Autonomous Assignment 3
Kalman Filter

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1 Kalman Filter Implementation

The Kalman filter is a recursive algorithm used for estimating the state of a dynamic system from a series of measurements. In the context of missile tracking, the Kalman filter is used to estimate the position and velocity of the target aircraft based on noisy measurements from a radar system.

The state of the target aircraft is represented by a 6-dimensional vector $\mathbf{x} = [x, y, z, v_x, v_y, v_z]$, where (x, y, z) is the position and (v_x, v_y, v_z) is the velocity. The system dynamics are modeled by the following linear state-space equations:

- **State Vector:** $\mathbf{x} = [x, y, z, v_x, v_y, v_z]$ (position and velocity).
- **System Dynamics:**

$$v_{t+1} - v_t = a_t \cdot \Delta t$$
$$\mathbf{x}_{t+1} - \mathbf{x}_t = \mathbf{v}_t \cdot \Delta t$$

  where a_t is the target's acceleration, provided as a control input.

1.1 Prediction Step

In the prediction step, the Kalman filter uses the known system dynamics and the previous state estimate to predict the current state of the system. The state transition matrix $\mathbf{A}$ and the control input matrix $\mathbf{B}$ are used to compute the predicted state:

- Uses system dynamics and previous state to predict the current state.
- State transition matrix $\mathbf{A}$ and control input matrix $\mathbf{B}$ are applied as:

$$\mathbf{x}_{\text{predicted}} = \mathbf{A} \cdot \mathbf{x}_{\text{estimated}} + \mathbf{B} \cdot \text{control\_input}$$
$$\mathbf{P}_{\text{predicted}} = \mathbf{A} \cdot \mathbf{P} \cdot \mathbf{A}^\top + \mathbf{Q}$$

1.2 Update Step

In the update step, the Kalman filter incorporates the new measurement from the radar system to update the state estimate. The measurement model is represented by the measurement matrix $\mathbf{H}$, which maps the state vector to the measurement vector. The Kalman gain $\mathbf{K}$ is computed based on the measurement covariance $\mathbf{R}$ and the predicted state covariance $\mathbf{P}_{\text{predicted}}$:

- Incorporates new radar measurement to refine state estimate.
- Measurement residual $\mathbf{y}$ and residual covariance $\mathbf{S}$:

$$\begin{aligned} \mathbf{y} &= \text{measurement} - \mathbf{H} \cdot \mathbf{x}_{\text{predicted}} \\ \mathbf{S} &= \mathbf{H} \cdot \mathbf{P}_{\text{predicted}} \cdot \mathbf{H}^{\top} + \mathbf{R} \end{aligned}$$

- Kalman Gain $\mathbf{K}$ and updated state:

$$\begin{aligned} \mathbf{K} &= \mathbf{P}_{\text{predicted}} \cdot \mathbf{H}^{\top} \cdot (\mathbf{S}^{-1}) \\ \mathbf{x}_{\text{estimated}} &= \mathbf{x}_{\text{predicted}} + \mathbf{K} \cdot \mathbf{y} \\ \mathbf{P} &= (\mathbf{I} - \mathbf{K} \cdot \mathbf{H}) \cdot \mathbf{P}_{\text{predicted}} \end{aligned}$$

2 Simulate Function

The 'simulate_trajectory' function simulates the target's trajectory to evaluate Kalman filter performance.

- **Inputs:**
  - 'time_steps': Number of simulation steps.
  - 'delta_t': Time interval per step.
  - 'eta': Prediction steps per measurement.
  - 'initial_position' and 'initial_velocity': Initial state of the target.
  - 'acceleration': Acceleration profile.
  - 'kalman_filter': An instance of 'MissileTrackerKalmanFilter'.
- **Working:** The function simulates the true trajectory of the target aircraft based on the given initial conditions and acceleration. At each step, the Kalman filter performs '`eta`' prediction steps and then updates the state estimate using the simulated measurement. The true positions, estimated positions, and Euclidean errors between them are recorded and returned.

  The simulated trajectory can be used to evaluate the performance of the Kalman filter under different conditions, such as varying levels of measurement noise or different acceleration profiles.

