



Variogram & Kriging


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Diwali Air Pollutant Concentration Data

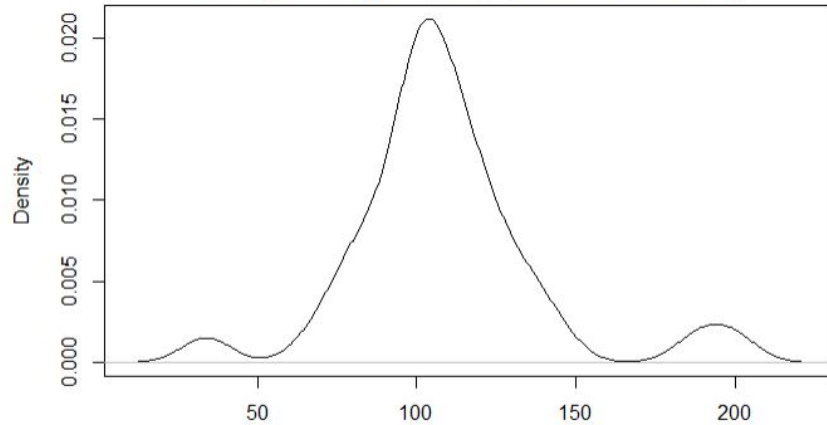
Dataset Source: CPCB

36 localities of Delhi * 13 pollutants (*PM_{2.5}, CO, NO, NO₂, NO_x, Ozone, SO₂, PM₁₀, NH₃, CH₄, CO₂, SPM, Black Carbon*) * 8 days (4th Nov, 2018 to 11th Nov, 2018)

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1. Data Cleaning and Structuring
 2. Variogram for PM_{2.5} concentration
 3. Fitting a Spherical Variogram
 4. Ordinary Kriging for PM_{2.5} concentration

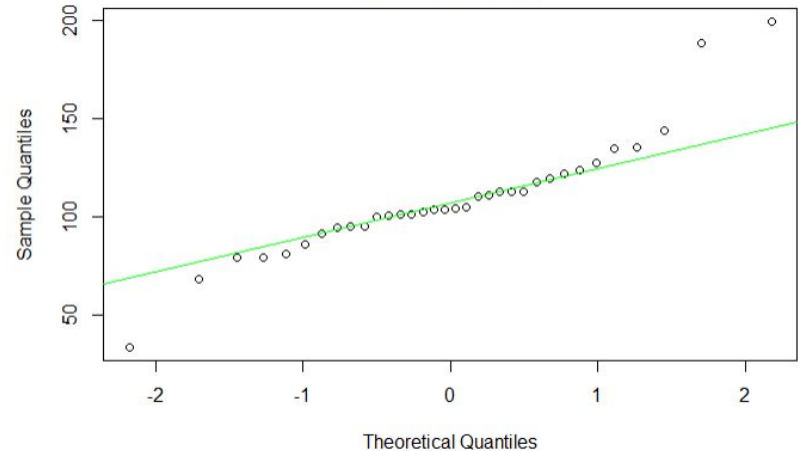
Variogram for PM_{2.5} concentration (on 4th Nov'18)

density.default(x = na.omit(data2\$`2018-11-04`))

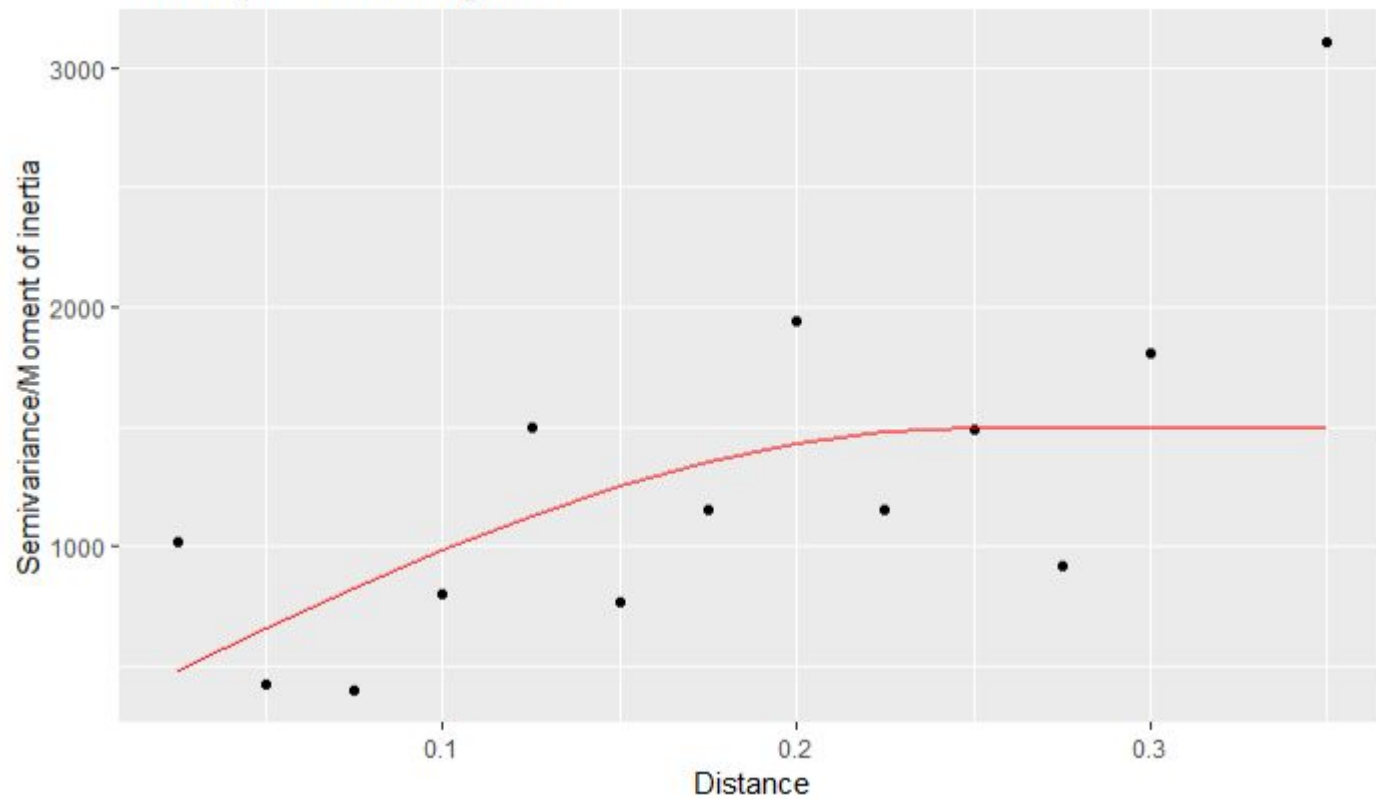


N = 34 Bandwidth = 7.804

Normal Q-Q Plot



Fitted Spherical Variogram





The results for variogram fitting are:

Range: $a = 0.25$

Nugget: $C_0 = 300$

Sill: $C_0 + C_1 = 1500$

Ordinary Kriging (B.L.U.E)

$$\hat{V}(x_0) = \sum_{i=1}^n w_i V(x_i) \quad R(x_0) = \sum_{i=1}^n w_i V(x_i) - V(x_0) \quad \sum_{i=1}^n w_i = 1$$

$$\tilde{\sigma}_R^2 = \tilde{\sigma}^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \tilde{C}_{ij} - 2 \sum_{i=1}^n w_i \tilde{C}_{i0}$$

results in

$$w = C^{-1} D \quad \underbrace{\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}}_{(n+1) \times 1} = \underbrace{\begin{bmatrix} \tilde{C}_{11} & \cdots & \tilde{C}_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{C}_{n1} & \cdots & \tilde{C}_{nn} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}}_{(n+1) \times (n+1)}^{-1} \underbrace{\begin{bmatrix} \tilde{C}_{10} \\ \vdots \\ \tilde{C}_{n0} \\ 1 \end{bmatrix}}_{(n+1) \times 1}$$

IDW Interpolation



$$\hat{V}(x_0) = \frac{\sum_{i=1}^n w_i V(x_i)}{\sum_{i=1}^n w_i} \quad \text{where, } w_i = \frac{1}{d(x_0, x_i)^2}$$

- In the general case, the power 2 can be replaced with any power depending on the kind of interpolation values required.
- Here, 2 is taken as the usual/default value.

