Probability Models - Sheldon M. Ross

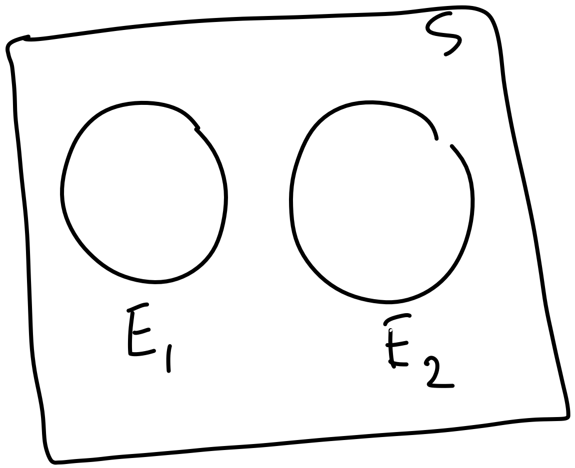
Chap-1

Sample Space, S

$S = \{H, T\}$, $S = \{1, 2, 3, 4, 5, 6\}$

Event, E = Any subset of sample space

$E = \{HH, HT, TH\}$



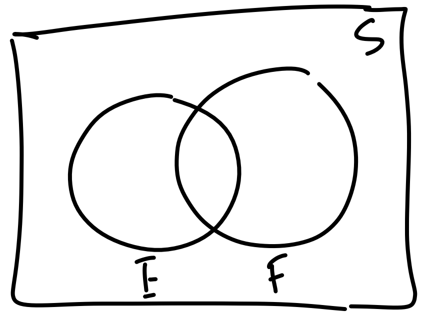
$$P(E) = \frac{|E|}{|S|}$$

(i) $0 \leq P(E) \leq 1$

(ii) $P(S) = 1$

(iii) For mutually exclusive events $P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$



$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

Ex: $E = \{HH, HT\}$, $F = \{HH, TH\}$

$$P(E \cap F) = \frac{1}{4}$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{2}{4} + \frac{2}{4} - \frac{1}{4}$$

$$= \frac{3}{4}$$

Conditional Probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(EF)}{P(F)}$$

Ex: $\{bb, bg, gb, gg\}$

E = both boys

F = at least one boy

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Independent Events :

$$P(E|F) = P(E) = \frac{P(EF)}{P(F)}$$

$$\therefore P(EF) = P(E)\,P(F)$$

Bayes Theorem:

$P(E|F)$

$$P(F|E) = \frac{P(EF)}{P(E)}$$

$F \cup F^c$

$$= \frac{P(EF)}{P(EF \cup EF^c)}$$

$$= \frac{P(EF)}{P(EF) + P(EF^c)}$$

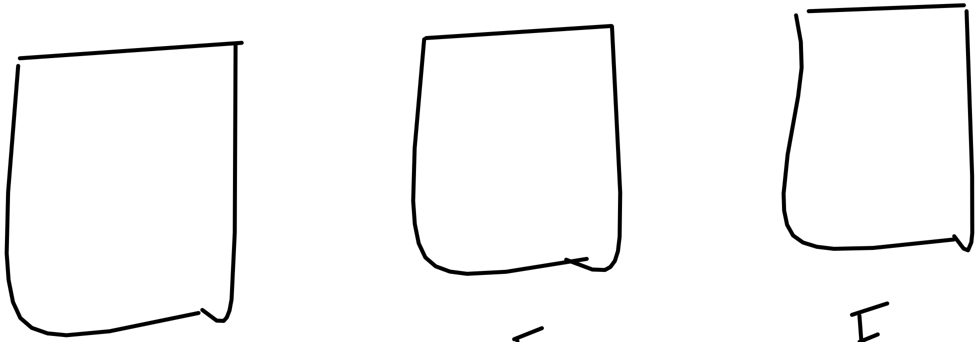
$$= \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^c)P(F^c)}$$

$F_1, F_2 \dots F_n$ mutually exclusive events

$$\bigcup_{i=1}^{n} F_i = S$$

$$P(F_j \mid E) = \frac{P(E \mid F_j)\, P(F_j)}{\sum_{i=1}^{n} P(E \mid F_i)\, P(F_i)}$$

Ex:



F_1 F_2 F_3

ℓ = word

$$P(F_1) = P(F_2) = P(F_3) = \frac{1}{3}$$

F_i = ℓ is in file i

α_i = Prob. of finding ℓ in i using QSearch

E = QSearched F_1 and did not find ℓ

$$P(F_1|E) = \frac{P(E|F_1)P(F_1)}{P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + P(E|F_3)P(F_3)}$$

$$= \frac{(1-\alpha_1)\frac{1}{3}}{(1-\alpha_1)\frac{1}{3} + 1\cdot\frac{1}{3} + 1\cdot\frac{1}{3}} = \frac{1-\alpha_1}{3-\alpha_1}$$