Chapter 5:
Adversarial Search
&
Game Playing

Typical assumptions

- Two agents whose actions alternate
- Utility values for each agent are the opposite of the other
  - creates the adversarial situation
- Fully observable environments
- In game theory terms:
  - “Deterministic, turn-taking, zero-sum games of perfect information”
- Can generalize to stochastic games, multiple players, non zero-sum, etc

Search versus Games

- Search – no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristics and CSP techniques can find *optimal* solution
  - Evaluation function: estimate of cost from start to goal through given node
  - Examples: path planning, scheduling activities

- Games – adversary
  - Solution is strategy (strategy specifies move for every possible opponent reply).
  - Time limits force an *approximate* solution
  - Evaluation function: evaluate “goodness” of game position
  - Examples: chess, checkers, Othello, backgammon

Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

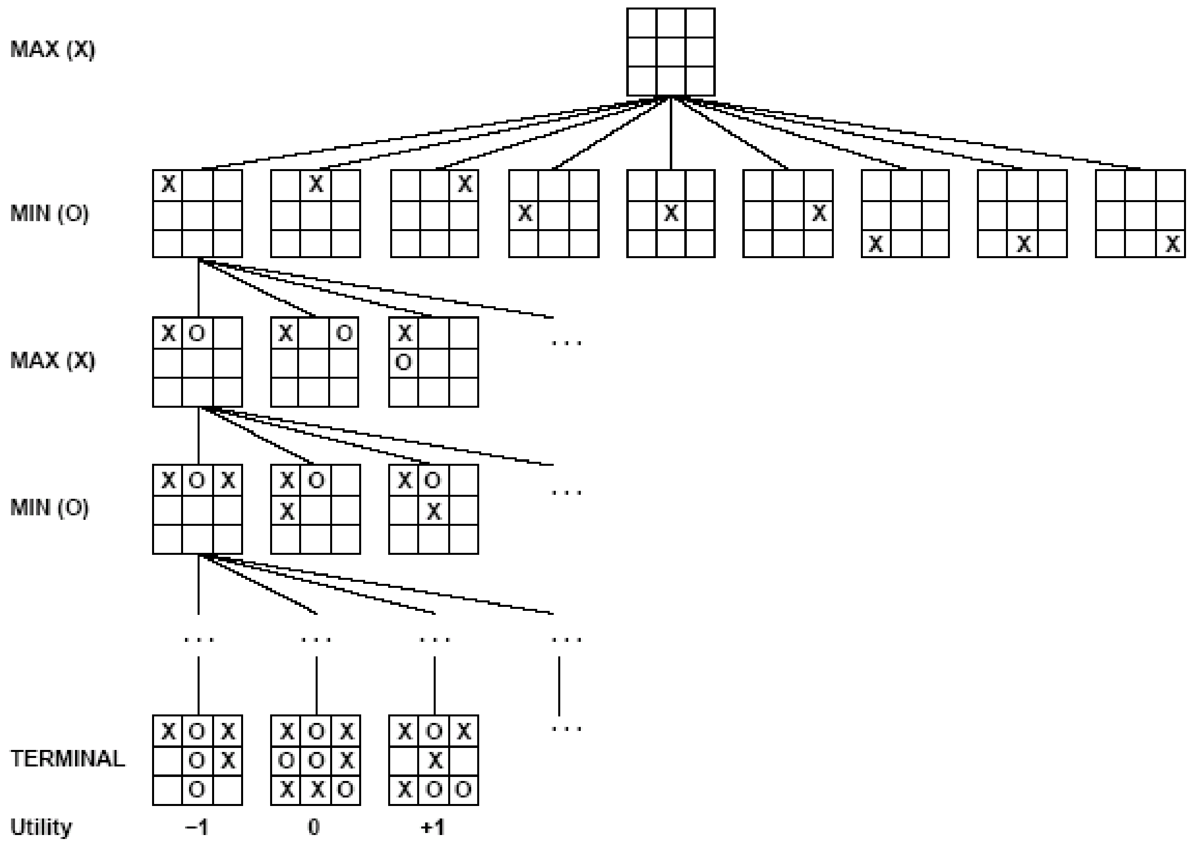
Game Setup

- Two players: MAX and MIN
- MAX moves first and they take turns until the game is over
  - Winner gets award, loser gets penalty.
- Games as search:
  - Initial state: e.g. board configuration of chess
  - Successor function: list of (move, state) pairs specifying legal moves.
  - Terminal test: Is the game finished?
  - Utility function: Gives numerical value of terminal states. E.g. win (+1), lose (-1) and draw (0) in tic-tac-toe or chess
- MAX uses search tree to determine next move.

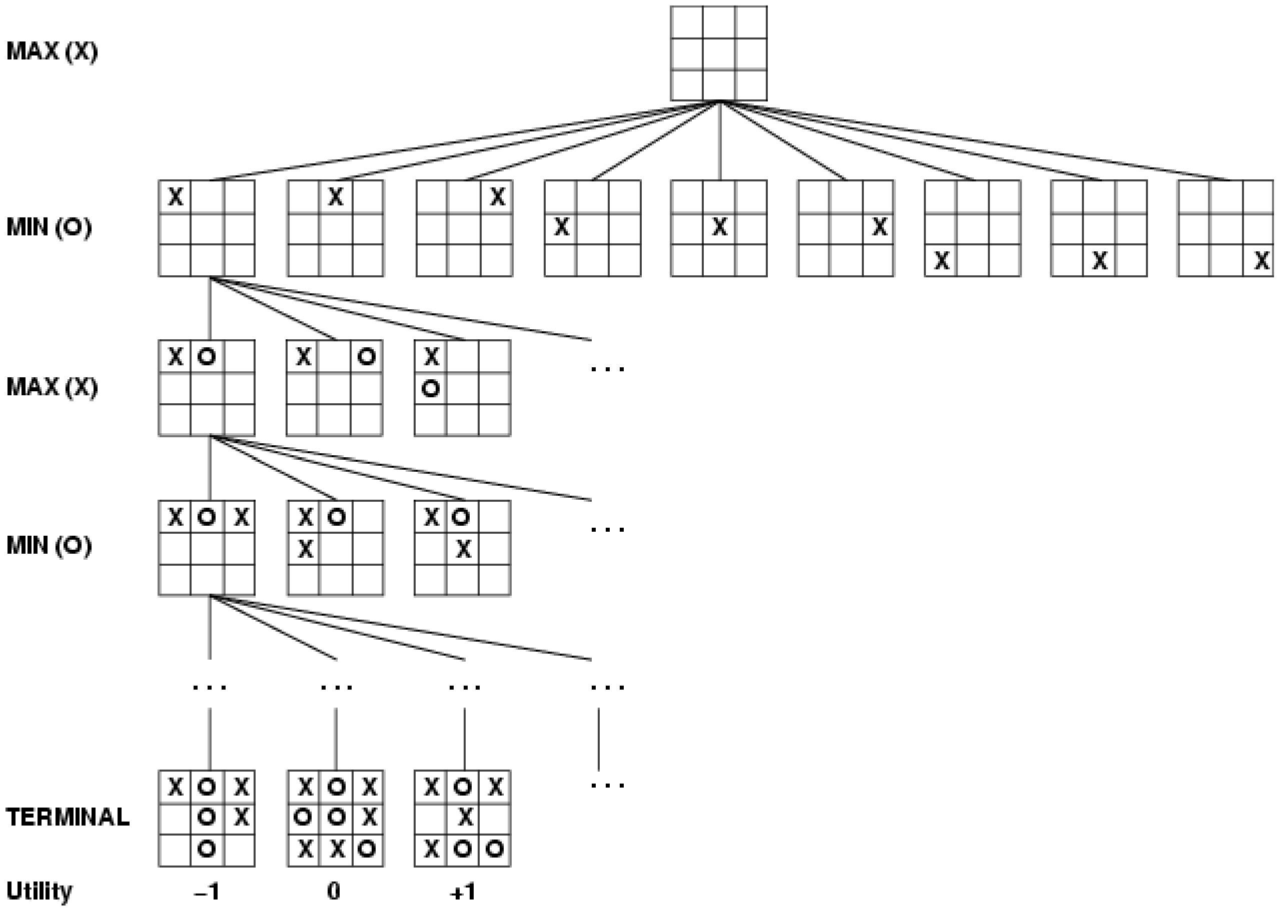
Size of search trees

- b = branching factor
- d = number of moves by both players
- Search tree is $O(b^d)$
- Chess
  - b ~ 35
  - d ~100
    - search tree is ~ 10^{154} (!!)
    - completely impractical to search this
- Game-playing emphasizes being able to make optimal decisions in a finite amount of time
  - Somewhat realistic as a model of a real-world agent
  - Even if games themselves are artificial

Partial Game Tree for Tic-Tac-Toe



Game tree (2-player, deterministic, turns)



How do we search this tree to find the optimal move?

