

# Vehicle scheduling

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# Introduction

**Definition (Vehicle scheduling (Schedule Blocking)).** Given a set of timetabled transit trips  $T$  and a set of transit vehicles  $V$  (possibly with varying capacities), find the assignment of trips to vehicle such as each trip in  $T$  is assigned to only one vehicle in  $V$  and spatial and temporal constraints of serving trips by any given vehicle is satisfied.

- In this step, we break down a schedule in form of **blocks**, which are the trips assigned to various vehicles to serve during a day.

## Definitions (TCRP 135)

**Definition (Block).** A vehicle (or train) assignment that includes the series of trips operated by each vehicle from the time it pulls out to the time it pulls in.

**Definition (Blocking).** The process in which trips are **hooked** together to form a vehicle assignment or block.

**Definition (Hooking).** The process of attaching the end of a trip in one direction to the beginning of a trip the other direction.

**Definition (Interlining).** The use of the same vehicle on a block operating on more than one route with the same operator, without returning to the garage during route changes.

**Definition (Through-routing).** A form of interlining in which a vehicle switches from inbound service on one route to outbound service on another route while continuing in service throughout the day.

## Simple blocking exercise

## Final output should look like this...

Block #	Pull Out	Eastbound					Westbound					Next Trip	Pull In
		A	B	C	D		D	C	B	A			
9703	05:55						06:15	06:26	06:40	06:48		07:00	
9701	05:50	06:00	06:08	06:22	06:33		06:45	06:56	07:10	07:18		07:30	
9702	06:20	06:30	06:38	06:52	07:03		07:15	07:26	07:40	07:48		08:00	
9703		07:00	07:08	07:22	07:33		07:45	07:56	08:10	08:18		08:30	
9701		07:30	07:38	07:52	08:03		08:15	08:26	08:40	08:48		09:00	
9702		08:00	08:08	08:22	08:33		08:45	08:56	09:10	09:18		09:30	
9703		08:30	08:38	08:52	09:03		09:15	09:26	09:40	09:48		10:00	
9701		09:00	09:08	09:22	09:33		09:45	09:56	10:10	10:18		10:30	
9702		09:30	09:38	09:52	10:03		10:15	10:26	10:40	10:48		11:00	
9703		10:00	10:08	10:22	10:33		10:45	10:56	11:10	11:18		11:30	
9701		10:30	10:38	10:52	11:03		11:15	11:26	11:40	11:48		12:00	
9702		11:00	11:08	11:22	11:33		11:45	11:56	12:10	12:18		12:30	
9703		11:30	11:38	11:52	12:03		12:15	12:26	12:40	12:48		13:00	
9701		12:00	12:08	12:22	12:33		12:45	12:56	13:10	13:18		13:30	
9702		12:30	12:38	12:52	13:03		13:15	13:26	13:40	13:48		14:00	
9703		13:00	13:08	13:22	13:33		13:45	13:56	14:10	14:18		14:30	
9701		13:30	13:38	13:52	14:03		14:15	14:26	14:40	14:48		15:00	
9702		14:00	14:08	14:22	14:33		14:45	14:56	15:10	15:18		15:30	
9703		14:30	14:38	14:52	15:03		15:15	15:26	15:40	15:48		16:00	
9701		15:00	15:08	15:22	15:33		15:45	15:56	16:10	16:18		16:30	
9702		15:30	15:38	15:52	16:03		16:15	16:26	16:40	16:48		17:00	
9703		16:00	16:08	16:22	16:33		16:45	16:56	17:10	17:18		17:30	
9701		16:30	16:38	16:52	17:03		17:15	17:26	17:40	17:48		18:00	
9702		17:00	17:08	17:22	17:33		17:45	17:56	18:10	18:18		18:30	
9703		17:30	17:38	17:52	18:03		18:15	18:26	18:40	18:48		19:00	
9701		18:00	18:08	18:22	18:33		18:45	18:56	19:10	19:18		19:30	
9702		18:30	18:38	18:52	19:03		19:15	19:26	19:40	19:48		19:58	
9703		19:00	19:08	19:22	19:33							19:53	

