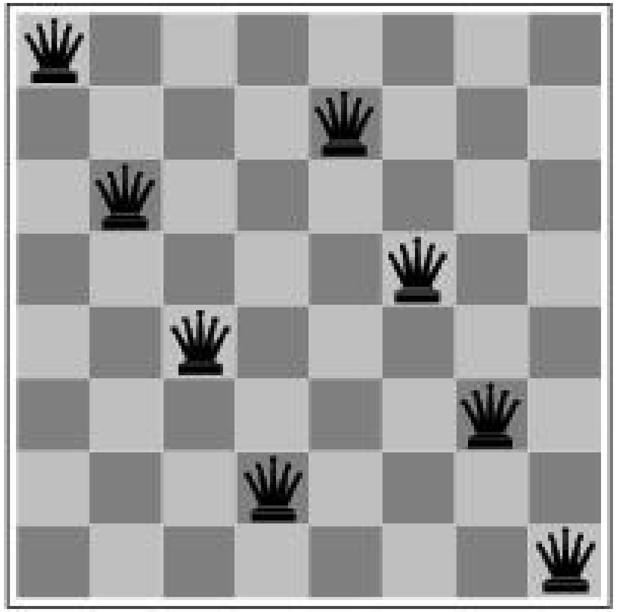
Chapter 4 (Section 4.3, …) 2nd Edition
or
Chapter 4 (3rd Edition)
Local Search and Optimization

Outline

- Local search techniques and optimization
  - Hill-climbing
  - Gradient methods
  - Simulated annealing
  - Genetic algorithms
  - Issues with local search

Local search and optimization

- Previously: systematic exploration of search space.
  - Path to goal is solution to problem
- YET, for some problems path is irrelevant.
  - E.g 8-queens
- Different algorithms can be used
  - Local search



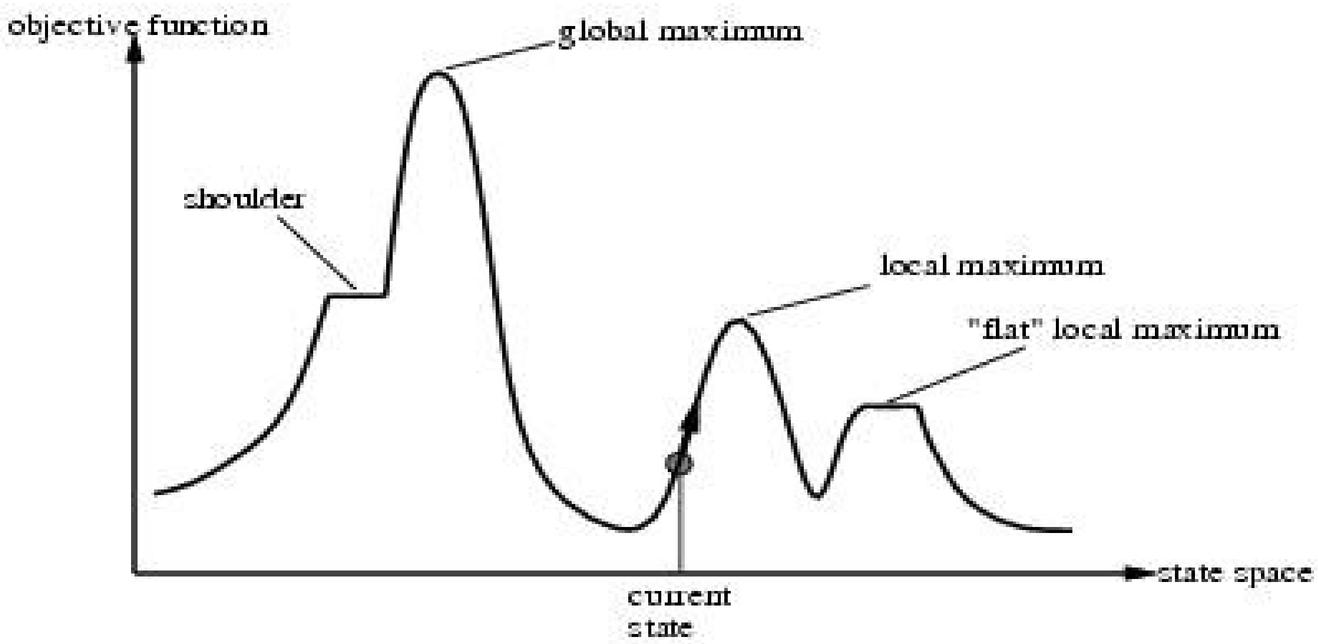
Local search and optimization

- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths

- Advantages:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.

