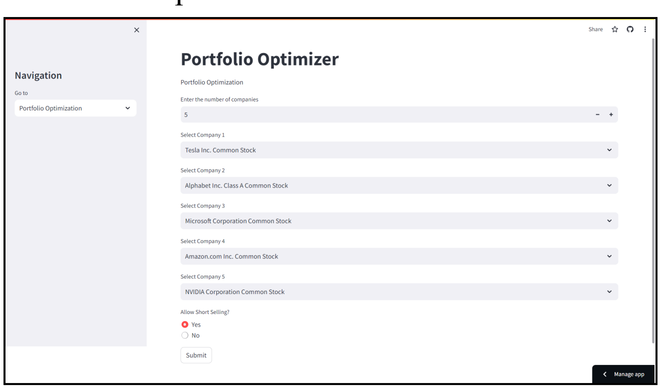
App Link - [Portfolio Optimizer]

Home Page :

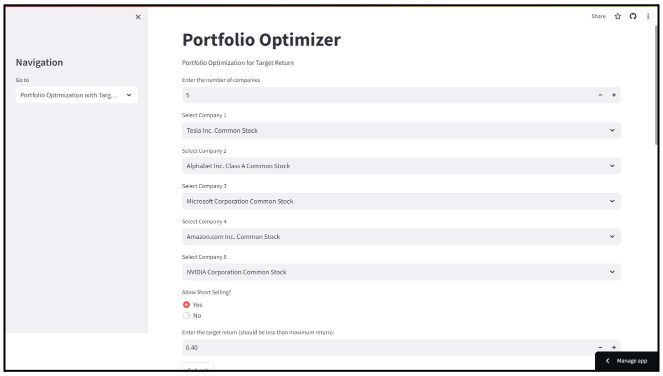


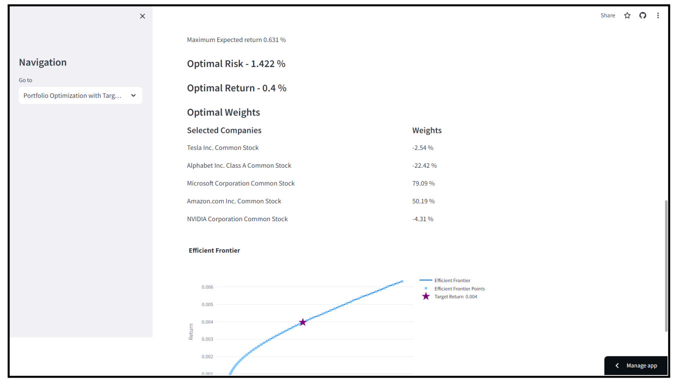
Side navigation bar in the home page contains following options :

1. Portfolio Optimization



2. Portfolio Optimization for given target return .





3. Portfolio Optimization for given risk tolerance level