Figure: Each color represents a block<sup>1</sup>

<sup>1</sup>Source: TCRP135

## Optimization approaches

Depending on the number of depots, we can classify the vehicle scheduling into two types:

- ▶ Single-depot vehicle scheduling problem (SD-VSP)
- ▶ Multi-depot vehicle scheduling problem (MD-VSP)

## Single-depot vehicle scheduling

## Minimal decomposition by Jahar Saha (1970)

Let  $T$  be the set of trips and a relation between any two trips  $i \alpha j$  which tells that whether trip  $j$  can be served after  $i$  by the same vehicle, i.e.,  $i \alpha j$  if

- ▶  $j$  starts at the same station as  $i$
- ▶  $j$ 's dispatching time is later than  $i$ 's arrival time.

Define  $C_{ij} = \begin{cases} 1 & \text{if } i \alpha j \\ -\infty & \text{otherwise} \end{cases}$



## Minimal decomposition by Jahar Saha (1970)

Decisions:  $X_{ij} = 1$  if  $j$  is performed right after  $i$

$$\underset{\mathbf{X}}{\text{maximize}} \quad \sum_{i \in T} \sum_{j \in T} C_{ij} X_{ij} \quad (1a)$$

$$\text{subject to} \quad \sum_{j \in T} x_{ij} \leq 1, \forall i \in T \quad (1b)$$

$$\sum_{i \in T} x_{ij} \leq 1, \forall j \in T \quad (1c)$$

$$X_{ij} = \{0, 1\}, \forall i \in T, \forall j \in T \quad (1d)$$

### Remark.

- ▶ It can be solved as max flow problem using Ford and Fulkerson algorithm.
- ▶ Final values of  $x_{ij}$  are used to find the vehicle blocks.
- ▶ It gives minimum fleet for serving the given number of trips.
- ▶ The formulation has following disadvantages:
  - Cost of operation not considered

## Assignment formulation by Freling et al. (2001)

Let  $b_i$  and  $e_i$  be the starting and ending locations, and let  $bt_i$  and  $et_i$  be the starting and ending times of a trip  $i$  respectively.

**Definition (Compatible trips).** Two trips  $i$  and  $j$  are said to be compatible pair of trips if the same vehicle can cover these trips in sequence, i.e., if  $et_i + t(e_i, b_j) \leq bt_j$ , where  $t(e_j, b_i)$  is the deadheading travel time from  $e_j$  to location  $b_i$ .

Let  $T$  be the set of trips and  $E = \{(i, j) : i \text{ and } j \text{ are compatible}\}$  be the set of compatible trips.

Consider a graph  $G(M, E^T)$ , where  $M = T \cup \{|T| + 1, |T| + 2, \dots, 2|T|\}$  and  $E^T = E \cup \{(i, |T| + i) \mid i = 1, \dots, |T|\}$ .

**Remark.** A path of the form  $\{i_1, \dots, i_k, |T| + i_k\}$  is to a feasible vehicle schedule leaving the depot to perform trips  $i_1, \dots, i_k$  and returning to the depot afterwards.

Let  $c_{ij}$  be the cost of serving  $j$  right after  $i$  (deadhead time). Further  $c_{sj}$  and  $c_{it}$  be the deadhead cost of going from depot to start of trip  $j$  and deadhead cost of going from end location of trip  $i$  to depot respectively. 10

## Assignment formulation by Freling et al. (2001)

**Remark.** If a vehicle is serving  $j$  after  $i$ , then it will not have to go to  $j$  from depot. Based on that, let us define

$$b_{ij} = \begin{cases} c_{ij} - c_{sj}, & \forall (i, j) \in E \\ c_{it}, & \forall (i, j) \in E^T \setminus E \end{cases}$$