Ordinary Kriging & IDW



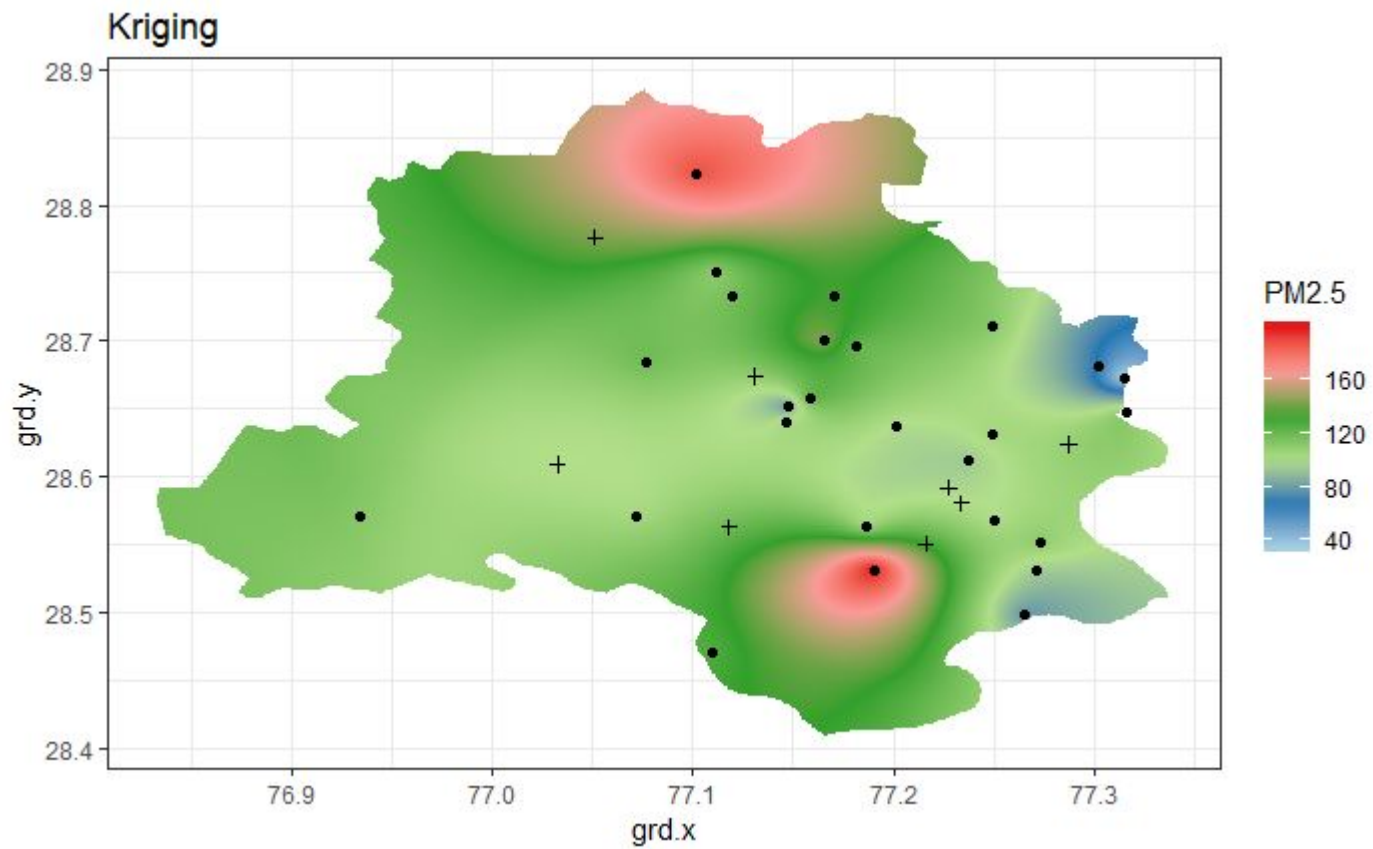
26 locations for building the kriging and the IDW models.

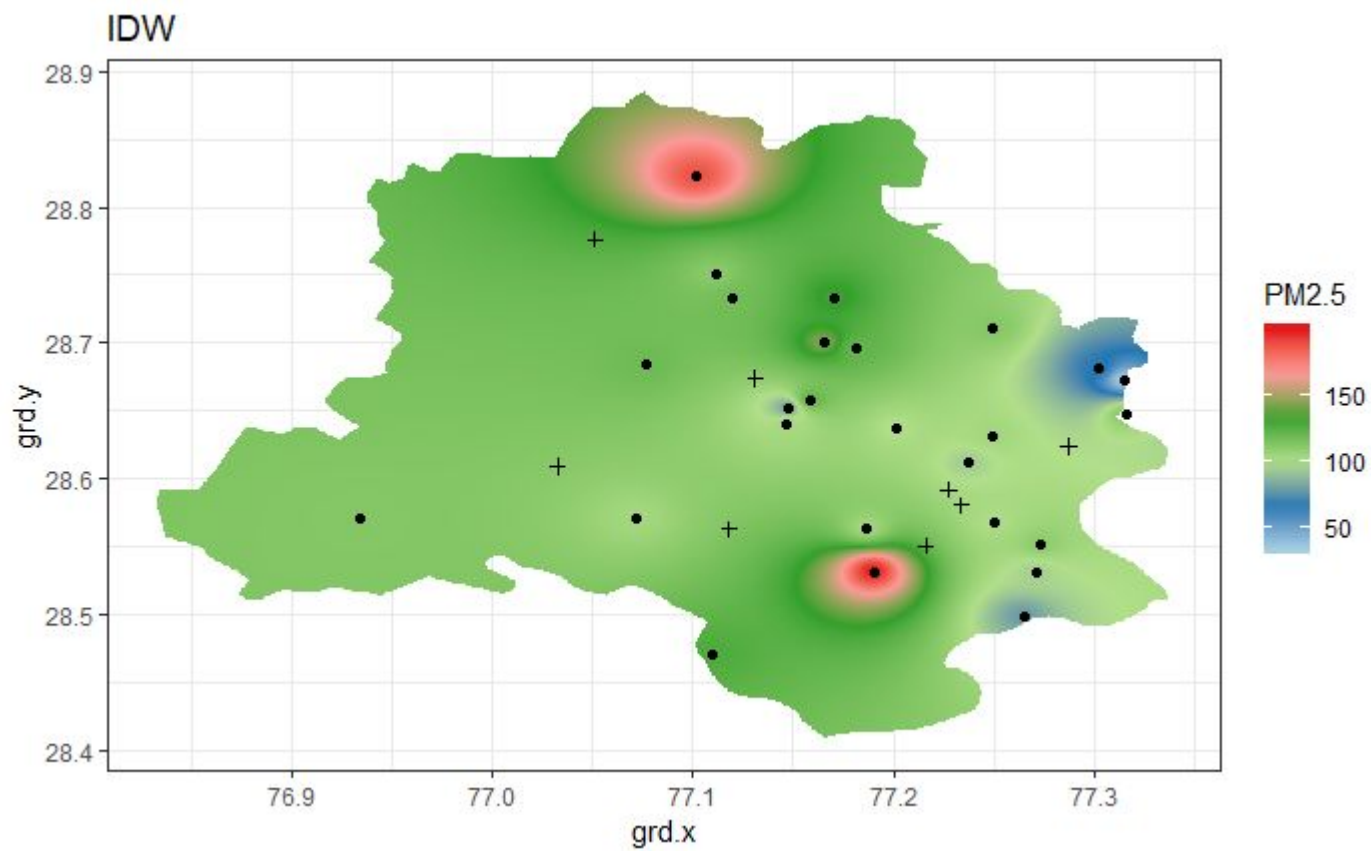
8 locations for prediction and cross validation using the built model.

Interpolation Results & Error Metrics

	X	Y	PM2.5	Predicted_PM2.5_Kriging	Predicted_PM2.5_IDW
1	77.05107	28.77620	134.79	143.05	127.02
2	77.11801	28.56278	102.81	119.65	114.05
3	77.23383	28.58028	95.53	101.76	104.72
4	77.22731	28.59182	79.18	98.41	103.42
5	77.03254	28.60909	100.79	100.97	112.08
6	77.28721	28.62375	91.58	108.03	99.13
7	77.13102	28.67404	101.39	110.57	110.79
8	77.21594	28.55042	85.91	131.35	122.68

	Ordinary Kriging	IDW Interpolation
Mean Absolute Error (MAE)	15.23	14.68
Root Mean Square Error (RMSE)	19.94	17.62
Mean Absolute Percentage Error (MAPE)	17%	16%





Delhi Air Pollution spatial plots showing $PM_{2.5}$ variation throughout the Diwali week