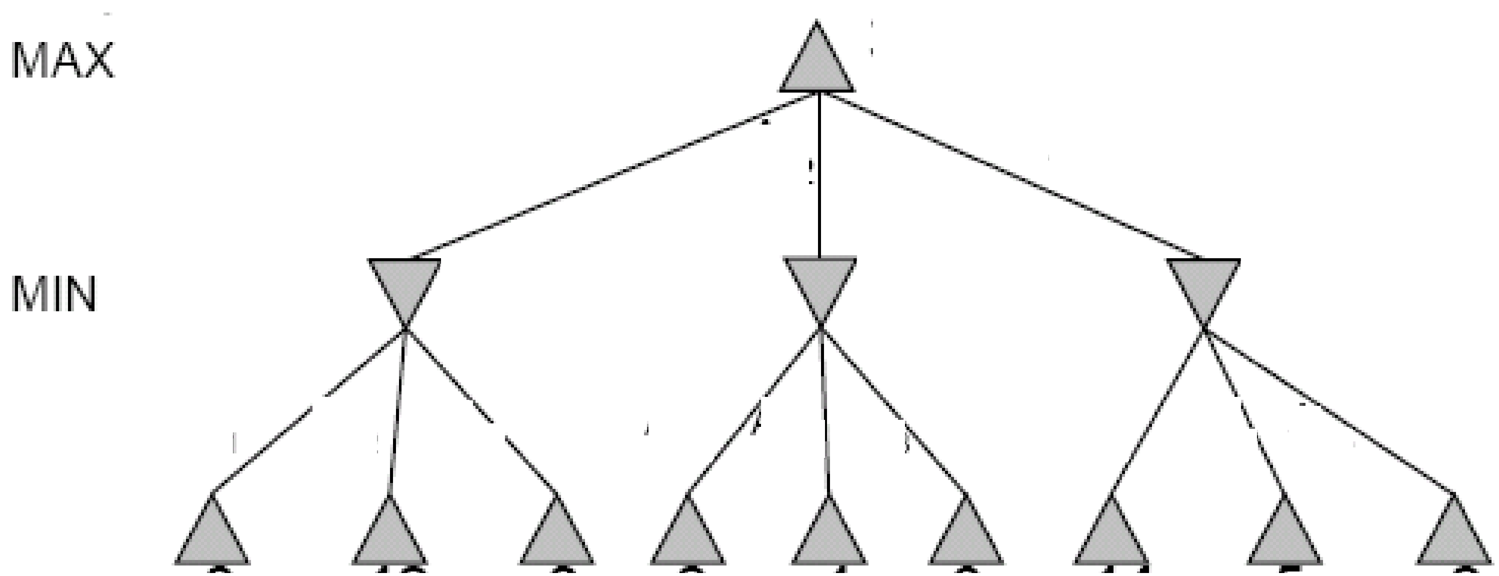
Minimax strategy

- Find the optimal *strategy* for MAX assuming an infallible MIN opponent
  - Need to compute this all the down the tree
- Assumption: Both players play optimally!
- Given a game tree, the optimal strategy can be determined by using the minimax value of each node:

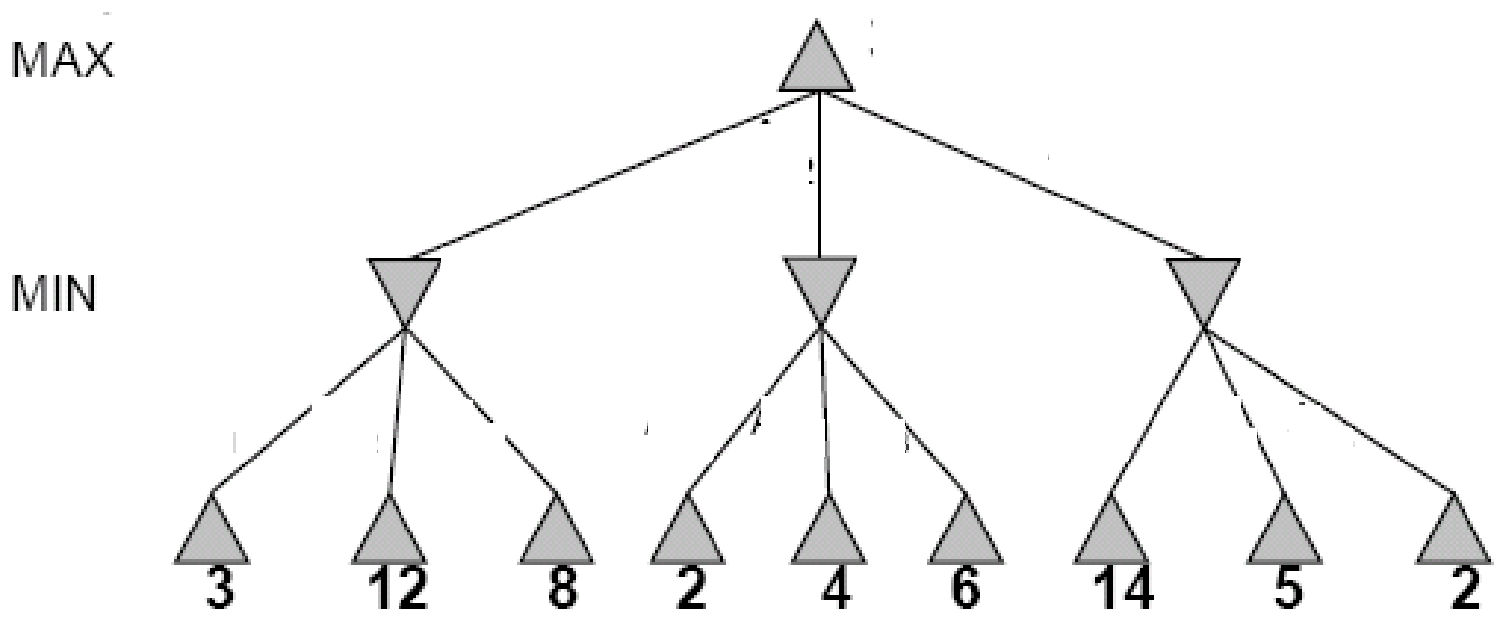
MINIMAX-VALUE(*n*)=

UTILITY(*n*)	**If *n* is a terminal**
$\max_{s \in successors(n)}$ MINIMAX-VALUE(*s*)	**If *n* is a max node**
$\min_{s \in successors(n)}$ MINIMAX-VALUE(*s*)	**If *n* is a min node**