Decisions:  $y_{ij} = 1$  if  $j$  is performed right after  $i$

$$\underset{\mathbf{y}}{\text{minimize}} \quad \sum_{(i,j) \in E^T} b_{ij} y_{ij} + \sum_{j \in N} c_{sj} \quad (2a)$$

$$\text{subject to} \quad \sum_{j: (i,j) \in E^T} y_{ij} = 1, \forall i \in T \quad (2b)$$

$$\sum_{i: (i,j) \in E^T} y_{ij} \leq 1, \forall j \in M \quad (2c)$$

$$y_{ij} = \{0, 1\}, \forall (i, j) \in E^T \quad (2d)$$

## Formulation by L. Bodin (1983)

Consider a digraph  $G = (N, A)$  where  $N = T \cup \{s, t\}$ , where  $s$  and  $t$  represent the depot and  $A = \{(s, i) \mid i \in T\} \cup \{(j, t) \mid j \in T\} \cup \{(i, j) : i, j \in T \text{ and } e_i^t + d_{ij} \leq b_j^t\} \cup \{(t, s)\}$ . Here,  $d_{ij}$  is the deadhead time between  $e_i$  and  $b_j$ .

$$\text{Define } c_{ij} = \begin{cases} d_{ij}, & \forall (i, j) \in A : i, j \in T \\ d_{si}, & \forall (s, i) : i \in T \\ d_{jt}, & \forall (j, t) : j \in T \\ c_0 \end{cases},$$

where  $c_0$  is the cost of adding extra vehicle. Also, assume  $p$  is the maximum number of vehicles allowed.

Decisions:  $x_{ij}$  = flow on arc  $(i, j)$

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{maximize}} & \sum_{(i,j) \in A} c_{ij} x_{ij} \end{array} \quad (3a)$$

$$\begin{array}{ll} \text{subject to} & \sum_{i:(i,j) \in A} x_{ij} - \sum_{i:(j,i) \in A} x_{ji} = 0, \forall j \in N \end{array} \quad (3b)$$

$$\sum_{i:(i,j) \in A} x_{ij} = 1, \forall j \in T \quad (3c)$$

$$x_{ts} \leq p \quad (3d)$$

$$x_{ij} \in \mathbb{Z}_+, \forall (i, j) \in A \quad (3e)$$

## Multi-depot vehicle scheduling

## Formulation by L. Bodin (1983)

Let  $K$  be the set of depots. Each depot  $k$  has stationed  $p_k$  vehicles. We construct  $|K|$  directed graphs  $G_k = (N_k, A_k)$ , where  $N_k = T \cup \{s_k\} \cup \{t_k\}$  and  $A_k = \{(s_k, i) \mid i \in T\} \cup \{(j, t_k) \mid j \in T\} \cup \{(i, j) : i, j \in T \text{ and } e_i^t + d_{ij} \leq b_j^t\} \cup \{(t_k, s_k)\}$ . Costs are defined as before.

Decisions:  $x_{ij}^k$  = flow on arc  $(i, j) \in A_k$

$$\underset{\mathbf{x}}{\text{maximize}} \quad \sum_{k \in K} \sum_{(i,j) \in A_k} c_{ij}^k x_{ij}^k \quad (4a)$$

$$\text{subject to} \quad \sum_{i:(i,j) \in A_k} x_{ij}^k - \sum_{i:(j,i) \in A_k} x_{ji}^k = 0, \forall j \in N_k, \forall k \in K \quad (4b)$$

$$\sum_{k \in K} \sum_{i:(i,j) \in A} x_{ij}^k = 1, \forall j \in T \quad (4c)$$

$$\sum_{j:(s_k,j) \in A_k} x_{s_k j} \leq p_k, \forall k \in K \quad (4d)$$

$$x_{ij} \in \mathbb{Z}_+, \forall (i, j) \in A \quad (4e)$$

This is NP-Hard. MDVSP with time windows is skipped from this lecture. 15

## Suggested reading

- ▶ Gkiotsalitis, Konstantinos. Public transport optimization, Chapter 11.
- ▶ TCRP Report 135
- ▶ Saha, J. L. "An algorithm for bus scheduling problems." Journal of the Operational Research Society 21.4 (1970): 463-474.
- ▶ Freling, Richard, Albert PM Wagelmans, and José M. Pinto Paixão. "Models and algorithms for single-depot vehicle scheduling." Transportation Science 35.2 (2001): 165-180.
- ▶ Bodin, Lawrence. "Routing and scheduling of vehicles and crews." Computer & Operations Research 10.2 (1983): 69-211.



Thank you!