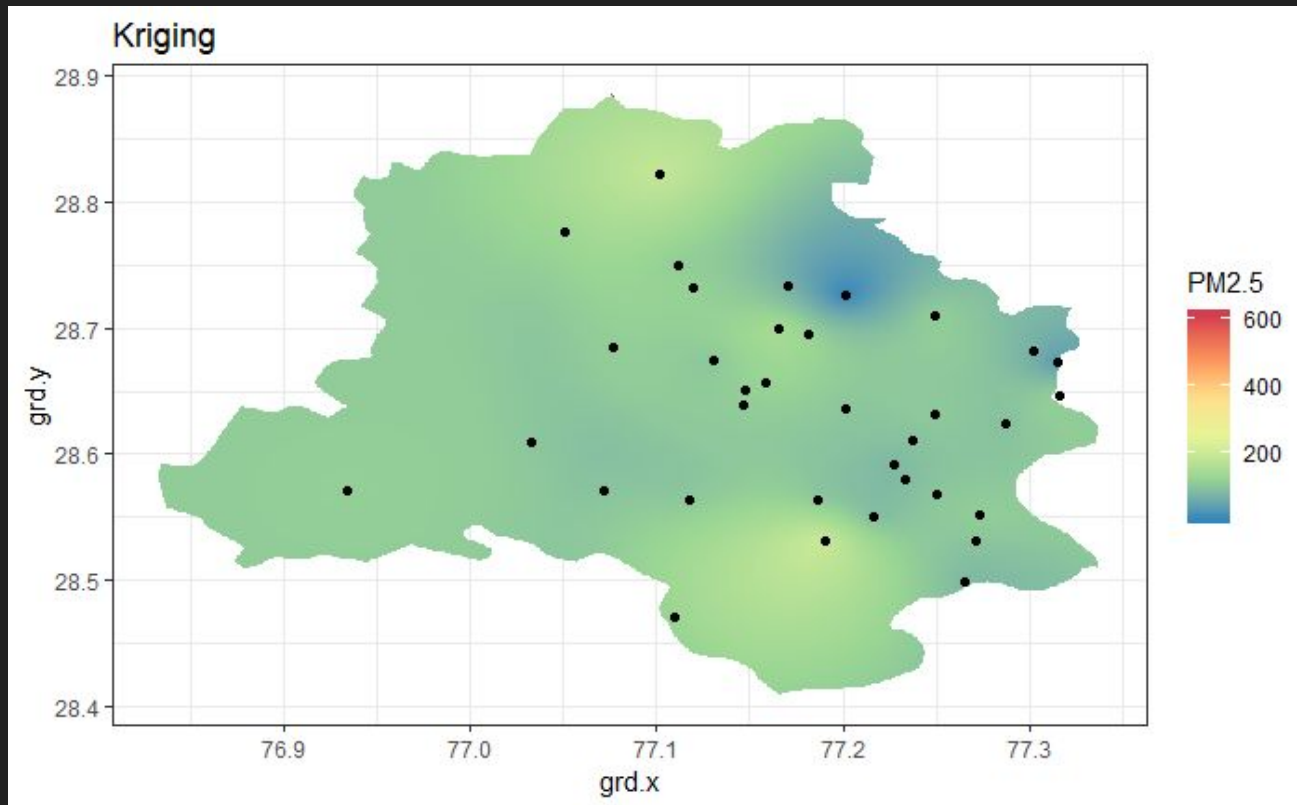
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Spatial Interpolation

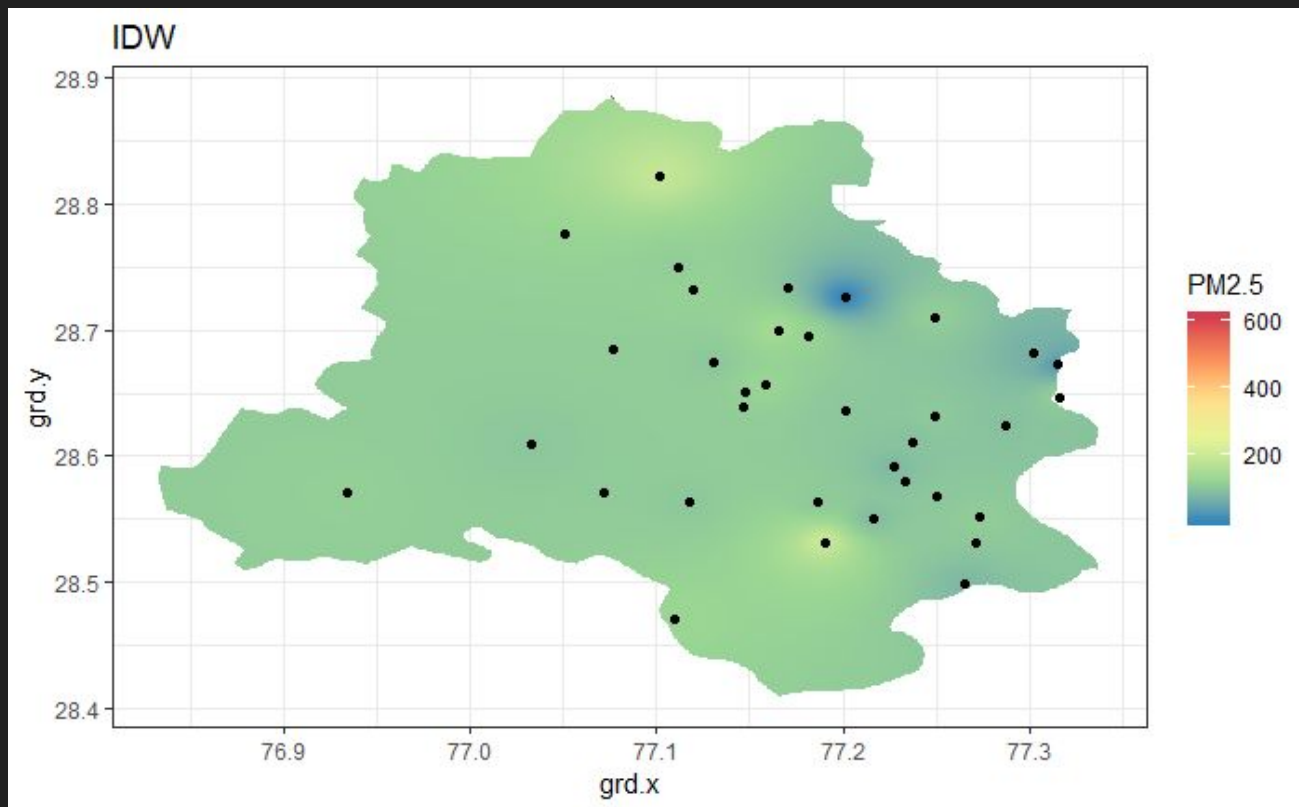
The next series of geographical plots will show the interpolated values across Delhi for a period of 8 days (the entire Diwali'18 week) interpolated by the 2 techniques:

