Exercise 11 – Differential equations

Exercise 11.1

Consider the differential equation: $y' = -x^2 y^2$

This equation can be analytically solved and the general solution is: $y = \frac{3}{A + x^3}$

If we apply the initial condition y(0)=1 the solution becomes: $y = \frac{3}{3 + x^3}$

Write a program which implements the three methods (Euler, Runge-Kutta II and Runge-Kutta IV) to solve the equation.

The program should ask as input:
- which method has to be used
- the range
- the stepsize

and write as output a file were evaluations of the function y at each step are performed.

Compare results obtained in the range [0,1] and stepsize Dx=0.1,0.01 with the exact solution.

Exercise 11.2

Modify your program in order to solve the differential equation:

$y' = y \sin x + \sin 2x$ with $y(0) = -2$

Compare your results with the exact solution:

$y = 2 - 2\cos x - 2e^{1-\cos x}$