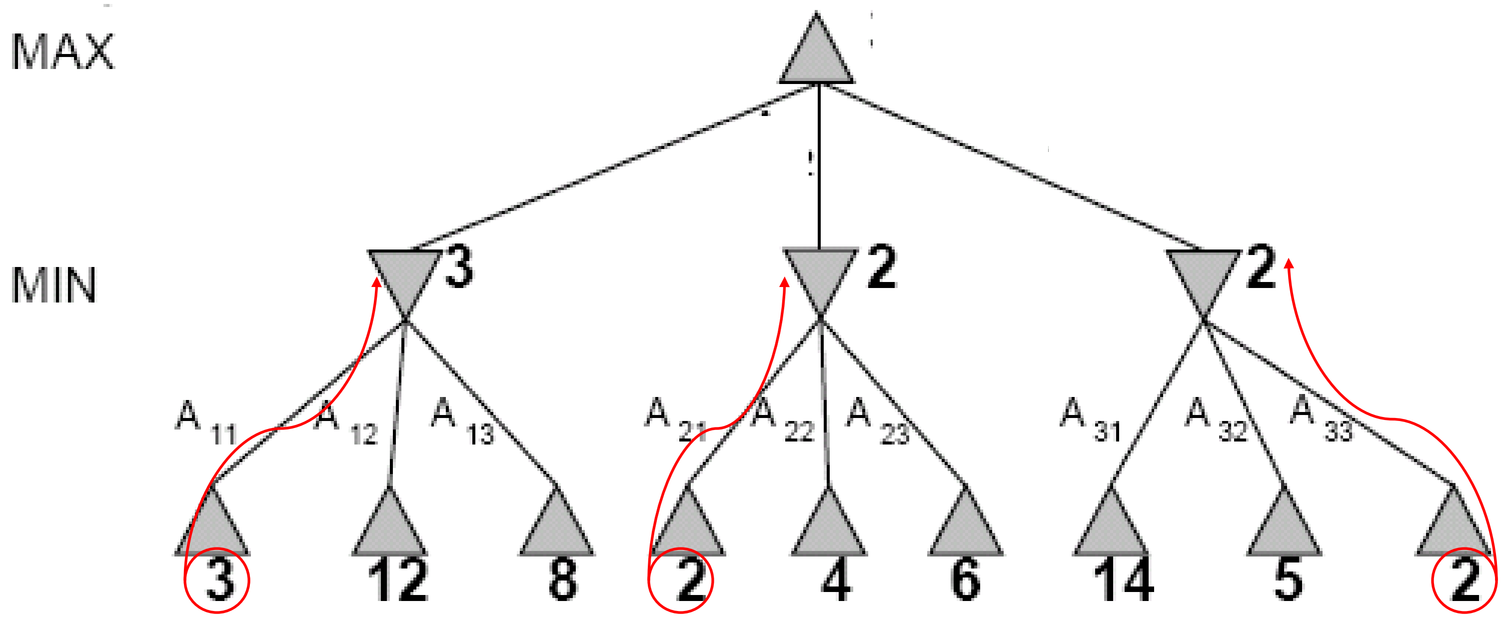
Two-Ply Game Tree



Two-Ply Game Tree

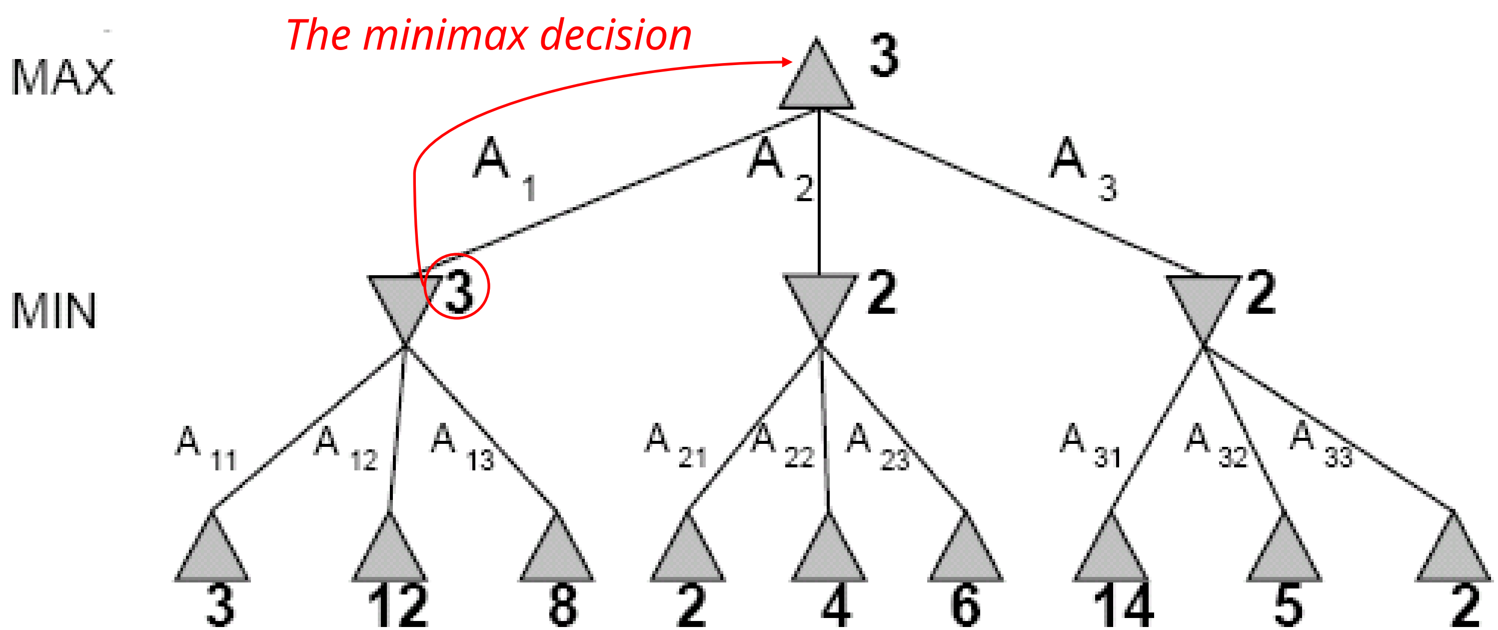


Two-Ply Game Tree



Two-Ply Game Tree

Minimax maximizes the utility for the worst-case outcome for max



What if MIN does not play optimally?

- Definition of optimal play for MAX assumes MIN plays optimally:
  - maximizes worst-case outcome for MAX
- But if MIN does not play optimally, MAX will do even better
  - Can prove this (Problem 6.2)

Minimax Algorithm

- Complete depth-first exploration of the game tree
- Assumptions:
  - Max depth = d, b legal moves at each point
  - E.g., Chess: d ~ 100, b ~35

Criterion	Minimax
Time	$O(b^m)$ ☹
Space	$O(bm)$ ☺

Pseudocode for Minimax Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
  inputs: *state*, current state in game
  v ← MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value *v*

---

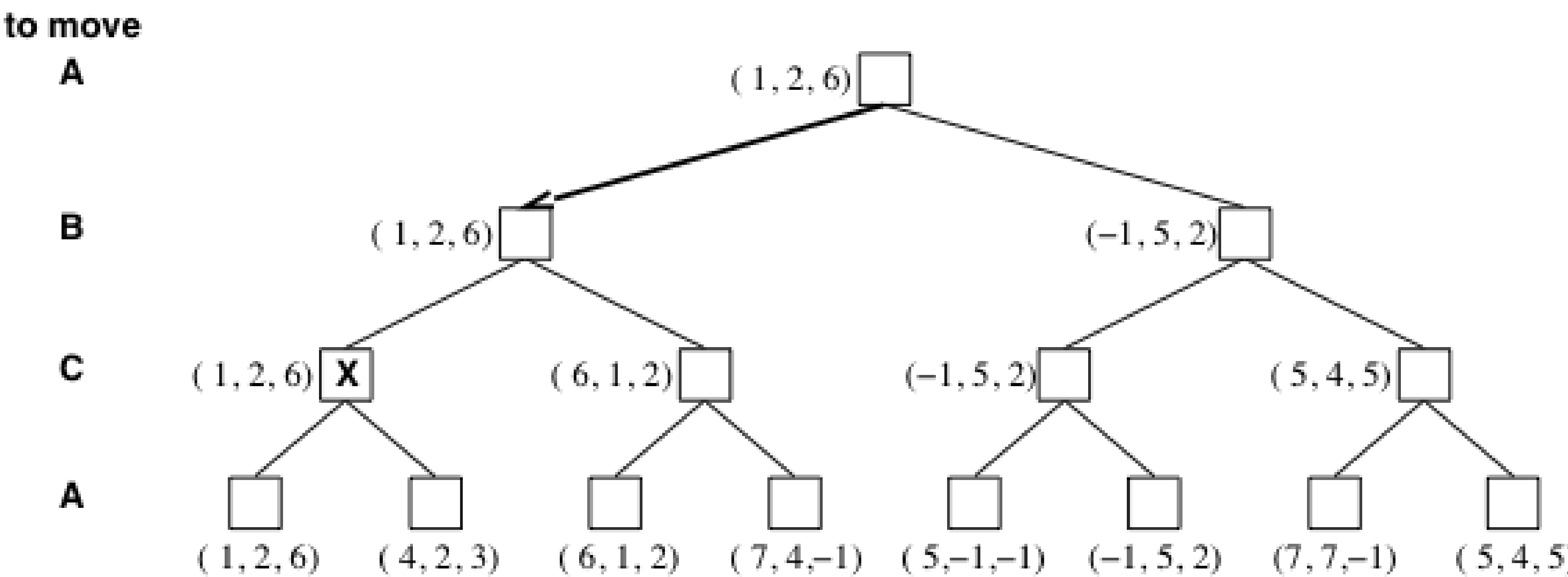
function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  v ← ∞
  for *a*,*s* in SUCCESSORS(*state*) **do**
    v ← MAX(*v*,MIN-VALUE(*s*))
  return *v*

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  v ← ∞
  for *a*,*s* in SUCCESSORS(*state*) **do**
    v ← MIN(*v*,MAX-VALUE(*s*))
  return *v*

Multiplayer games

- Games allow more than two players
- Single minimax values become vectors



Aspects of multiplayer games

- Previous slide (standard minimax analysis) assumes that each player operates to maximize only their own utility