- Ordinary Kriging
- IDW Interpolation

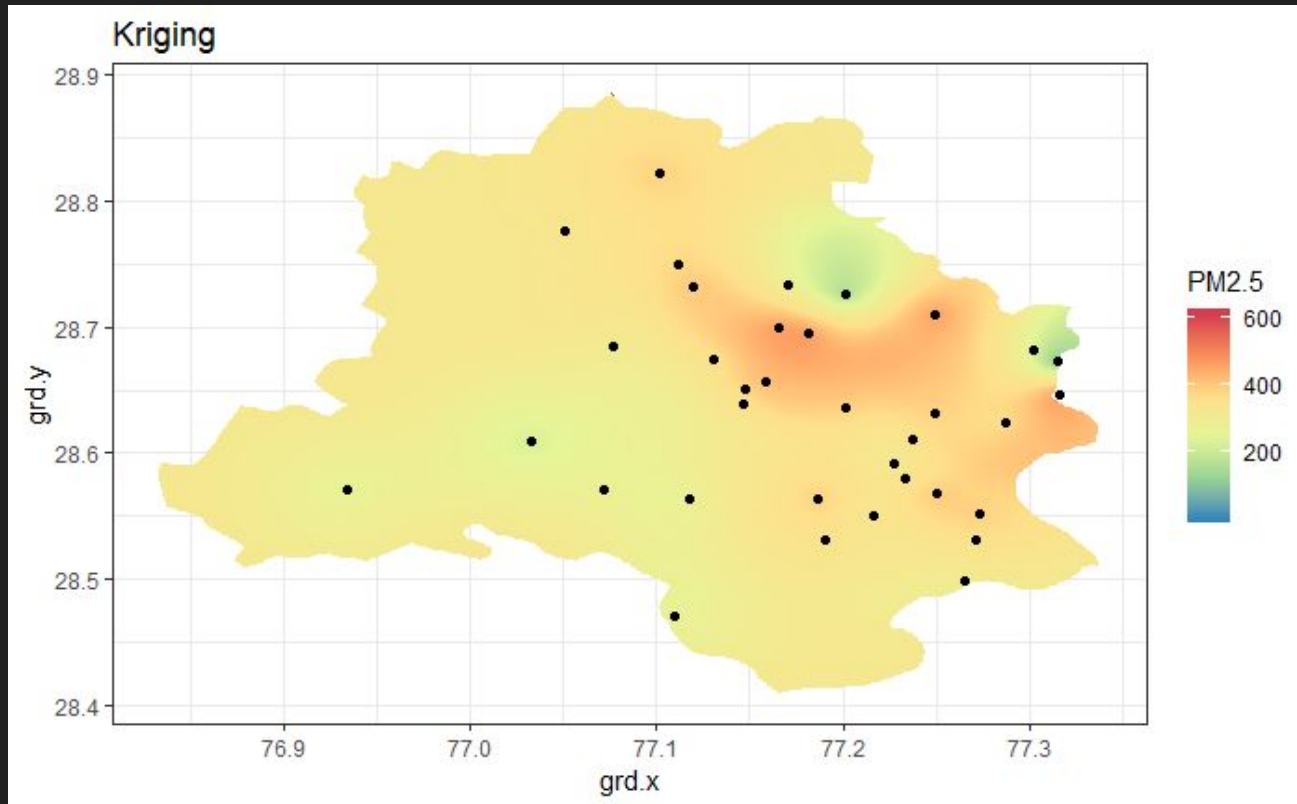
4th Nov 2018



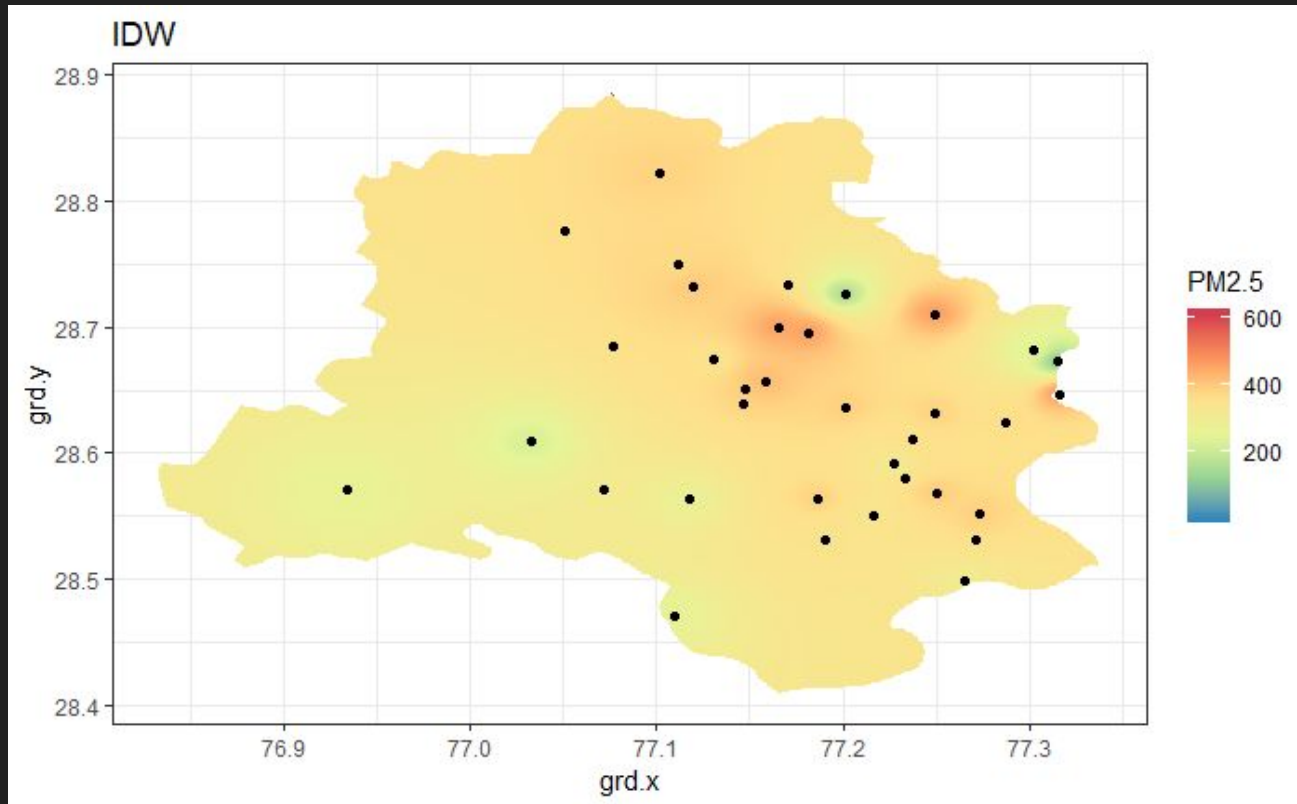
4th Nov 2018



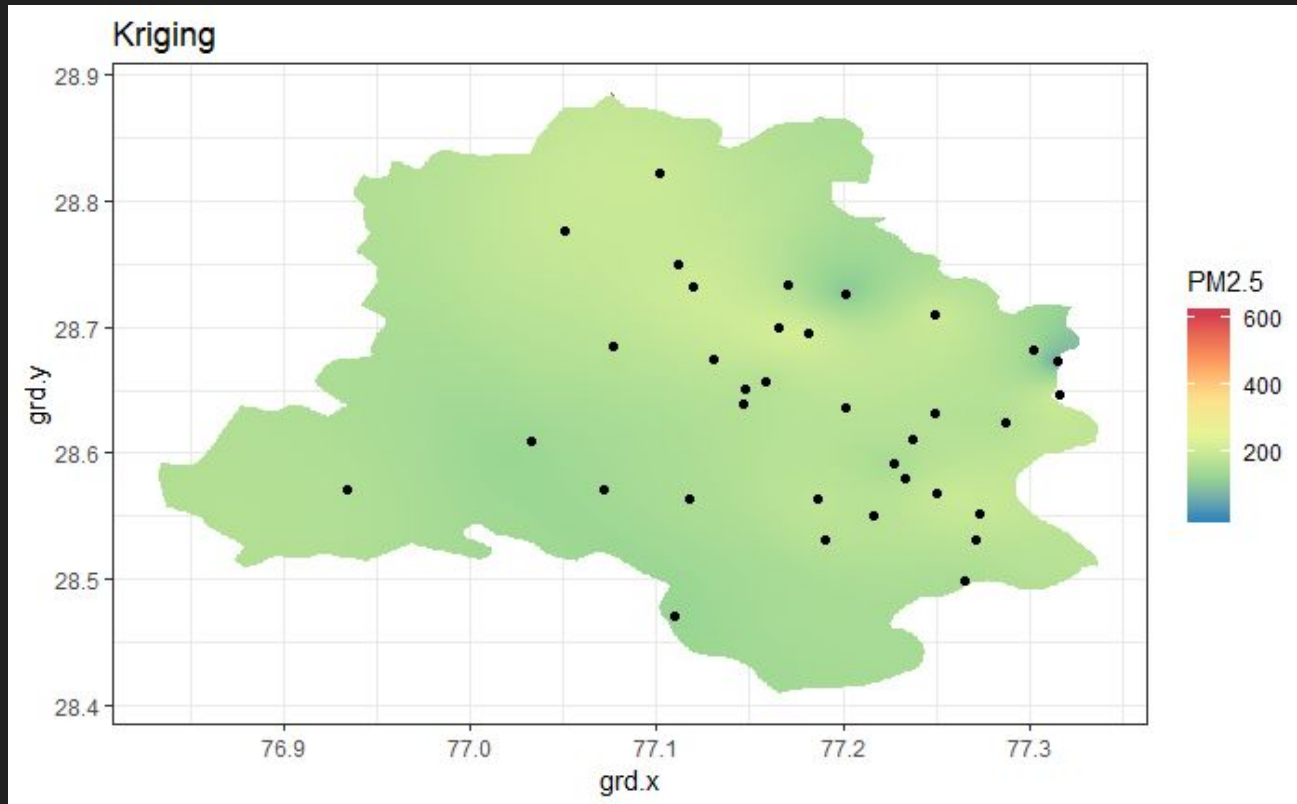
5th Nov 2018