- "Pure optimization" problems
  - All states have an objective function
  - Goal is to find state with max (or min) objective value
  - Does not quite fit into path-cost/goal-state formulation
  - Local search can do quite well on these problems.

“Landscape” of search



Hill-climbing search

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum
  input: *problem*, a problem
  local variables: *current*, **a node.**
    neighbor, **a node.**

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  loop do
    neighbor ← a highest valued successor of *current*
    if VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]
    current ← *neighbor*

Hill-climbing search

- “a loop that continuously moves in the direction of increasing value”
  - terminates when a peak is reached
  - Aka greedy local search

- Value can be either
  - Objective function value
  - Heuristic function value (minimized)

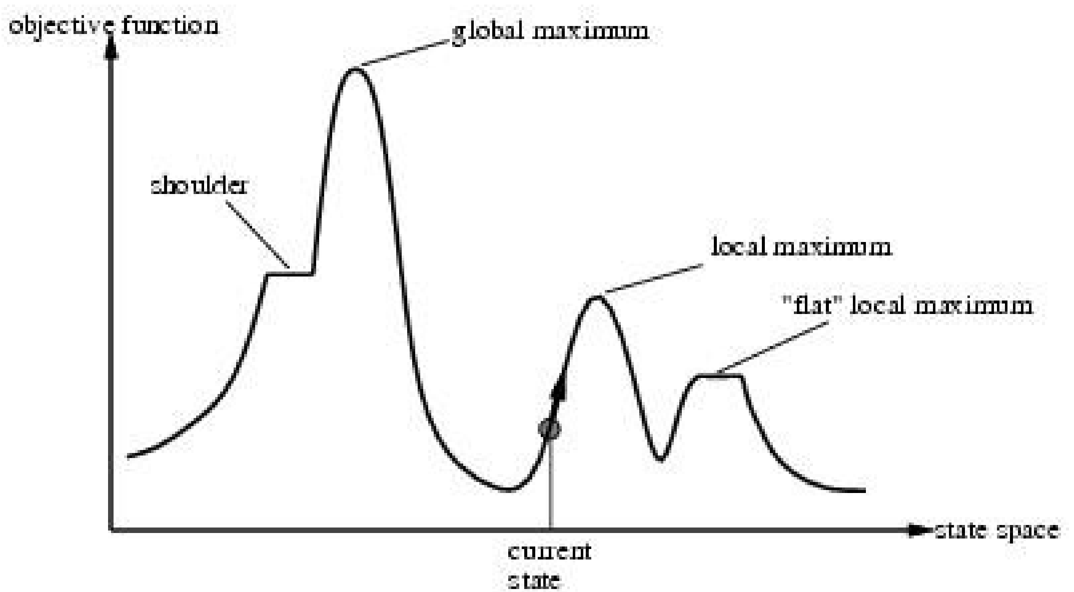
- Hill climbing does not look ahead of the immediate neighbors of the current state.

- Can randomly choose among the set of best successors, if multiple have the best value

- Characterized as “trying to find the top of Mount Everest while in a thick fog”

Hill climbing and local maxima

- When local maxima exist, hill climbing is suboptimal
- Simple (often effective) solution
  - Multiple random restarts



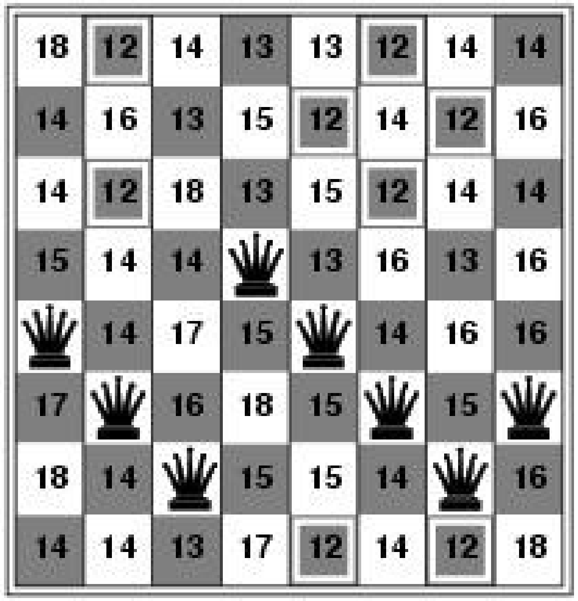
Hill-climbing example

- 8-queens problem, complete-state formulation
  - All 8 queens on the board in some configuration

- Successor function:
  - move a single queen to another square in the same column.

- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other (directly or indirectly)
  - (so we want to minimize this)

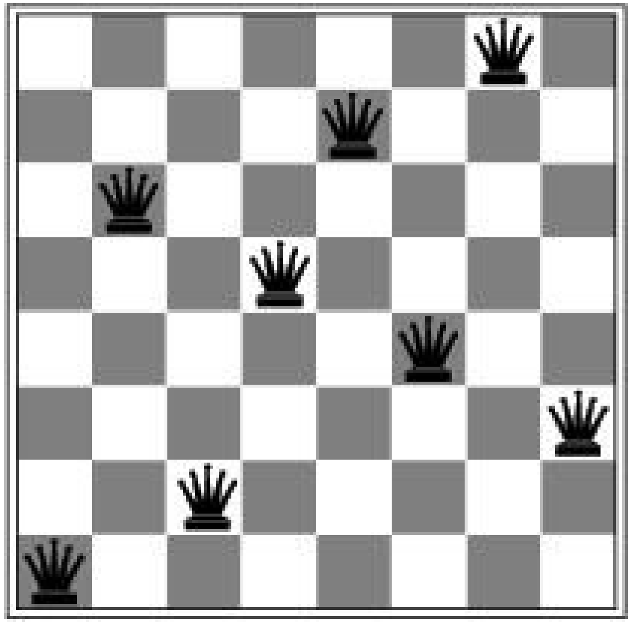
Hill-climbing example



Current state: h=17

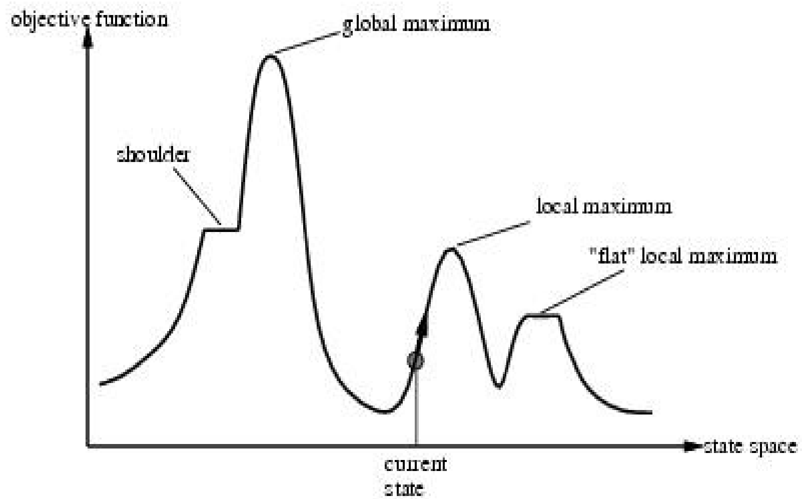
Shown is the h-value for each possible successor in each column

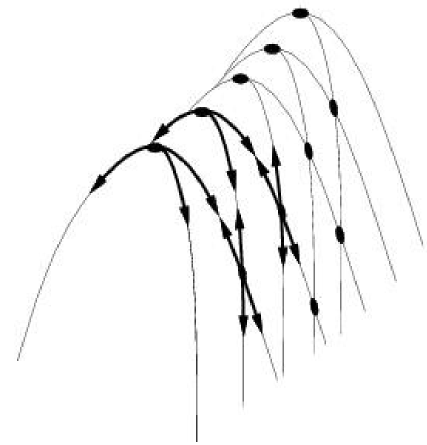
A local minimum for 8-queens



A local minimum in the 8-queens state space (h=1)

Other drawbacks





- Ridge = sequence of local maxima difficult for greedy algorithms to navigate

- Plateau = an area of the state space where the evaluation function is flat.