- In practice, players make alliances
  - E.g, C strong, A and B both weak
  - May be best for A and B to attack C rather than each other

- If game is not zero-sum (i.e., utility(A) = - utility(B) then alliances can be useful even with 2 players
  - e.g., both cooperate to maximum the sum of the utilities

Practical problem with minimax search

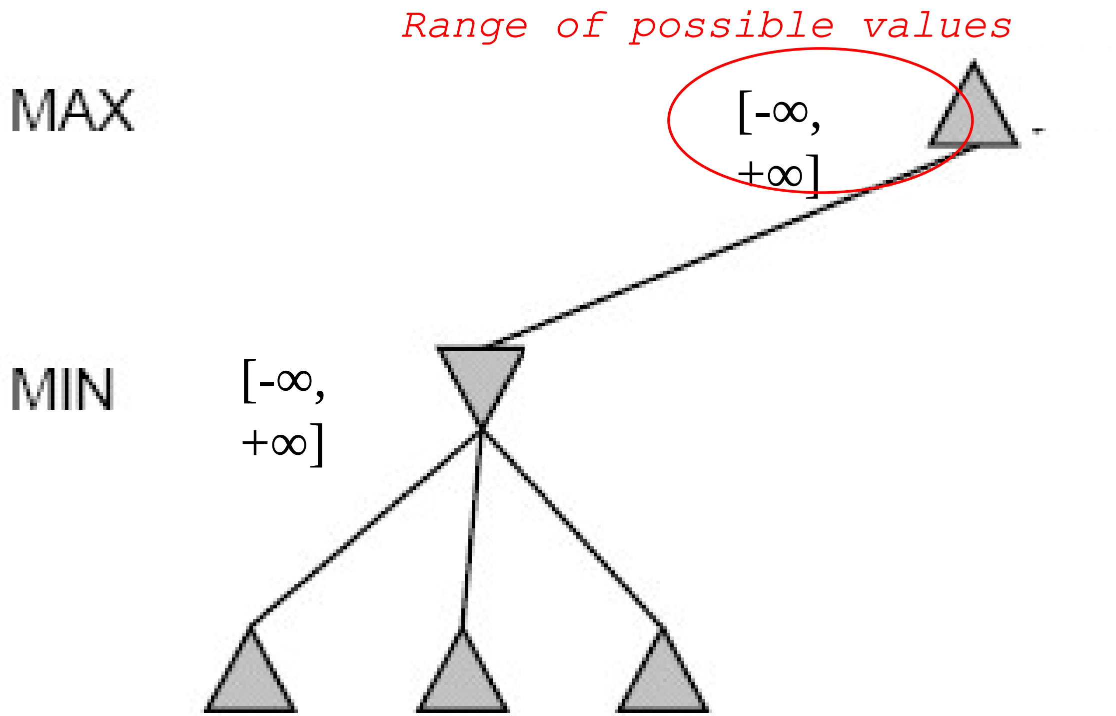
- Number of game states is exponential in the number of moves.
  - Solution: Do not examine every node