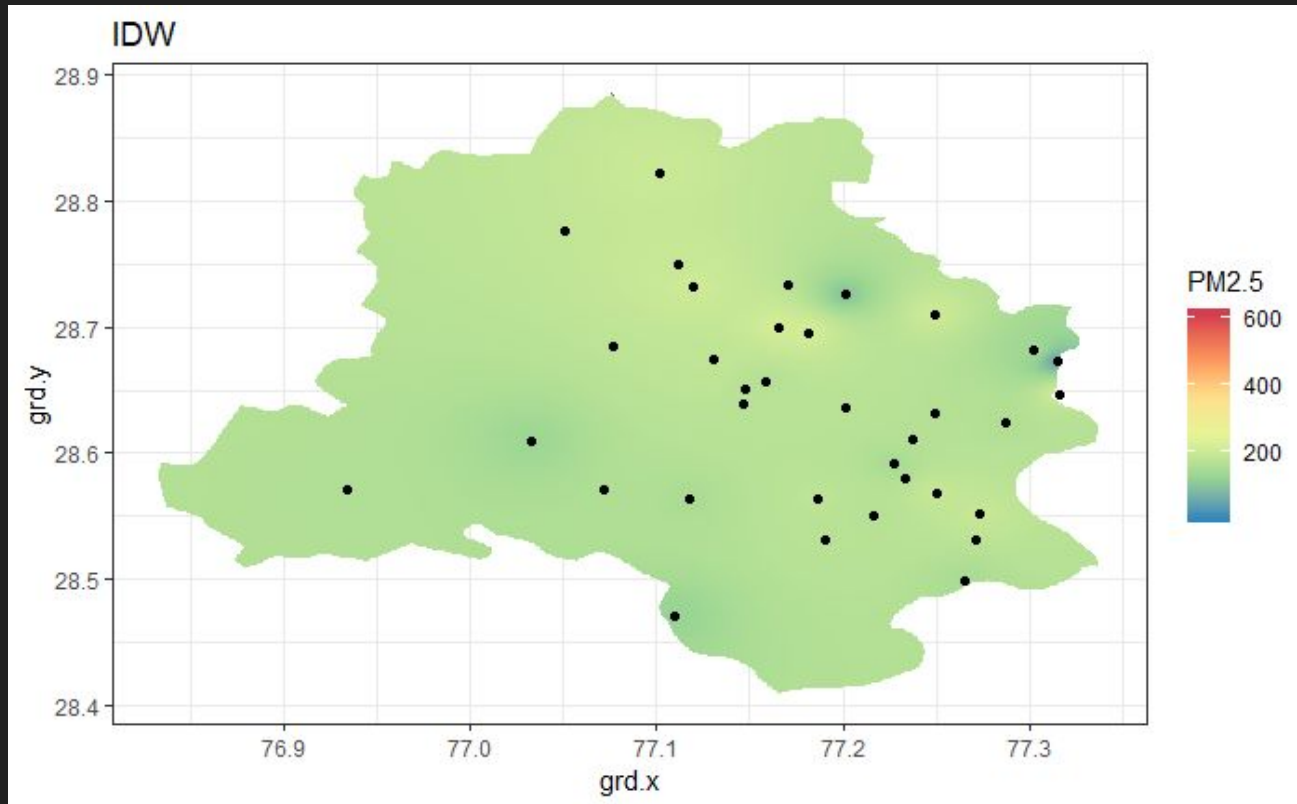
5th Nov 2018



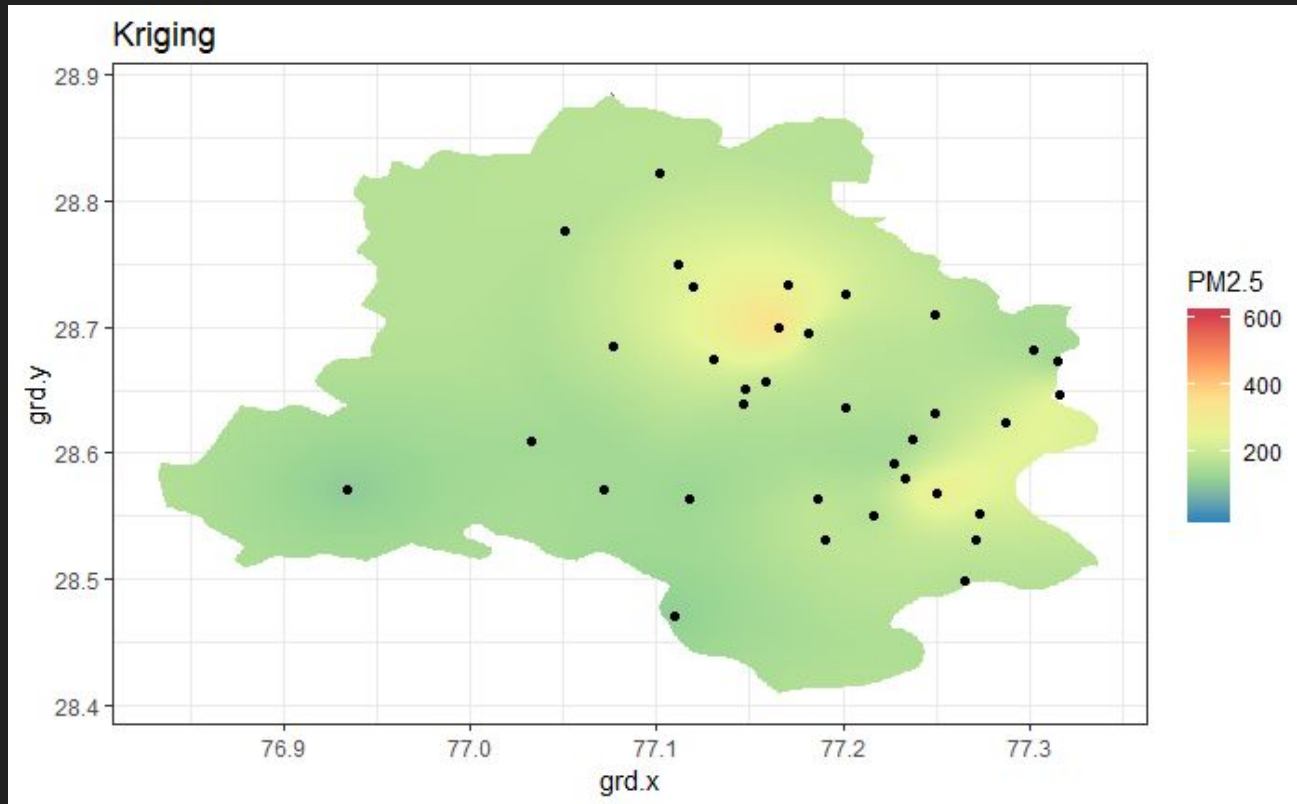
6th Nov 2018



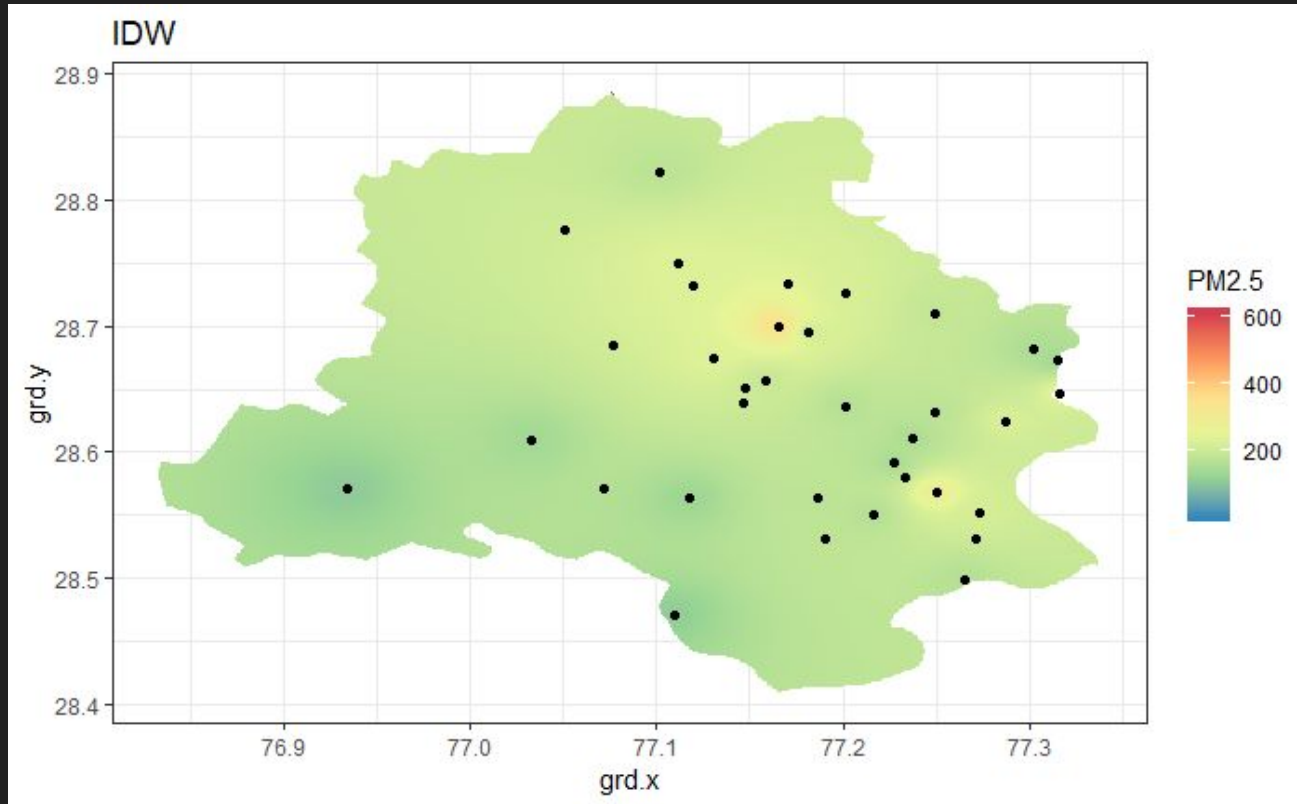
6th Nov 2018



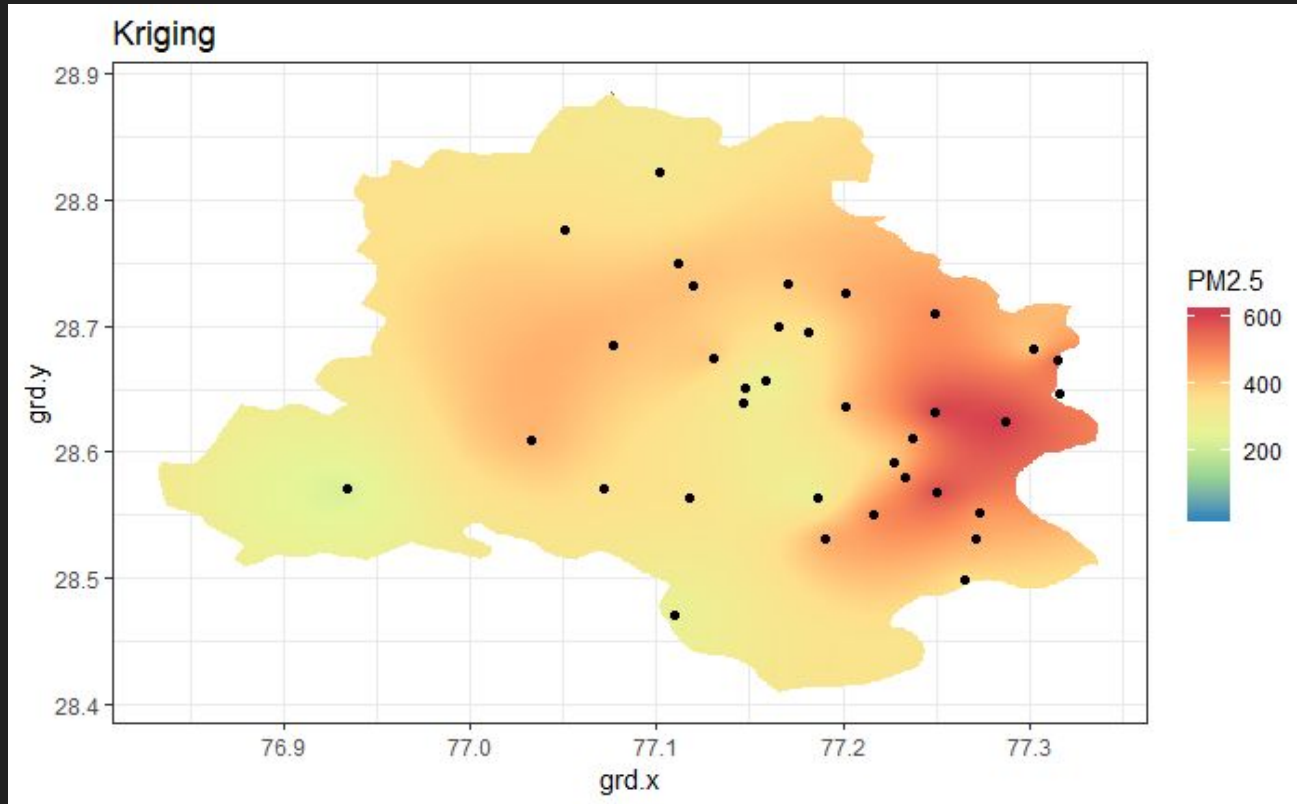