Performance of hill-climbing on 8-queens

- Randomly generated 8-queens starting states…

- 14% the time it solves the problem

- 86% of the time it get stuck at a local minimum

- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with ~17 million states)

Possible solution…sideways moves

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Need to place a limit on the possible number of sideways moves to avoid infinite loops

- For 8-queens
  - Now allow sideways moves with a limit of 100
  - Raises percentage of problem instances solved from 14 to 94%

  - However….
    - 21 steps for every successful solution
    - 64 for each failure

Hill-climbing variations

- Stochastic hill-climbing
  - Random selection among the uphill moves.
  - The selection probability can vary with the steepness of the uphill move.
- First-choice hill-climbing
  - stochastic hill climbing by generating successors randomly until a better one is found
  - Useful when there are a very large number of successors
- Random-restart hill-climbing
  - Tries to avoid getting stuck in local maxima.

Hill-climbing with random restarts

- Different variations
  - For each restart: run until termination v. run for a fixed time
  - Run a fixed number of restarts or run indefinitely

- Analysis
  - Say each search has probability p of success
    - E.g., for 8-queens, p = 0.14 with no sideways moves

  - Expected number of restarts?
  - Expected number of steps taken?

Expected number of restarts

- Probability of Success = p
- Number of restarts = 1 / p
- This means 1 successful iteration after (1/p – 1) failed iterations
- Let avg. number of steps in a failure iteration = f

  and avg. number of steps in a successful iteration = s

Therefore, expected number of steps in random-restart hill climbing = 1 * s + (1/p – 1) f

So for 8-queens, p = 14%, s = 4, f = 3,

Expected no of moves = 1 * 4 + (1/0.14 -1) * 3 = 22

With sideways moves, p = 94%, s = 21, f = 64

Expected no of moves = 1 * 21 + (1/0.94 -1) * 64 = 25

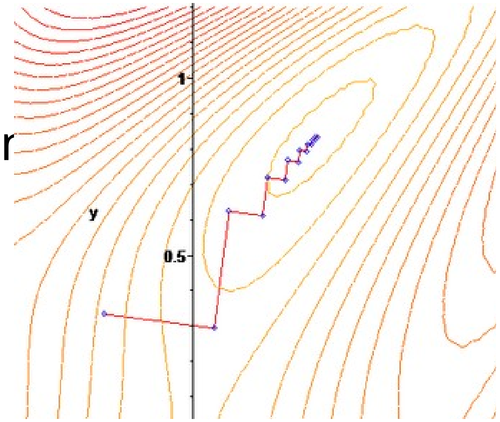
Local beam search

- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal → finished
  - Else select *k* best from successors and repeat.

- Major difference with random-restart search
  - Information is shared among *k* search threads.

- Can suffer from lack of diversity.
  - Stochastic beam search
    - choose k successors proportional to state quality.

Gradient Descent



Assume we have some cost-function: $C(x_1,\ldots,x_n)$
and we want minimize over continuous variables X1,X2,..,Xn

1. Compute the *gradient :* $\frac{\partial}{\partial x_i} C(x_1,\ldots,x_n) \quad \forall i$

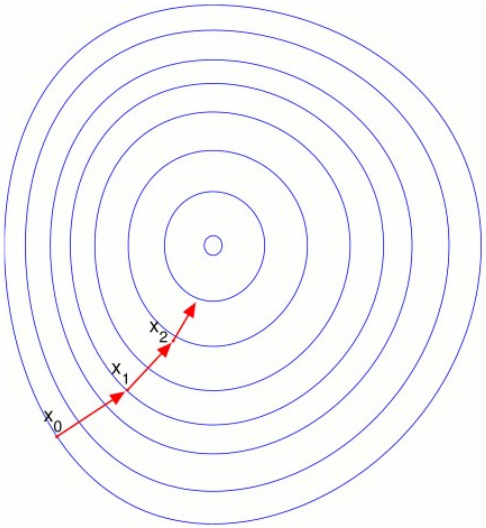
2. Take a small step downhill in the direction of the gradient:

$$x_i \to x'_i = x_i - \lambda \frac{\partial}{\partial x_i} C(x_1,\ldots,x_n) \quad \forall i$$

3. Check if $C(x_1,..,x'_i,..,x_n) < C(x_1,..,x_i,..,x_n)$

4. If true then accept move, if not reject.

5. Repeat.



Learning as optimization

- Many machine learning problems can be cast as optimization

- Example:
  - Training data D = $\{(\underline{x}_1,c_1),\ldots\ldots\ldots(\underline{x}_n, c_n)\}$

    where $\underline{x}_i$ = feature or attribute vector

    and c_i = class label (say binary-valued)