  => pruning
    - Remove branches that do not influence final decision

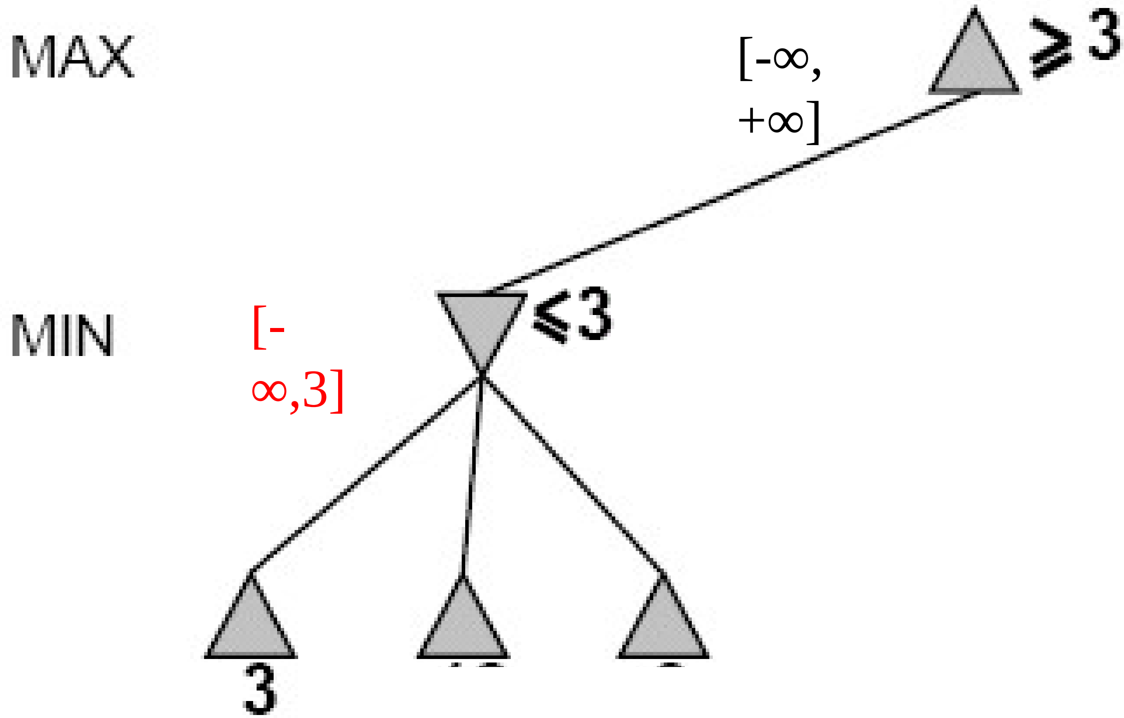
- Revisit example …

Alpha-Beta Example

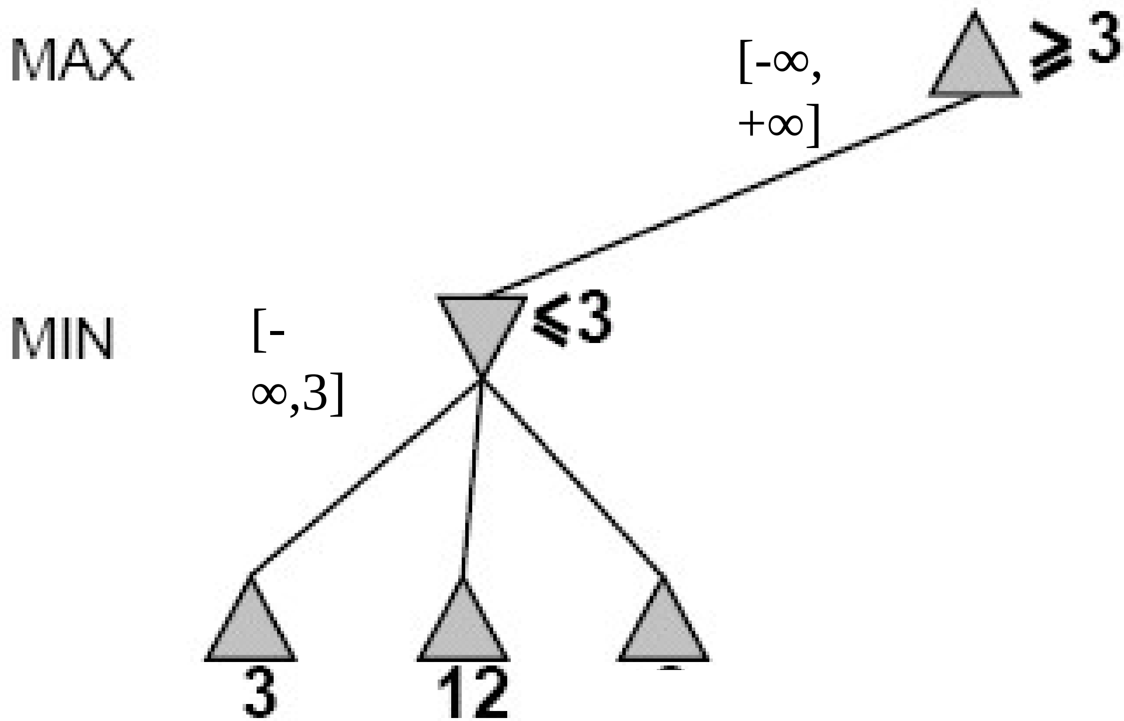
Do DF-search until first leaf



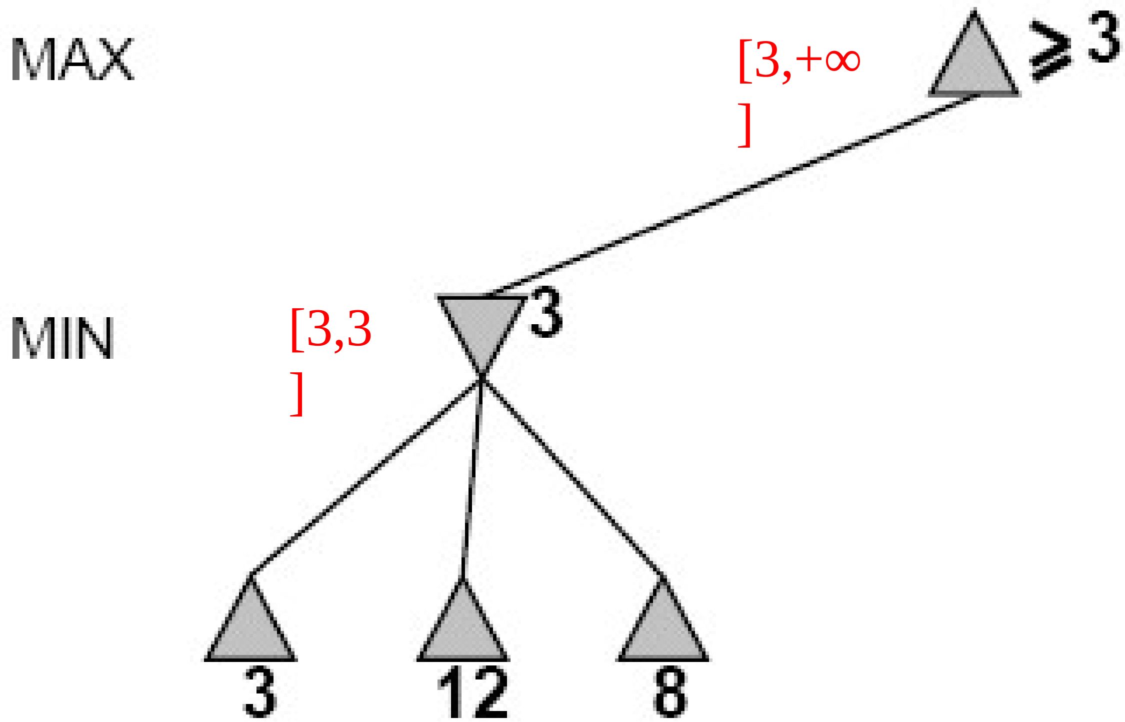
Alpha-Beta Example (continued)



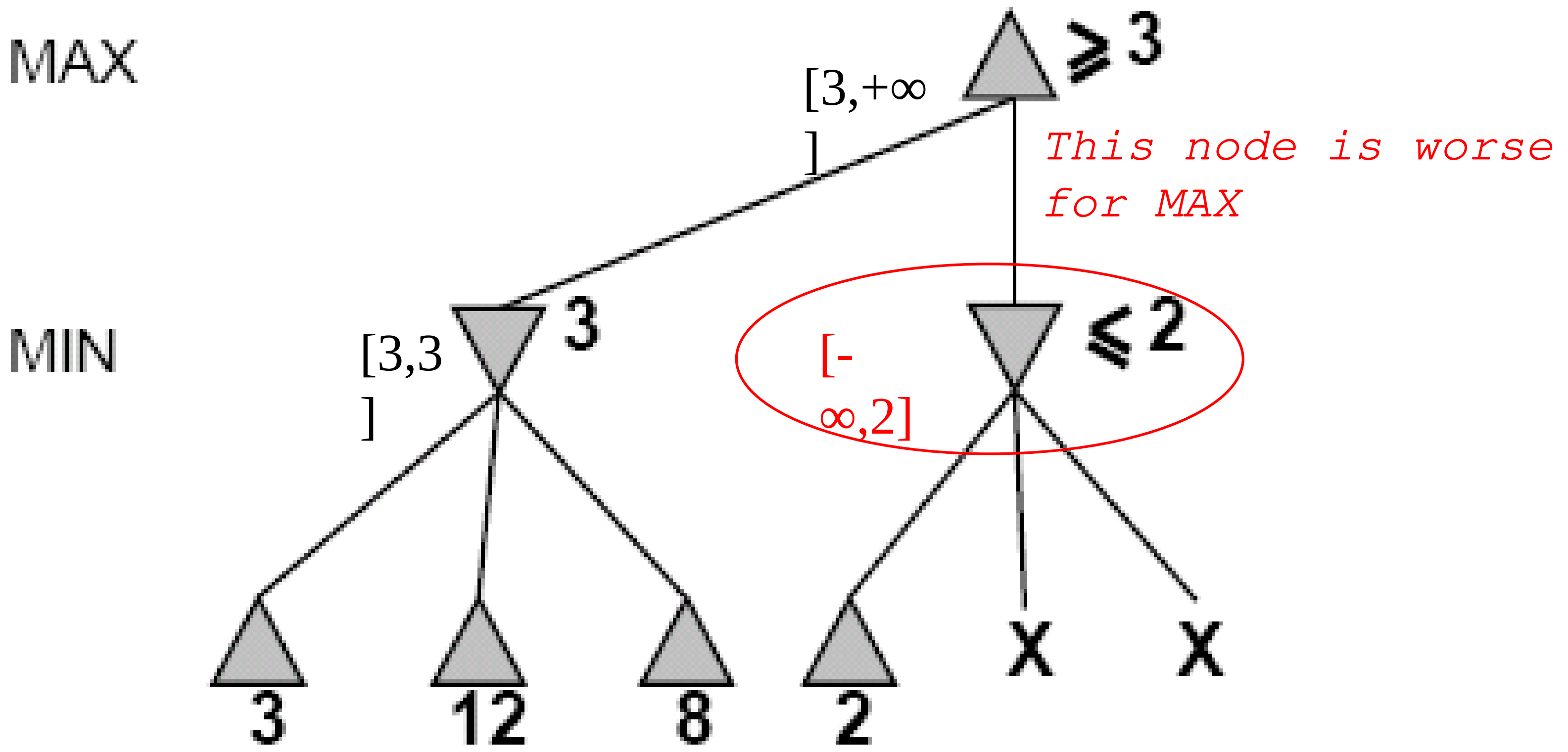
Alpha-Beta Example (continued)



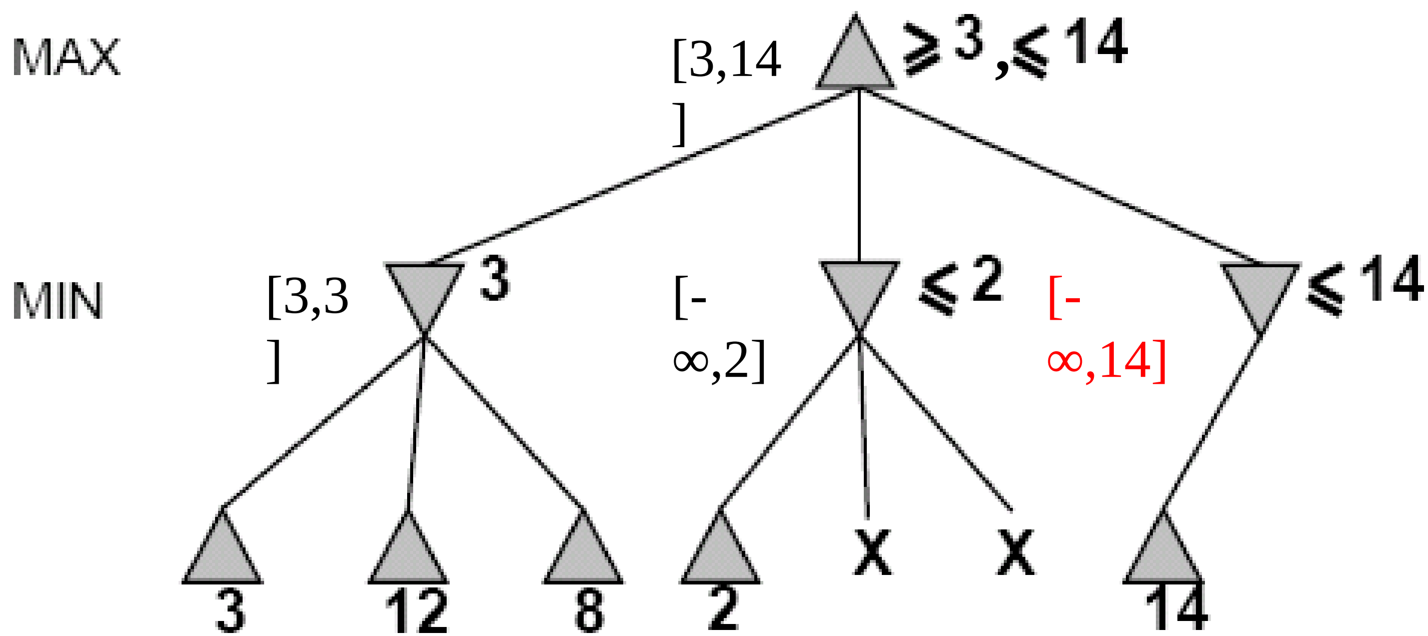
Alpha-Beta Example (continued)