7th Nov 2018 (Today is Diwali)



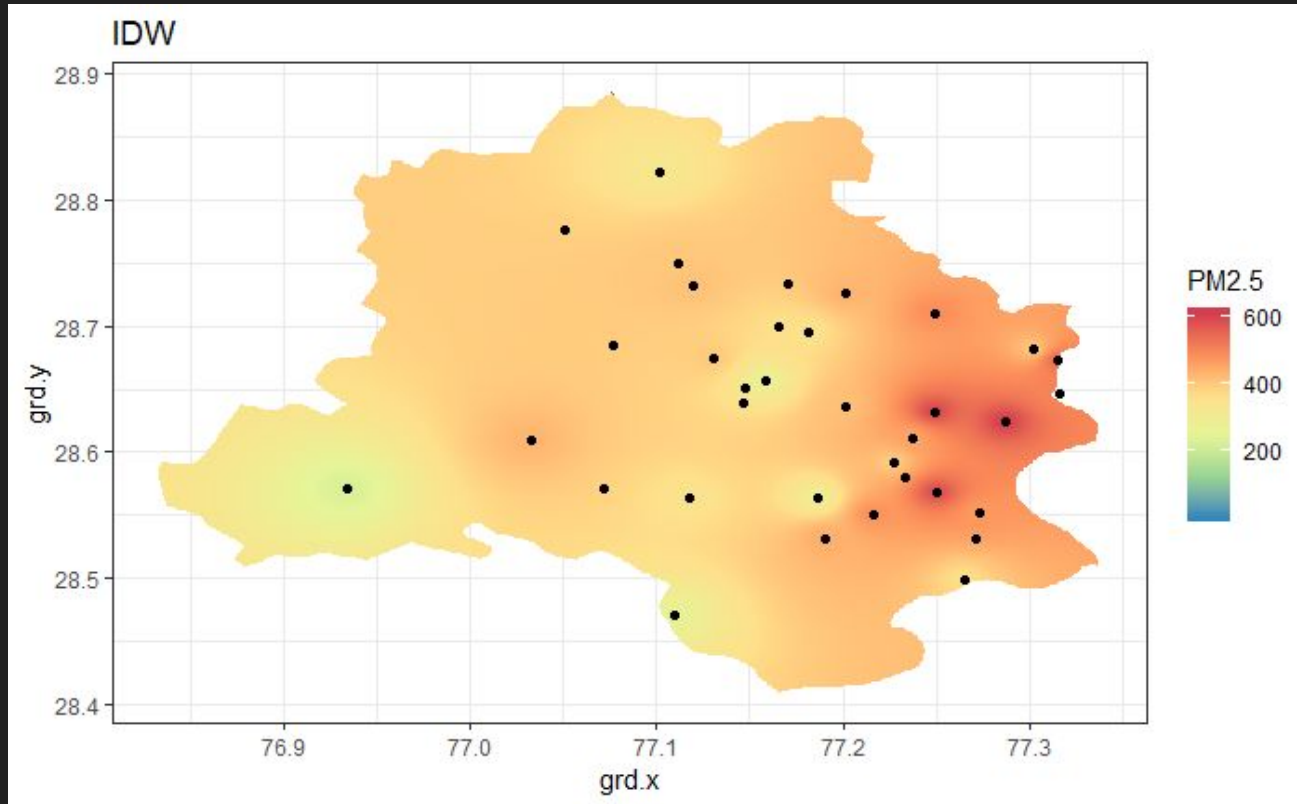
7th Nov 2018 (Today is Diwali)



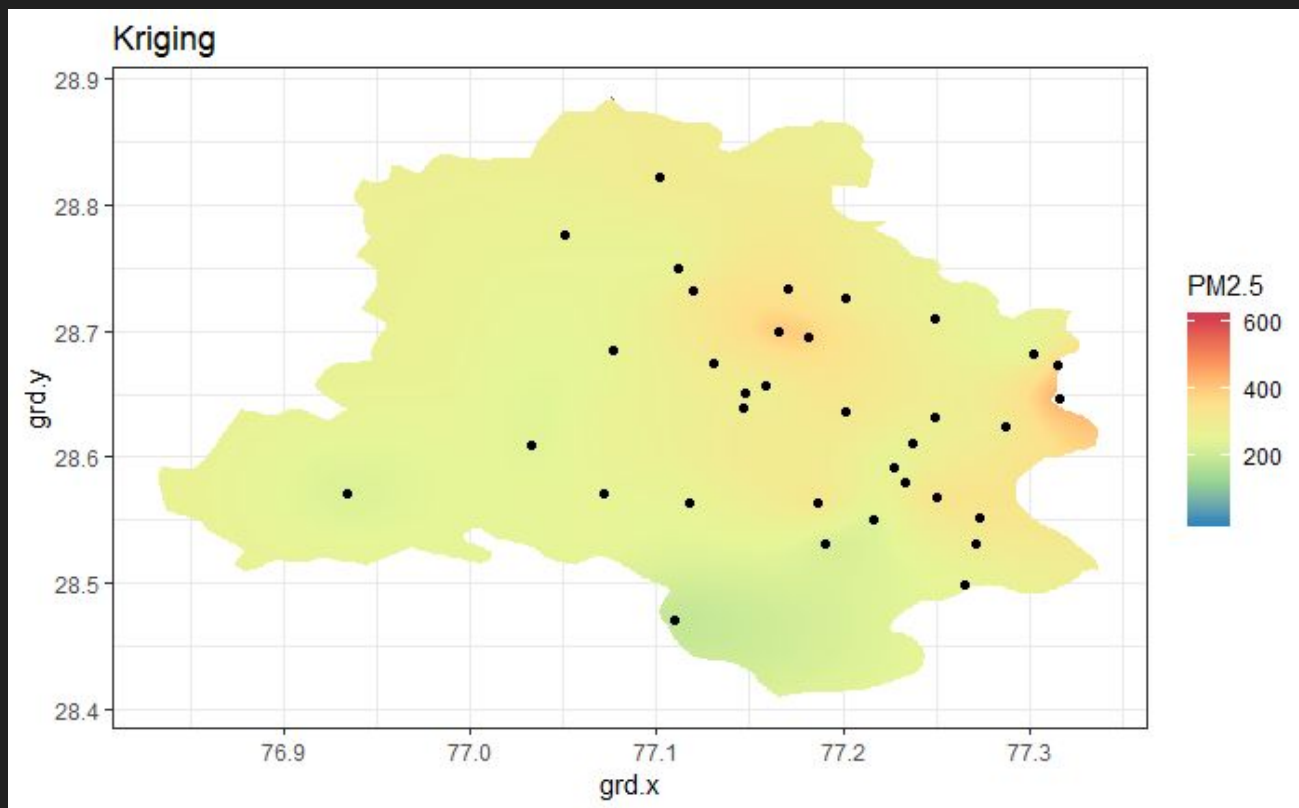
8th Nov 2018 (Observe Diwali After-effects!)