  - We have a model (a function or classifier) that maps from x to c

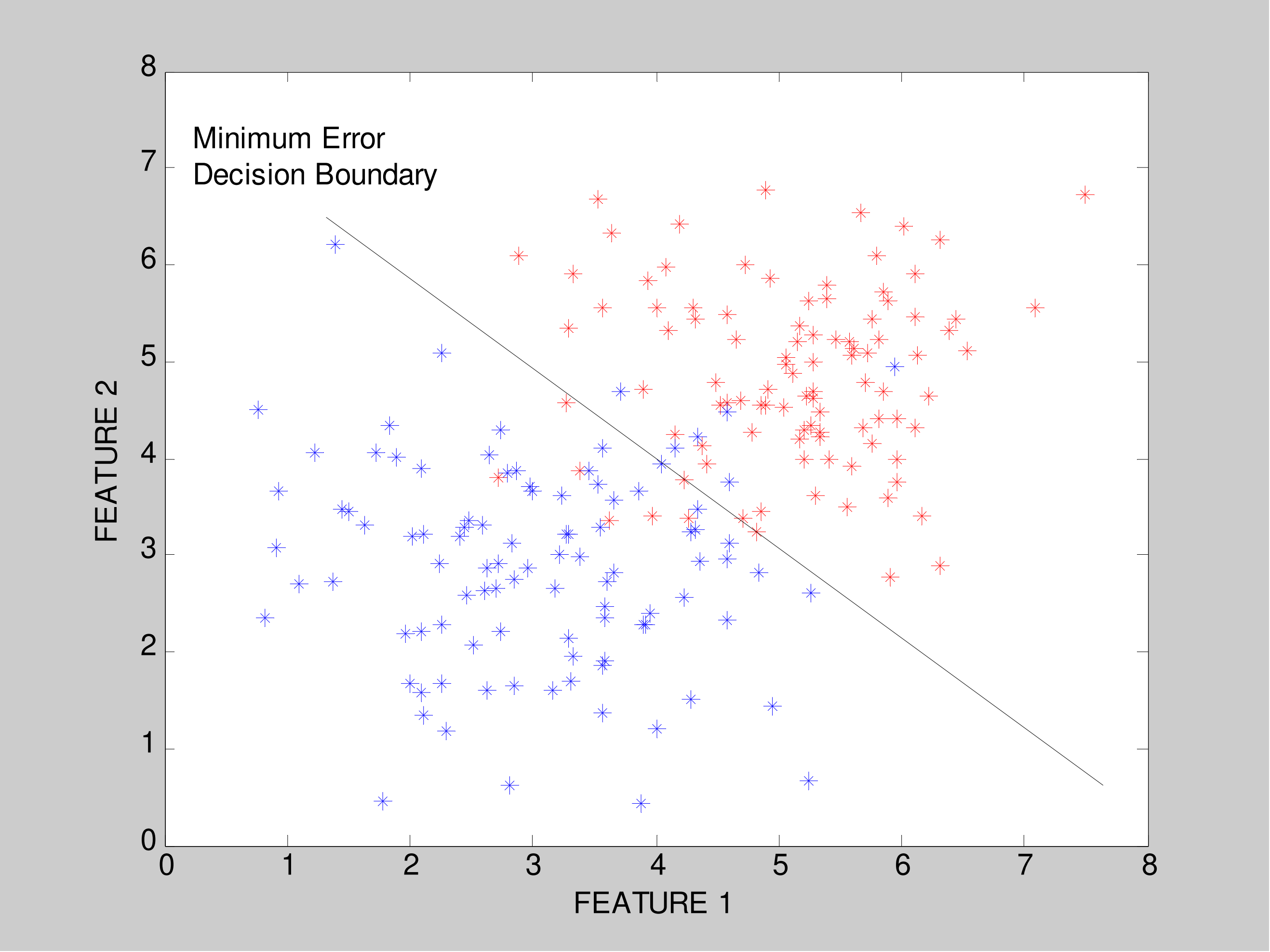
    e.g., $\text{sign}(\ \underline{w}.\ \underline{x}'\ ) = \{-1, +1\}$

  - We can measure the error $E(\underline{w})$ for any settig of the weights $\underline{w}$, and given a training data set D

  - Optimization problem: find the weight vector that minimizes $\underline{E(w)}$

(general idea is "empirical error minimization")