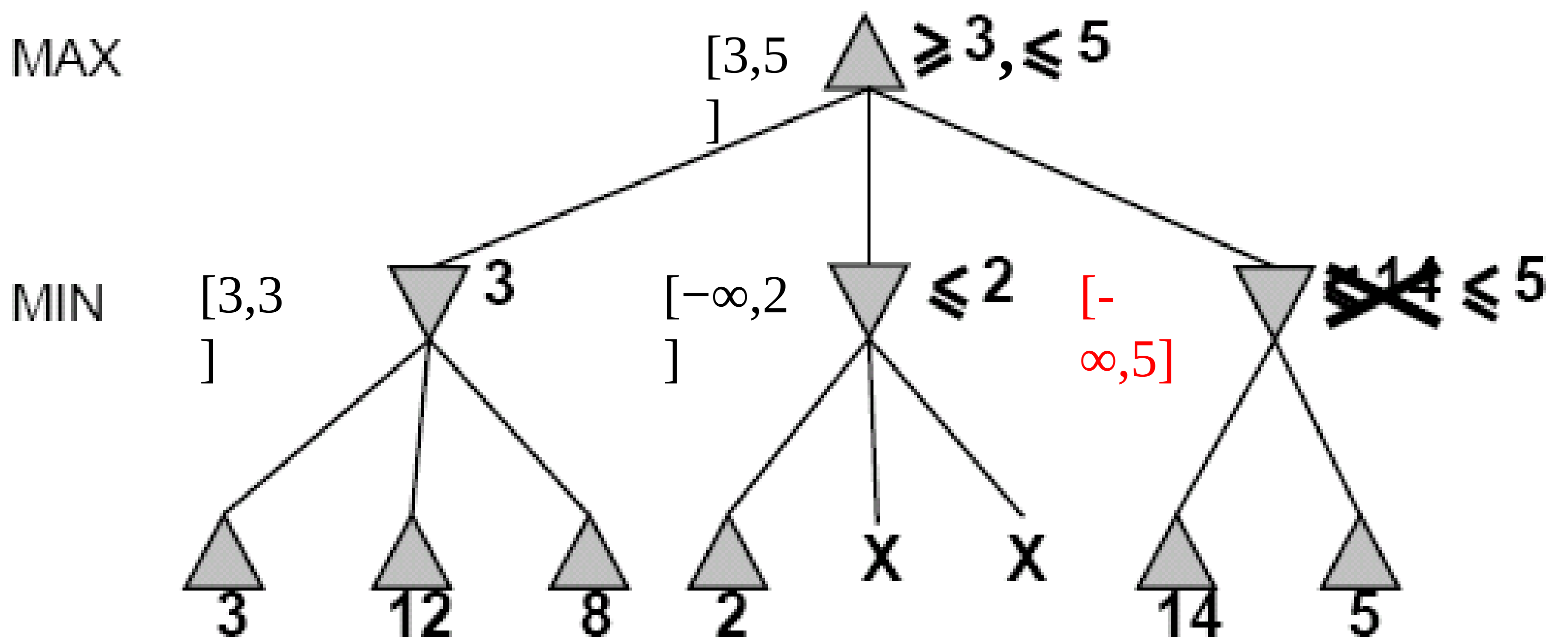
Alpha-Beta Example (continued)



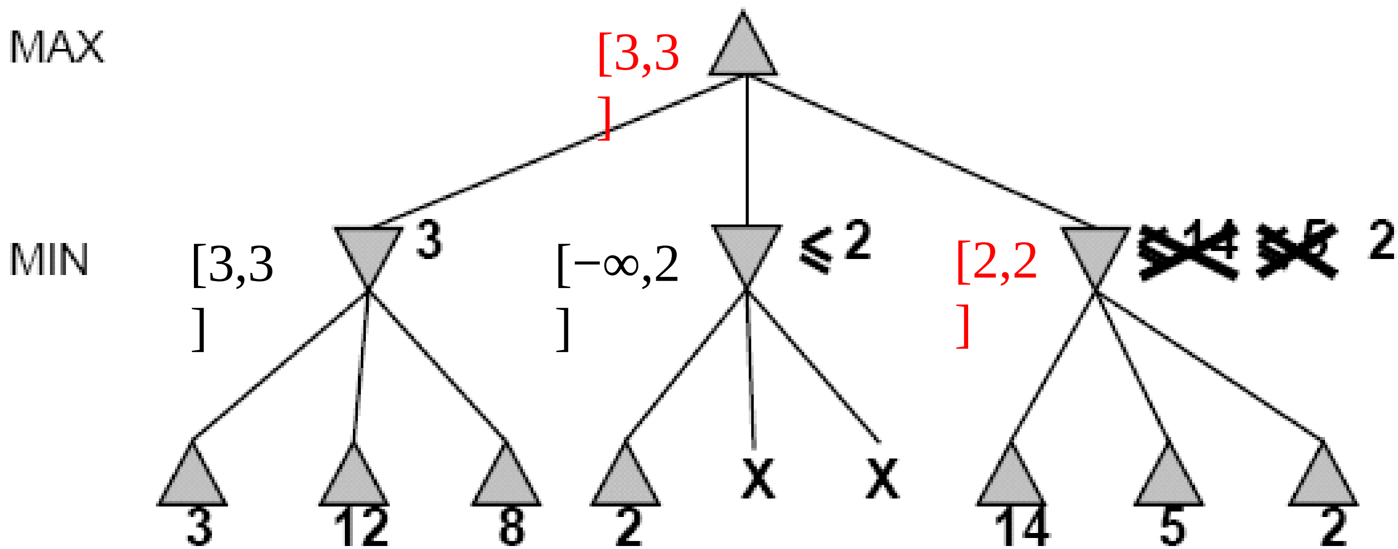
Alpha-Beta Example (continued)



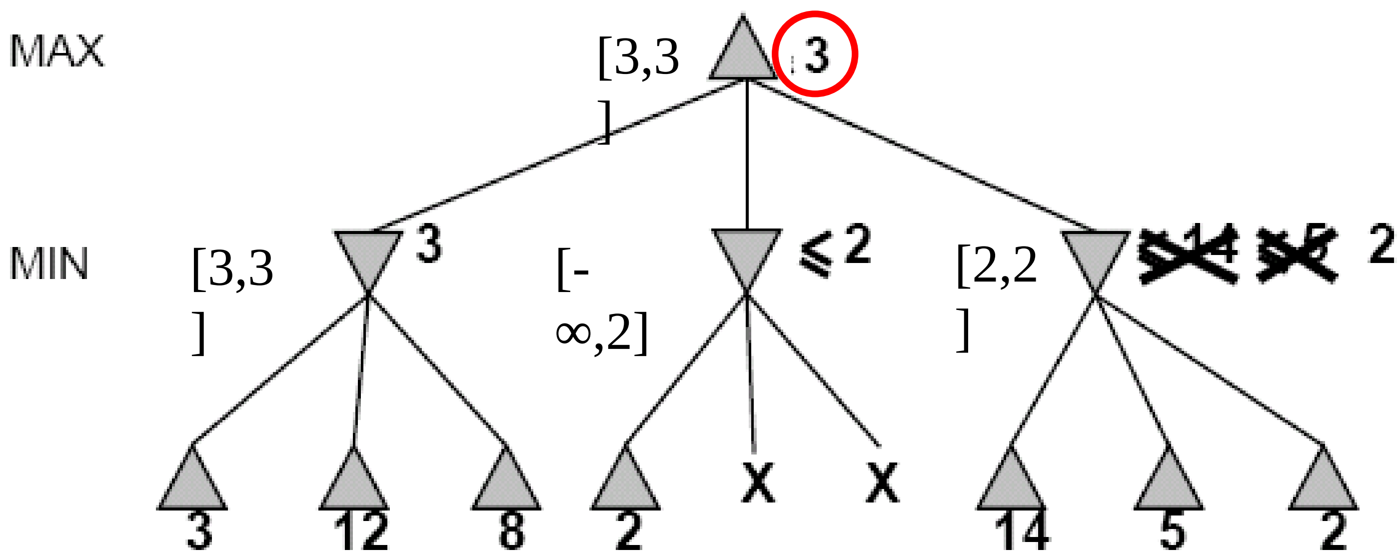
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