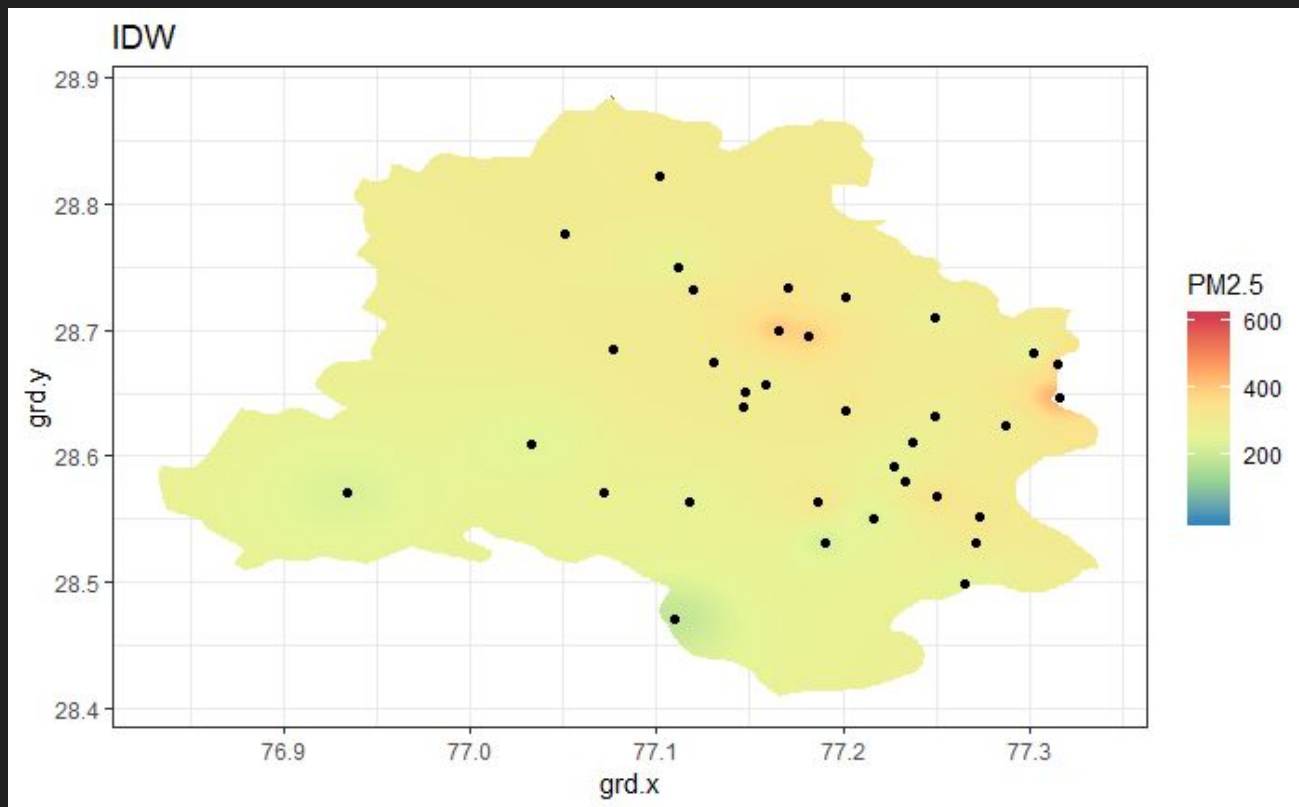
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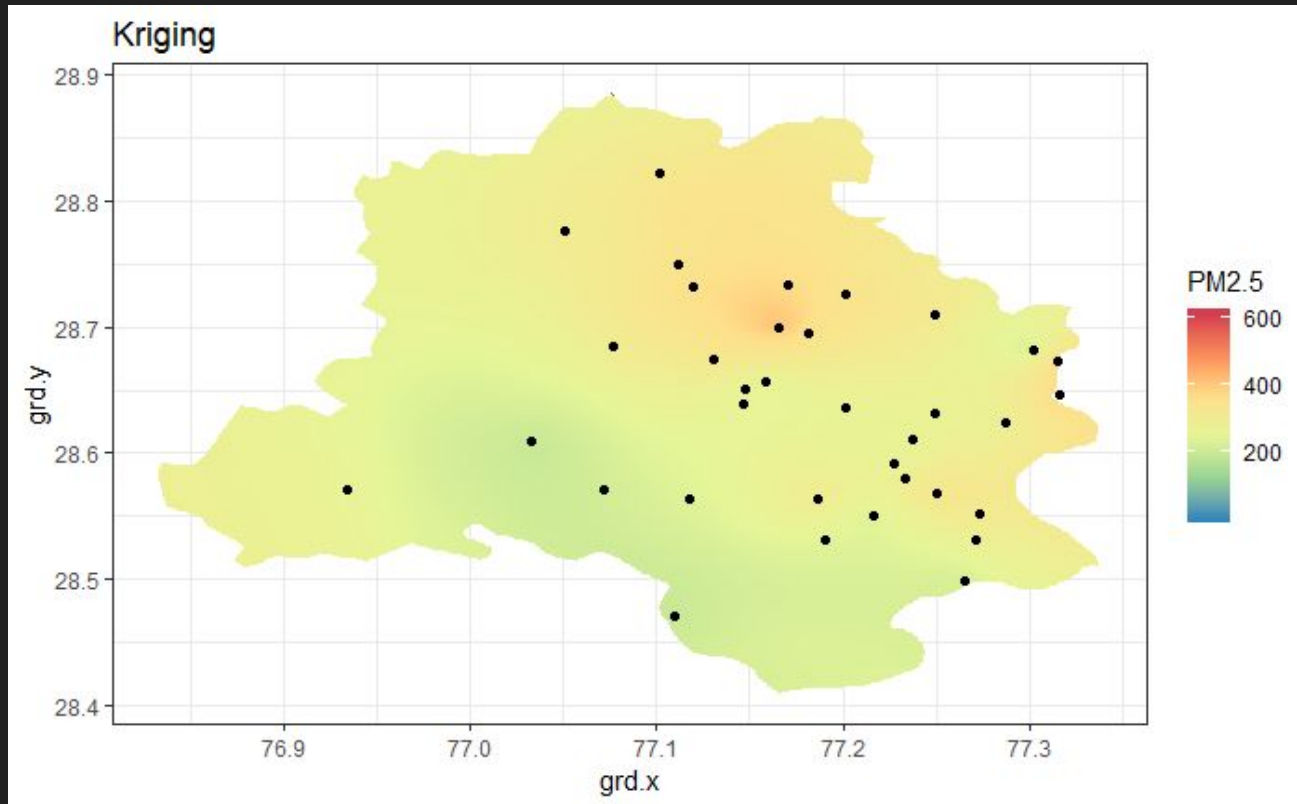
9th Nov 2018



