Exam of Introduction to Programming and Computational Physics

June 23th, 2017 09.00-11.30

General information

The use of recommended books, slides and solutions to the exercises solved during the course is allowed.

The use of Internet, email, Skype, smartphones and similar is strictly forbidden. **No internet browser can be opened during the exam.**

When you are ready to hand in, please call one of the assistants and (**supervised by her/him**) send an email to the address Ciro.Pistillo@cern.ch attaching your source code (do not use zip file, attach your .c files one by one). Please make sure that the email is correctly received by Dr. Pistillo before leaving.

Time limit is **11:30**. After that time you have **15** more minutes to review your code and prepare for handing in (if not done yet). **After 11:45 you will not be allowed to use the computer anymore.** Then please wait for an assistant to hand in your code.

Please write your name and matriculation number in any source file. Please also indicate an email address where you can be contacted if further clarifications are needed or we need to get in touch with you for some reason.

Exercise 1

The following iterative sequence is defined for the set of positive integers:

$n \rightarrow n/2$ (if n is even)
$n \rightarrow 3n + 1$ (if n is odd)

And the iteration is terminated when the number 1 is met.
Using the rule above and starting for example with 13, the chain:
$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$
is generated, containing 10 terms.
Write a program to calculate which starting number, under 10^6, produces the longest chain. Print this starting number and the length of the related chain to the screen.

NOTE: Once a chain starts the terms happen to go well above one million

Exercise 2

Consider the following rule, belonging to the family of the Newton-Cotes formulas. It is applied to four consecutive strips and its formula is:

$$\int_{x_0}^{x_4} f(x)dx \cong \tfrac{2}{45} h(7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4))$$

where $h = x_1 - x_0$

Write a program which implements iteratively the rule (subdividing more and more the strips) to evaluate the value of the integral

$$\int_0^{\pi/4} \frac{3\tan(x)}{1+\cos^2(x)} dx$$

with a precision of 10^{-6}

$$\left(\quad |I(iteration\_n) - I(iteration\_n+1)| < 10^{-6} \quad \right)$$

Print the value of the integral to the screen

Exercise 3

Write a program to perform the following Monte Carlo simulation.
A particle is generated with initial coordinates (x_0,y_0,z_0) uniformly distributed in a sphere with center (0,0,0) and radius 1. This is obtained by generating u_1, u_2, u_3 from a uniform distribution U(0,1) using the standard C function srand() then first delivering the coordinates as spherical coordinates:

$$\vartheta_0 = 2\pi u_1$$
$$\phi_0 = \cos^{-1}(2u_2 - 1)$$
$$r_0 = u_3^{1/3}$$

and then converting them to cartesian coordinates:

$$x_0 = r_0 \cos(\vartheta_0)\sin(\phi_0)$$
$$y_0 = r_0 \sin(\vartheta_0)\sin(\phi_0)$$
$$z_0 = r_0 \cos(\vartheta_0)$$

Assign the particle an initial velocity (vx_0,vy_0,vz_0) with each component generated from a Gaussian distribution with μ=0 and σ=0.02 using the Box-Muller method. Assume that this constant speed is kept for one second and evaluate the new position of the particle. Assign the particle a new velocity (vx_1,vy_1,vz_1) in the same way and evaluate its position after one more second. Repeat the operation until the particle leave the sphere.

Repeat the whole simulation for 10^5 particles, calculate the time needed on average with the error (standard deviation of the mean) to leave the sphere, print the result to the screen.