Homework 3

COMS W3261, Summer B 2021

This homework is due **Monday, 7/19/2021, at 11:59PM EST**. Submit to GradeScope (course code: X3JEX4).

Grading policy reminder: LaTeX is preferred, but neatly typed or handwritten solutions are acceptable. Your TAs may dock points for indecipherable writing. Proofs should be complete; that is, include enough information that a reader can clearly tell that the argument is rigorous.

The tool http://madebyevan.com/fsm/ may be useful for drawing finite state machines.

If a question is ambiguous, please state your assumptions. This way, we can give you credit for correct work. (Even better, post on Ed so that we can resolve the ambiguity.)

1 Problem 1 (12 points)

1. (6 points.) Prove that the language

$$A = \{w \mid \text{ For all } y \in \{0,1\}^*, w \neq yy\}$$

over the alphabet $\Sigma = \{0,1\}$ is nonregular. You may use the pumping lemma and/or closure properties.

2. (6 points.) Prove that the language

$$B = \{1^n 0^m 1^n \mid n \geq 0, m \geq 1\}$$

over the alphabet $\Sigma = \{0,1\}$ is nonregular. You may use the pumping lemma and/or closure properties.
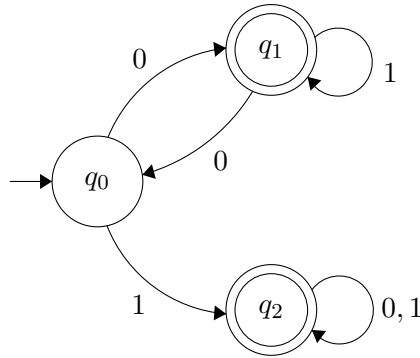
2 Problem 2 (8 points)

1. (8 points.) Is the language

$$C = \{a^i b^j c^k \mid i, j, k \geq 0; i \leq j\} \cup \{a^i b^j c^k \mid i, j, k \geq 0; j \leq k\} \cup \{a^i b^j c^k \mid i, j, k \geq 0; k \leq i\}$$

over the alphabet $\Sigma = \{a, b, c\}$ a regular language? Prove your answer.

3 Problem 3 (10 points)

1. (4 points). Convert the DFA below into a GNFA state diagram using the procedure outlined in class. (This procedure is also outlined in the textbook on page 71.)

2. (6 points). Use the procedure $CONVERT(G)$ outlined in class (and on page 73 of the textbook) to compute the values of the transitions $\delta'(q_{start}, q_2)$, $\delta'(q_{start}, q_{accept})$, and $\delta'(q_2, q_{accept})$ after removing state q_1 from the GNFA below. Hint: Recall that $\emptyset^*$ evaluates to the language $\{\varepsilon\}$.
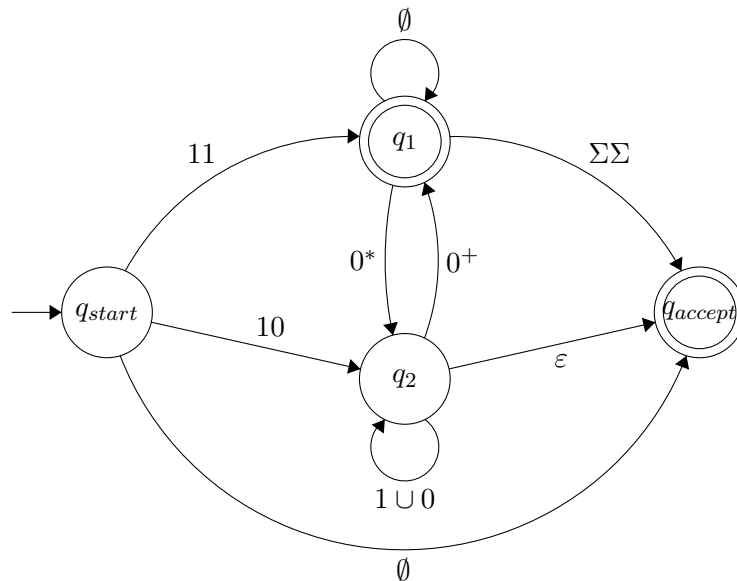
4 Problem 4 (12 points)

1. (3 points). What is the language of the grammar G_1 below? Here S, A, and B are the variables and 0 and 1 are the terminals. Explain your reasoning.

$$S \to 1A1$$
$$A \to S \mid B$$
$$B \to 0B \mid \varepsilon$$

2. (3 points). What is the language of the grammar G_2 below? Here A and B are the variables and x, y, and z are the terminals. Explain your reasoning.

$$A \to xAx \mid yAy \mid zAz \mid B$$
$$B \to x \mid y \mid z \mid \varepsilon$$

3. (3 points). Design a grammar for the language

$$D = \{a^i b^j c a^j b^i \mid i, j \geq 1\}$$

and explain why your grammar produces D.

4. (3 points). Design a grammar for the language

$$L = I(saw \cup met \cup loved)(the \cup a)(very)^*(large \cup tiny \cup red)(frog \cup dog)$$

and explain why your grammar produces L. (You can treat each word as a single terminal symbol.)