Learning a minimum error decision boundary



Search using Simulated Annealing

- Simulated Annealing = hill-climbing with non-deterministic search
- Basic ideas:
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the change in objective function is δ
  - if δ is positive, then move to that state
  - otherwise:
    - move to this state with probability proportional to δ
    - thus: worse moves (very large negative δ) are executed less often
  - however, there is always a chance of escaping from local maxima
  - over time, make it less likely to accept locally bad moves
  - (Can also make the size of the move random as well, i.e., allow “large” steps in state space)

Physical Interpretation of Simulated Annealing

- A Physical Analogy:
  - imagine letting a ball roll downhill on the function surface
    - this is like hill-climbing (for minimization)
  - now imagine shaking the surface, while the ball rolls, gradually reducing the amount of shaking
    - this is like simulated annealing

- Annealing = physical process of cooling a liquid or metal until particles achieve a certain frozen crystal state
  - simulated annealing:
    - free variables are like particles
    - seek “low energy” (high quality) configuration
    - get this by slowly reducing temperature T, which particles move around randomly

Simulated annealing

```
function SIMULATED-ANNEALING( problem, schedule) return a solution state
    input: problem, a problem
           schedule, a mapping from time to temperature
    local variables: current, a node.
                     next, a node.
                     T, a "temperature" controlling the probability of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] - VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

More Details on Simulated Annealing

- Lets say there are 3 moves available, with changes in the objective function of d1 = -0.1, d2 = 0.5, d3 = -5. (Let T = 1).
- pick a move randomly:
  - if d2 is picked, move there.
  - if d1 or d3 are picked, probability of move = exp(d/T)
  - move 1: prob1 = exp(-0.1) = 0.9,
    - i.e., 90% of the time we will accept this move
  - move 3: prob3 = exp(-5) = 0.05
    - i.e., 5% of the time we will accept this move

- T = “temperature” parameter
  - high T => probability of “locally bad” move is higher
  - low T   => probability of “locally bad” move is lower
  - typically, T is decreased as the algorithm runs longer
    - i.e., there is a “temperature schedule”

Simulated Annealing in Practice

- method proposed in 1983 by IBM researchers for solving VLSI layout problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).
    - theoretically will always find the global optimum (the best solution)

- useful for some problems, but can be very slow