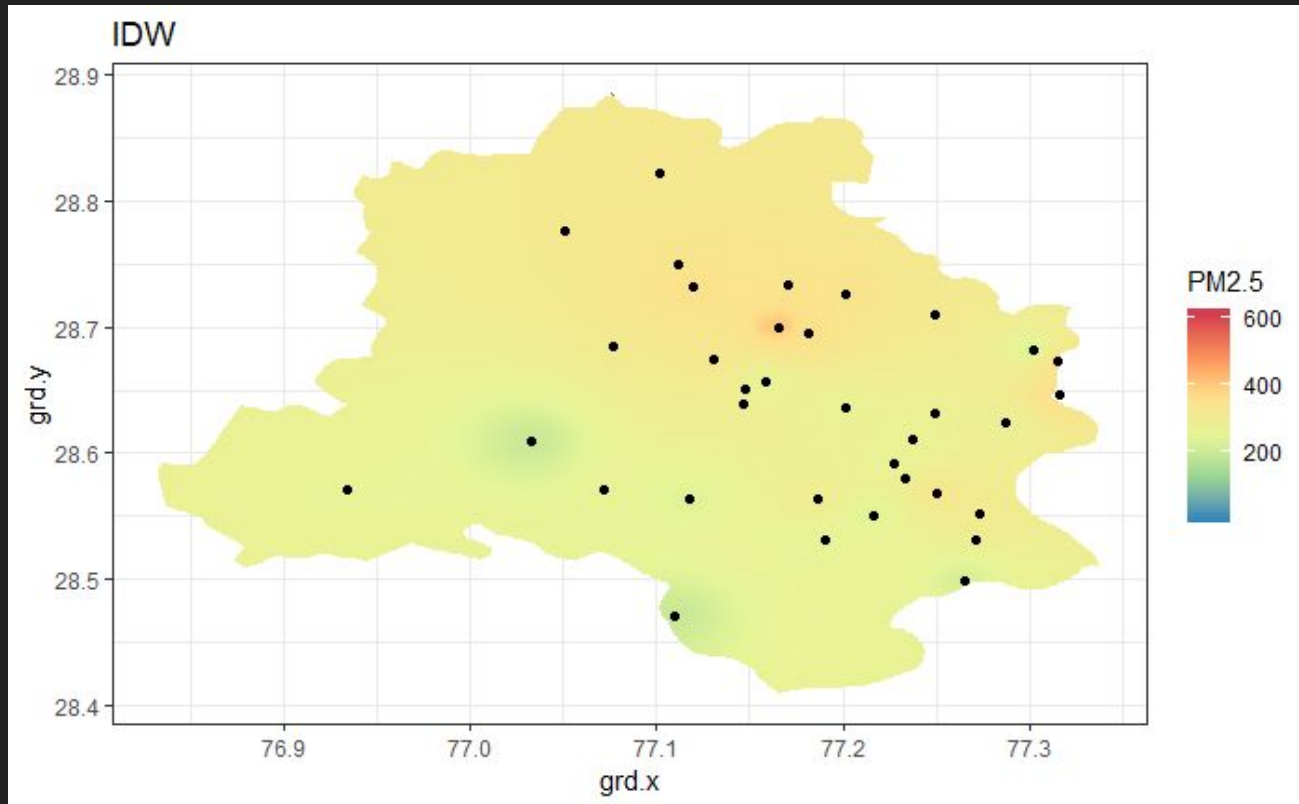
9th Nov 2018



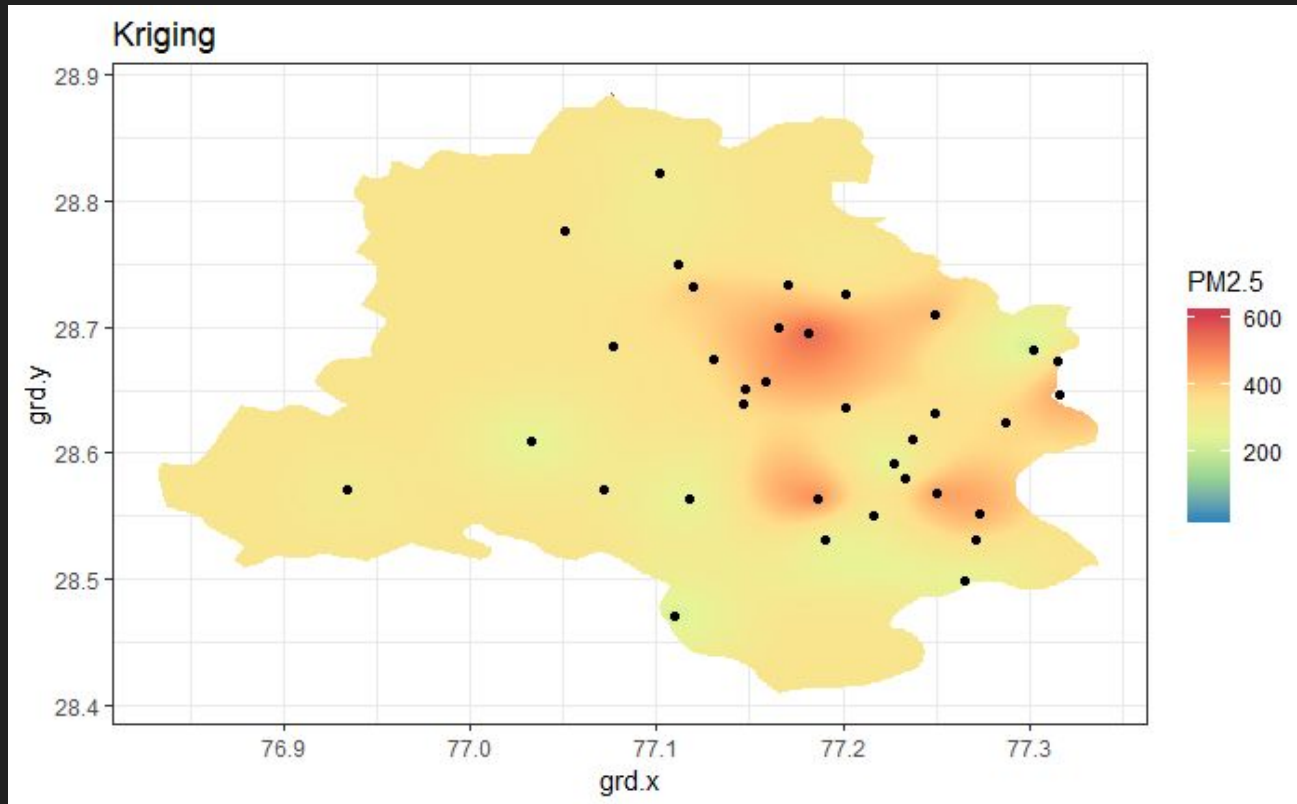
10th Nov 2018



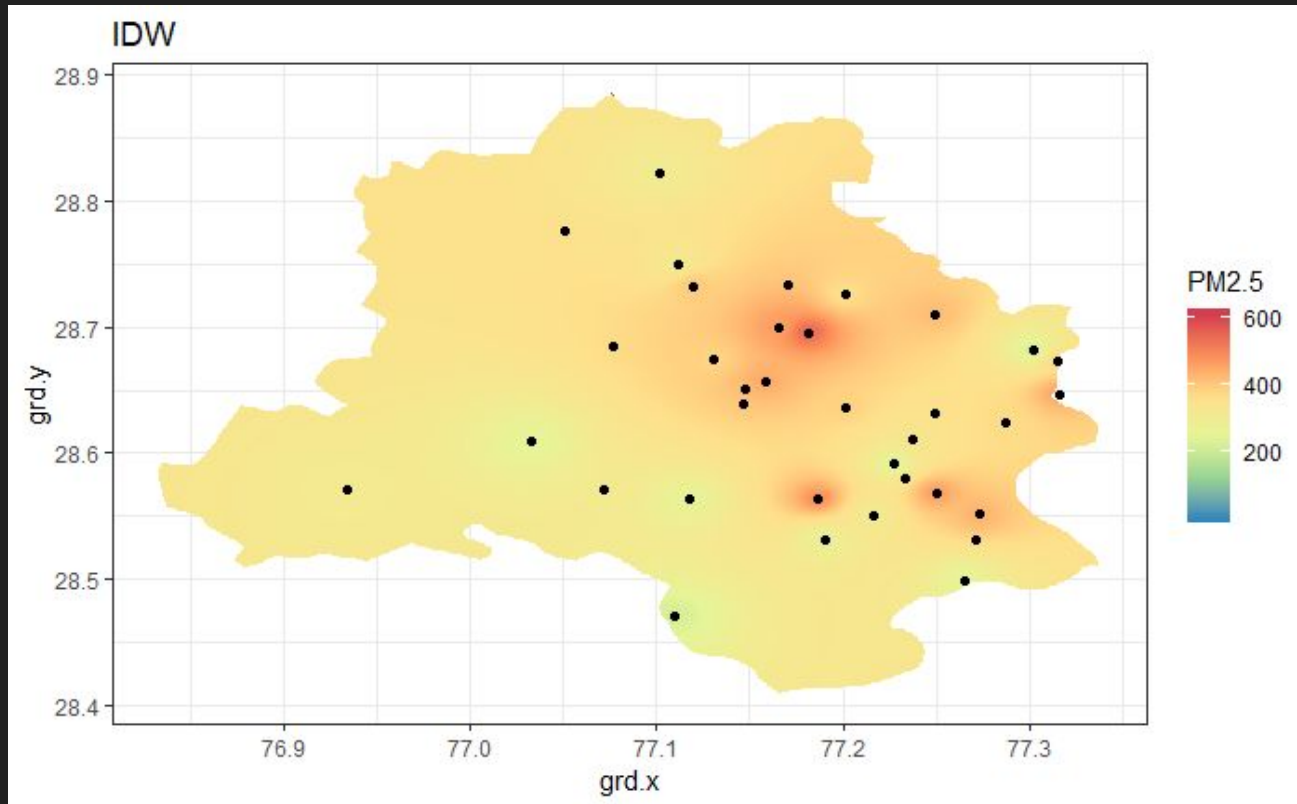
10th Nov 2018



11th Nov 2018



11th Nov 2018



Thank You!