3 Results & Observations

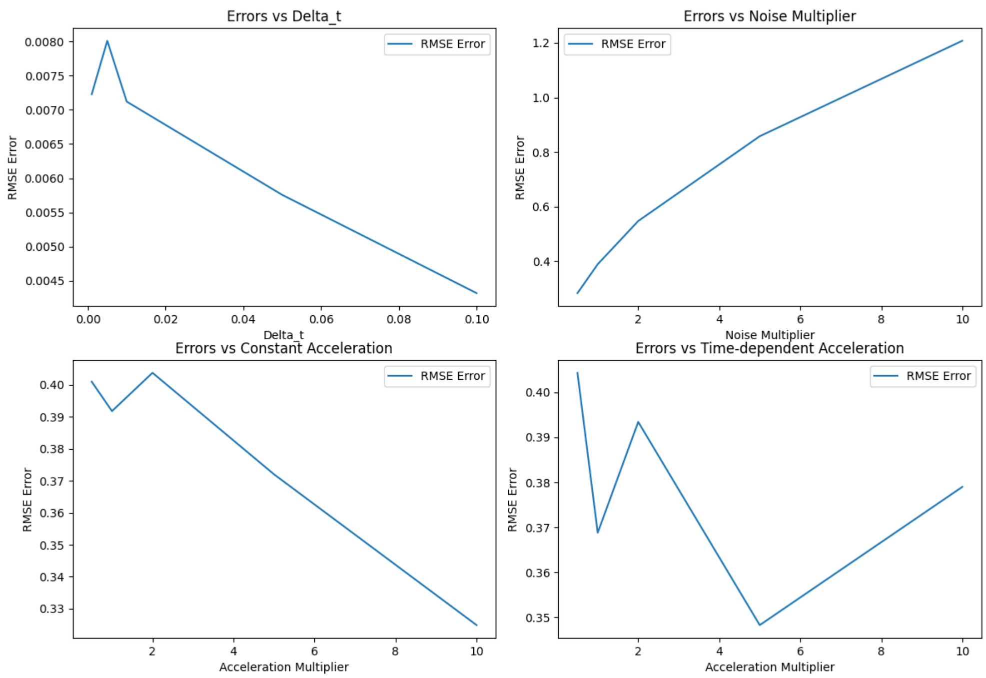

Figure 1: Performance Observations

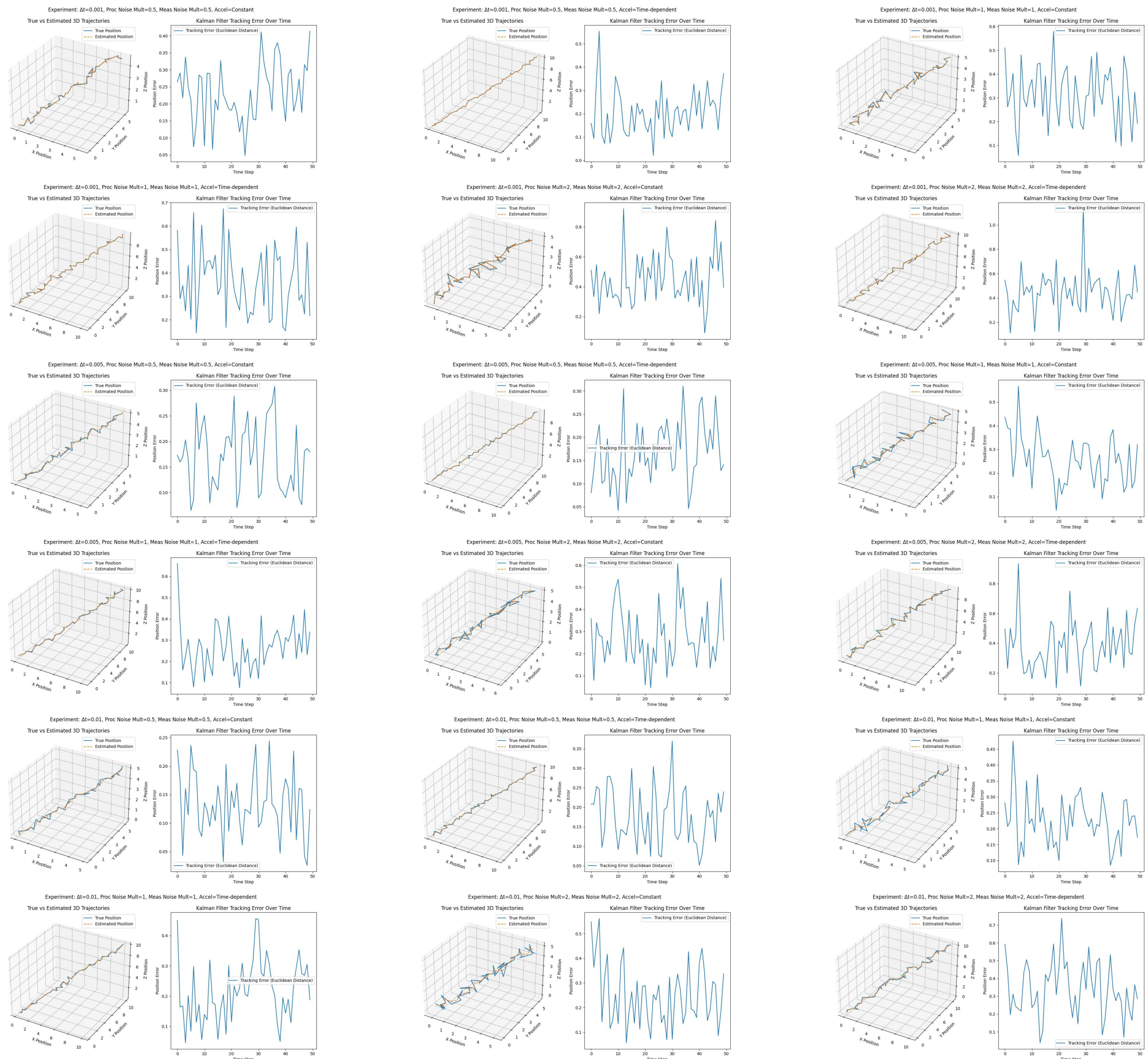

Figure 2: Simulated Trajectories (Collage)

- We observe that the Scaled RMSE decreases with an increase in Δt, which is expected as the parameters change frequently, and the filter cannot keep up with the rapid changes.
- The Scaled RMSE increases with increased measurement noise, as expected, due to the increased uncertainty in measurements.
- The results in both constant and variable acceleration do not reveal much about the filter's performance, as the RMSE is unpredictable and does not follow a specific pattern.