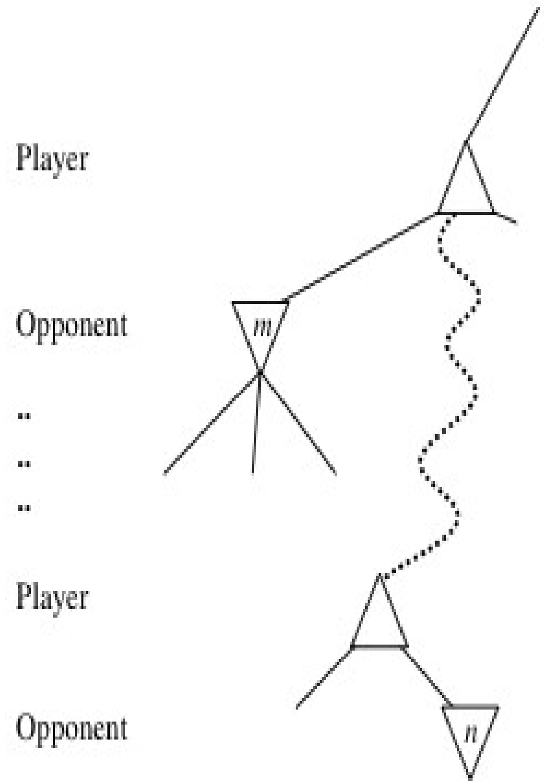
General alpha-beta pruning

- Consider a node *n* somewhere in the tree

- If player has a better choice at
  - Parent node of n
  - Or any choice point further up

- *n* will **never** be reached in actual play.

- Hence when enough is known about *n*, it can be pruned.



Alpha-beta Algorithm

- Depth first search – only considers nodes along a single path at any time

α = highest-value choice we have found at any choice point along the path for MAX

β = lowest-value choice we have found at any choice point along the path for MIN

- update values of α and β during search and prunes remaining branches as soon as the value is known to be worse than the current α or β value for MAX or MIN

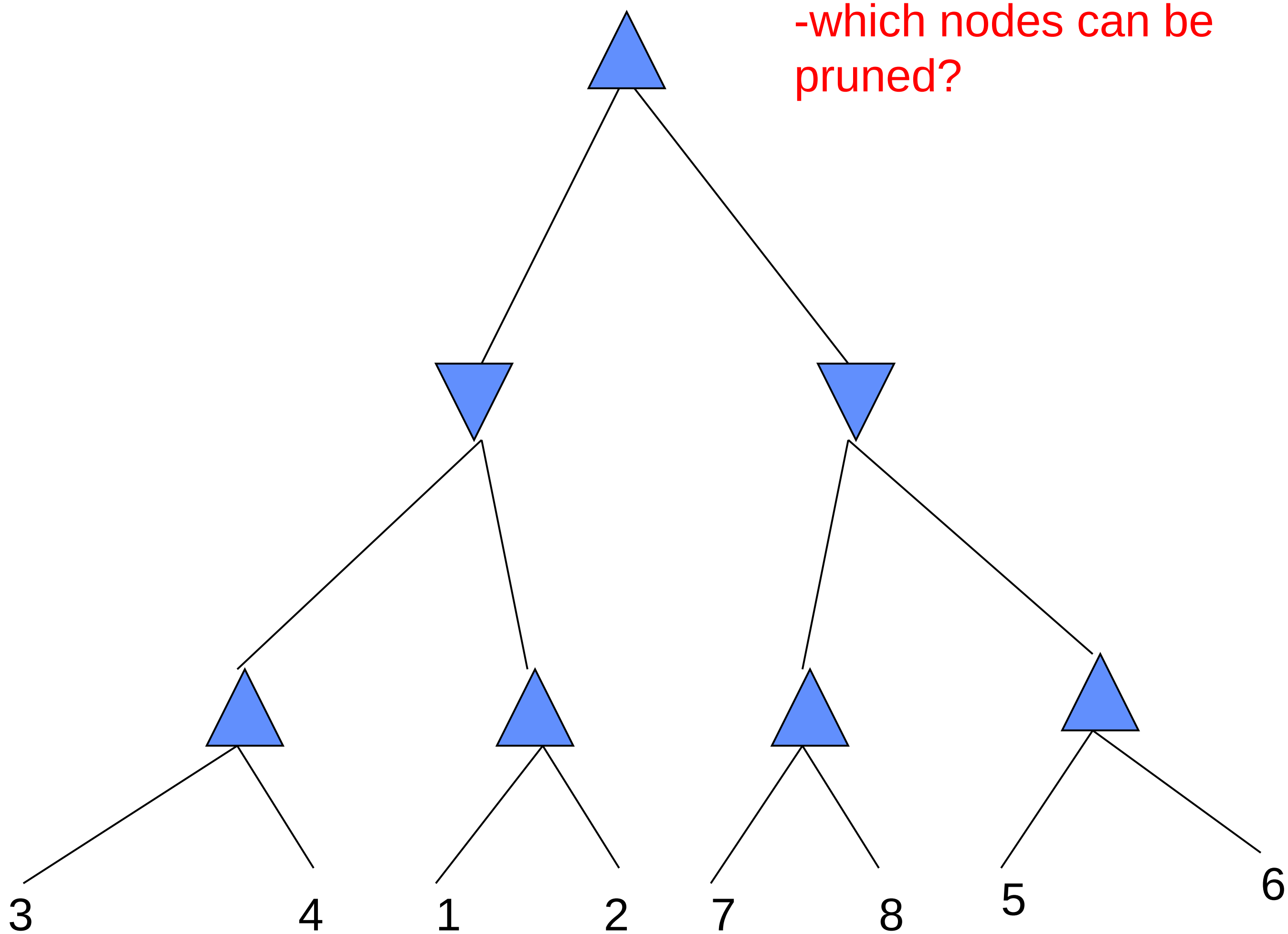
Effectiveness of Alpha-Beta Search

- Worst-Case
  - branches are ordered so that no pruning takes place. In this case alpha-beta gives no improvement over exhaustive search
- Best-Case
  - each player's best move is the left-most alternative (i.e., evaluated first)
  - in practice, performance is closer to best rather than worst-case
- In practice often get $O(b^{(d/2)})$ rather than $O(b^d)$
  - this is the same as having a branching factor of sqrt(b),
    - since $(\mathrm{sqrt}(b))^d = b^{(d/2)}$
    - i.e., we have effectively gone from b to square root of b
  - e.g., in chess go from b ~ 35 to b ~ 6
    - this permits much deeper search in the same amount of time

Final Comments about Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned.
- Good move *ordering* improves effectiveness of pruning
- Repeated states are again possible.
  - Store them in memory = transposition table

Example



-which nodes can be pruned?

Practical Implementation

How do we make these ideas practical in real game trees?

Standard approach:

- cutoff test: (where do we stop descending the tree)
  - depth limit
  - better: iterative deepening
  - cutoff only when no big changes are expected to occur next (quiescence search).

- evaluation function
  - When the search is cut off, we evaluate the current state by estimating its utility. This estimate if captured by the evaluation function.

Static (Heuristic) Evaluation Functions

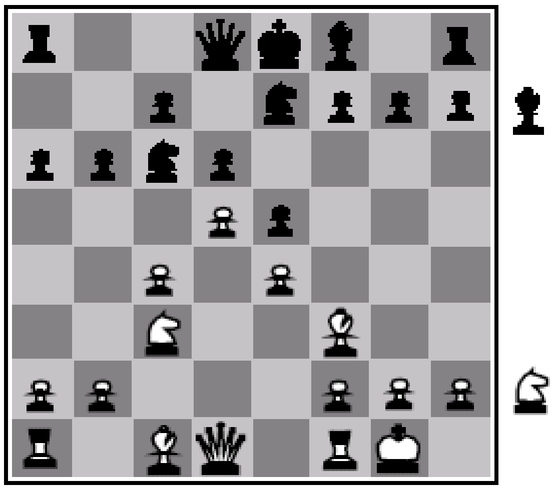
- An Evaluation Function:
  - estimates how good the current board configuration is for a player.
  - Typically, one figures how good it is for the player, and how good it is for the opponent, and subtracts the opponents score from the players
  - Othello: Number of white pieces - Number of black pieces
  - Chess: Value of all white pieces - Value of all black pieces

- Typical values from -infinity (loss) to +infinity (win) or [-1, +1].