        - slowness comes about because T must be decreased very gradually to retain optimality
    - In practice how do we decide the rate at which to decrease T? (this is a practical problem with this method)

Genetic algorithms

- Different approach to other search algorithms
  - A successor state is generated by combining two parent states
- A state is represented as a string over a finite alphabet (e.g. binary)
  - 8-queens
    - State = position of 8 queens each in a column
    => 8 x log(8) bits = 24 bits (for binary representation)
- Start with *k* randomly generated states (population)
- Evaluation function (fitness function).
  - Higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
- Produce the next generation of states by "simulated evolution"
  - Random selection
  - Crossover
  - Random mutation
  -

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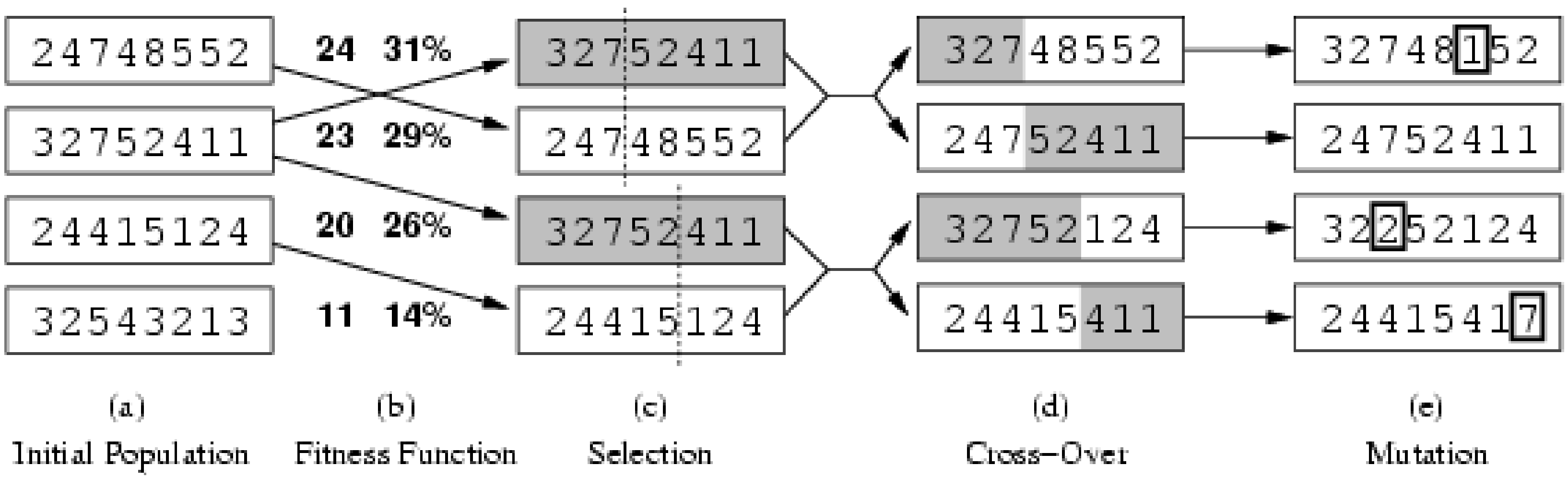
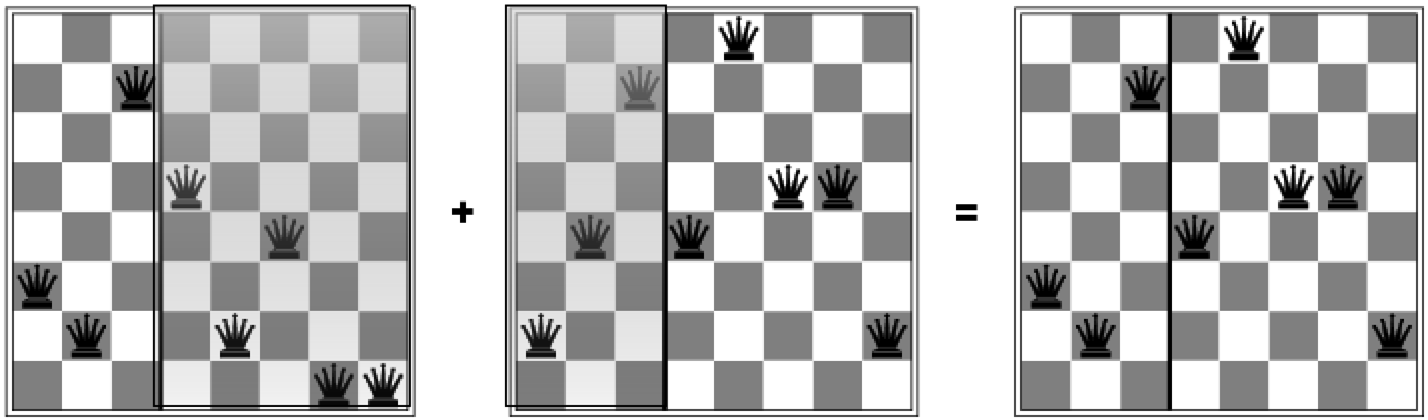
Genetic algorithms



- 4 states for 8-queens problem
- 2 pairs of 2 states randomly selected based on fitness. Random crossover points selected
- New states after crossover
- Random mutation applied

Genetic algorithms



Has the effect of "jumping" to a completely different new part of the search space (quite non-local)

Genetic algorithm pseudocode

function GENETIC_ALGORITHM(*population,* FITNESS-FN) **return** an individual
    input: *population*, a set of individuals
        FITNESS-FN, a function which determines the quality of the individual
    repeat
        new_population ← empty set
        loop for i **from** 1 **to** SIZE(*population*) **do**
            x ← RANDOM_SELECTION(*population*, FITNESS_FN)
    y ← RANDOM_SELECTION(*population*, FITNESS_FN)
            child ← REPRODUCE(*x,y*)
            if (small random probability) **then** *child* ← MUTATE(*child*)
            add *child* to *new_population*
        population ← *new_population*
    until some individual is fit enough or enough time has elapsed
    return the best individual

Comments on genetic algorithms

- Positive points
  - Random exploration can find solutions that local search can't
    - (via crossover primarily)
  - Appealing connection to human evolution
    - E.g., see related area of genetic programming

- Negative points
  - Large number of "tunable" parameters
    - Difficult to replicate performance from one problem to another

  - Lack of good empirical studies comparing to simpler methods

  - Useful on some (small?) set of problems but no convincing evidence that GAs are better than hill-climbing w/random restarts in general

Summary

- Local search techniques and optimization
  - Hill-climbing
  - Gradient methods
  - Simulated annealing
  - Genetic algorithms
  - Issues with local search