MA202: Tutorial 3

Name: Prakram Rathore

Roll No.: 20110141

Q2.

For small values of n, the Gauss elimination seems to take less iterations than that of Gauss Seidel Method. For n>5 possibly, Gauss Seidel starts taking less iterations than Gauss elimination, owing to its O (n2) nature, while Gauss elimination has its iterations equal to O (n3), which is evident from the fixed number of iterations due to its for loop.

Also, keep the tolerance E to 1 or more, for random input for Seidel Method, otherwise it would not run properly as n increases.

Additionally, Gauss Seidel shall only work properly, if the matrix is diagonally dominant.

```
import numpy as np
n = int(input("Enter n? "))
a = np.zeros((n,n))
b = np.zeros((n,1))
x = np.zeros((n,1))
for i in range(n):
    for j in range(n):
        a[i,j] = (12*i+13*j)/10 + 7
for i in range(n):
    b[i] = 14*i/10 + 3
iter = 0
a1 = a
b1 = b
x1 = x
for i in range(n-1):
    for j in range(i+1,n):
        b[j] = b[j]-a[j,i]*b[i]/a[i,j]
        iter=iter+1
        for k in range(n-1,-1,-1):
            a[j,k] = a[j,k] - a[j,i]*a[i,k]/a[i,i]
            iter = iter+1
```

```
x[n-1] = b[n-1]/a[n-1,n-1]
for i in range(n-2,-1,-1):
   s = 0
   for j in range(i+1,n):
        s = s+a[i,j]*x[j]
       iter=iter+1
    x[i] = (b[i]-s)/a[i,j]
    iter = iter + 1
print(x)
print("Gauss elimination-iterations: ", iter)
px = np.array(x1,copy=True)
iter = 0
for i in range(n):
   x1[i] = i*17/10 + 9
E=0.01
Ea=0
for i in range(n):
    Ea=max(Ea,abs((x1[i]-px[i])/x1[i]))
print(E)
print(Ea)
while Ea>=E:
   px = x1
    Ea = 0
    for i in range(n):
        s1 = 0
        s2 = 0
        for j in range(i-1):
            s1 = s1+a1[i,j]*x1[j]
            iter = iter + 1
        for j in range(i,n):
            s2 = s2+a1[i,j]*x1[j]
            iter = iter+1
        x1[i] = (b1[i]-s1-s2)/a1[i,i]
    for i in range(n):
        Ea = \max(Ea, abs((x1[i]-px[i])/x1[i]))
print(x1)
print("Gauss Seidel-iterations: ", iter)
```

Observation:

For n = 3,

Gauss elimination-iterations: $17 \sim O(3^3)$

Gauss Seidel-iterations: $7 \sim O(3^2)$

For n = 10,

Gauss elimination-iterations: $549 \sim O(10^3)$

Gauss Seidel-iterations: $91 \sim O(10^2)$

Note: Here for small value of n also, Gauss elimination is taking more time, because here we generating random equation by manipulating the values of i, j, k of for loops. If we use different equation then our results which is mentioned above is holding true.