- If the board evaluation is X for a player, it's -X for the opponent

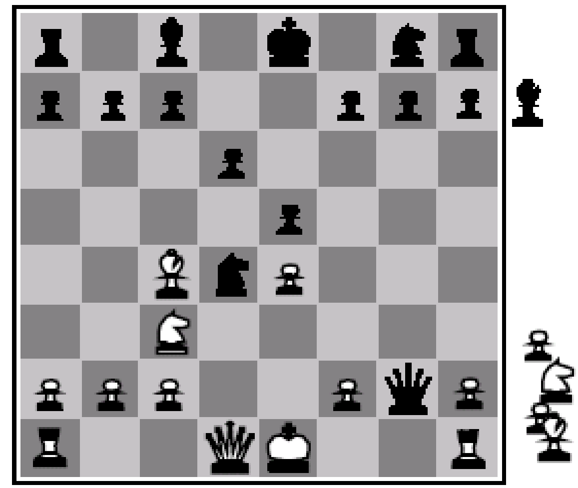
- Example:
  - Evaluating chess boards,
  - Checkers
  - Tic-tac-toe

Evaluation functions



Black to move

White slightly better



White to move

Black winning

For chess, typically *linear* weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

Iterative (Progressive) Deepening

- In real games, there is usually a time limit T on making a move

- How do we take this into account?
  - using alpha-beta we cannot use “partial” results with any confidence unless the full breadth of the tree has been searched
  - So, we could be conservative and set a conservative depth-limit which guarantees that we will find a move in time < T
    - disadvantage is that we may finish early, could do more search

- In practice, iterative deepening search (IDS) is used
  - IDS runs depth-first search with an increasing depth-limit
  - when the clock runs out we use the solution found at the previous depth limit

Heuristics and Game Tree Search

- The Horizon Effect
  - sometimes there's a major "effect" (such as a piece being captured) which is just "below" the depth to which the tree has been expanded
  - the computer cannot see that this major event could happen
  - it has a "limited horizon"
  - there are heuristics to try to follow certain branches more deeply to detect to such important events
  - this helps to avoid catastrophic losses due to "short-sightedness"

- Heuristics for Tree Exploration
  - it may be better to explore some branches more deeply in the allotted time
  - various heuristics exist to identify "promising" branches

The State of Play

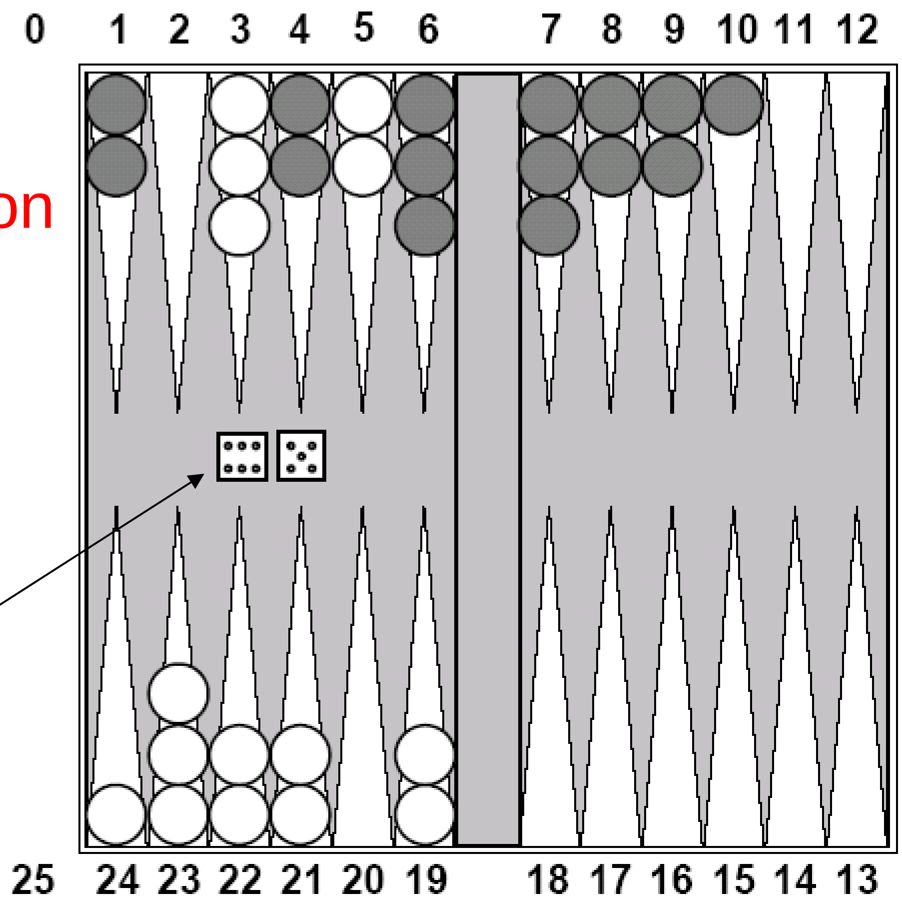
- Checkers:
  - Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- Chess:
  - Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997.
- Othello:
  - human champions refuse to compete against computers: they are too good.
- Go:
  - human champions refuse to compete against computers: they are too bad
  - $b > 300$ (!)
- See (e.g.) http://www.cs.ualberta.ca/~games/ for more information

Deep Blue

- 1957: Herbert Simon
  - “within 10 years a computer will beat the world chess champion”
- 1997: Deep Blue beats Kasparov
- Parallel machine with 30 processors for “software” and 480 VLSI processors for “hardware search”
- Searched 126 million nodes per second on average
  - Generated up to 30 billion positions per move
  - Reached depth 14 routinely
- Uses iterative-deepening alpha-beta search with transpositioning
  - Can explore beyond depth-limit for interesting moves

Chance Games.

Backgammon

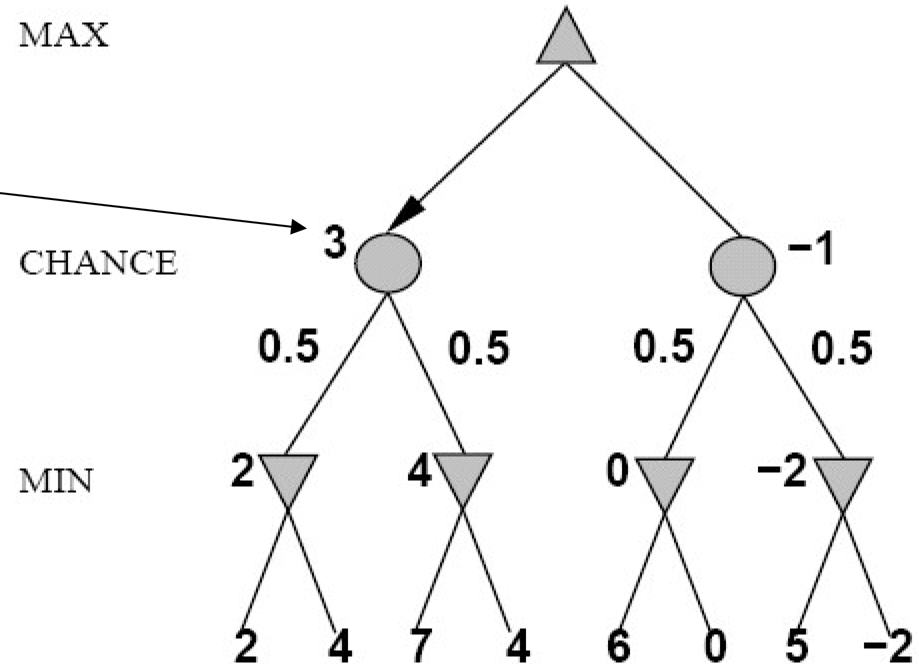


your element of chance

Expected Minimax

$$v = \sum_{\text{chance nodes}} P(n) \times Minimax(n)$$

$$3 = 0.5 \times 4 + 0.5 \times 2$$



Interleave chance nodes with min/max nodes

Again, the tree is constructed bottom-up

Summary

- Game playing can be effectively modeled as a search problem
- Game trees represent alternate computer/opponent moves
- Evaluation functions estimate the quality of a given board configuration for the Max player.
- Minimax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- For many well-known games, computer algorithms based on heuristic search match or outperform human world experts.