

# **Timetable development**

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# Introduction

**Definition (Timetabling (Schedule building)).** Given a set of transit lines  $L$  and their associated frequencies  $f : L \mapsto \mathbb{R}$ , find the exact dispatching time of each trip of each line and their arrival and departure time at every stop they visit so as to optimize a given objective.

- ▶ Timetable should be responsive to demand patterns
  - An agency's assumption about passengers adjusting to timetable instead of planners adjusting the timetable to passenger demand could become a major source of unreliable service.
- ▶ Common objectives of timetabling are:
  - Even distribution of passenger loads across all trips of any transit line
  - Even headway
  - Maximizing the number of transfer synchronization events
  - Minimizing the passenger wait time
- ▶ Irregular time headway of consecutive trips is justified in case of
  - varying demand patterns
  - transfer time synchronization

## Examples

பேருந்துநிலைய கால அட்டவணை																			
வெளிநிலையம் Perundurai					வெளிநிலையம் Chennai					வெளிநிலையம் Chennai					வெளிநிலையம் Chennai				
12.5-4.40	12.0-9.35	PTS-200	PTS-635	59.100	ATC-12.45	C5-10.30	SRL-2.40	C5A-8.20	7-1.30	P4-7.20	LNT-12.00	B2-11.10	B12-10.20	B12-10.40	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-4.55	SSM-9.50	SIS-2.40	51-6.42	20.2.10	ATC-12.45	C5-10.30	C5-2.55	C4-8.45	7-1.40	7-6.20	P5-12.20	8E-12.00	B12-10.40	B12-10.40	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
31-5.10	12-10.05	12.0-2.20	PTS-650	39-2.30	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TSP-5.20	P7/12-10.15	31-2.55	12.0-7.05	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
PTS-5.40	SIS-10.25	PTS-2.40	12.0-7.15	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.6-5.50	12.0-10.30	12.0-2.45	TSP-7.25	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12-6.00	31-10.55	12.0-2.50	PTS-7.55	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.25	PTS-10.40	12.0-5.02	12.0-7.40	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.35	PTS-10.50	12.0-5.15	SSM-7.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
PTS-6.40	12.0-11.00	TSP-3.22	12.0-8.00	20.7.45	C5A-6.30	C-6.30	PSB-9.26	சென்னை-15	7.05-6.35	7.05-6.35	19A-3.50	LNT-4.25	PSB-5.25	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.55	12.0-11.15	12.0-5.30	12-8.35	39A-9.10	C5A-7.00	C5SS-7.00	C5SS-4.35	7A-6.40	7-4.30	P4-4.20	P5-4.50	17-6.55	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-7.15	12.0-11.25	P7/12-5.35	PTS-8.40	39-9.10	C5A-7.10	C5SS-7.10	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-7.22	TSP-11.30	SSM-5.30	12.0-9.32	39-9.10	C5A-7.10	C5SS-7.10	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TSP-7.30	12.0-11.45	12.0-5.55	PTS-9.35	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
31-7.40	SSM-11.50	12.0-6.15	12-10.00	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
SSM-7.45	P7/12-11.55	PTS-4.15	12-10.00	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
2-8.05	12-12.05	SIS-4.25	39-9.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.20	31-12.15	PTS-4.55	39-9.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.30	SIS-12.25	12A-3.00	VRS-6.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.40	12.0-12.35	12.0-5.10	39A-6.50	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.45	PTS-12.40	12.0-5.15	39-7.30	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.00	12.0-13.00	12.0-5.30	39-9.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.10	12.0-13.15	SSM-5.50	39-9.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.15	12.0-13.30	12.0-5.50	39-9.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.25	P7/12-14.00	12-6.15	20.8.10	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.30	SSM-14.55	SSM-6.25	39-12.00	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.35	12-1.50	12-6.30	39-12.00	39-9.10	ATC-7.30	C5A-12.20	C5A-4.45	7.05-7.20	7-4.45	K1-5.50	P5-5.40	B2-7.10	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20

Figure: Perundurai to Chennimalai bus timings <sup>1</sup>

<sup>1</sup><http://kkthecommonman.blogspot.com/2017/09/perundurai-to-chennimalai-bus-timings.html>

# Examples

## ► Southbound To Farragut Square

### Monday thru Friday — De Lunes a viernes





Route Number	Chevy Chase Circle NW	Connecticut & Nebraska Aves. NW	Connecticut Ave. & Veazey Terr. NW (Van Ness-UDC)	Connecticut Ave. & Porter St. NW (Cleveland Park)	Calvert St. & Connecticut Ave. NW (Woodley Park)	Connecticut Ave. & Leroy Pl. NW (Florida Ave.)	17th St. (E) & I St NW (FARRAGUT N&W)
							
<b>AM Service — Servicio matutino</b>							
<b>L2</b>	5:05	5:09	5:12	5:14	5:19	5:24	5:29
<b>L2</b>	5:35	5:39	5:42	5:44	5:49	5:54	5:59
<b>L2</b>	5:55	6:00	6:04	6:07	6:13	6:19	6:25
<b>L2</b>	6:10	6:15	6:19	6:22	6:28	6:34	6:40
<b>L2</b>	6:24	6:29	6:33	6:36	6:42	6:48	6:54
<b>L2</b>	6:36	6:41	6:45	6:48	6:54	7:00	7:06
<b>L2</b>	6:48	6:53	6:57	7:00	7:06	7:12	7:18
<b>L2</b>	7:00	7:05	7:09	7:12	7:18	7:24	7:30
<b>L2</b>	7:10	7:16	7:21	7:24	7:31	7:39	7:47
<b>L2</b>	7:20	7:26	7:31	7:34	7:41	7:49	7:57
<b>L2</b>	7:30	7:36	7:41	7:44	7:51	7:59	8:07
<b>L2</b>	7:40	7:47	7:53	7:57	8:04	8:12	8:21
<b>L2</b>	7:50	7:57	8:03	8:07	8:14	8:22	8:31
<b>L2</b>	8:00	8:07	8:13	8:17	8:24	8:32	8:41
<b>L2</b>	8:10	8:17	8:23	8:27	8:34	8:42	8:51
<b>L2</b>	8:20	8:27	8:33	8:37	8:44	8:52	9:01

Figure: L2 Connecticut Avenue Line in Washington D.C.<sup>2</sup>

<sup>2</sup>[https://www.wmata.com/schedules/timetables/upload/L2\\_231217.pdf](https://www.wmata.com/schedules/timetables/upload/L2_231217.pdf)

## Definitions (TCRP 135)

**Definition (Layover time).** The time between the scheduled arrival and departure of a vehicle at a transit terminal<sup>3</sup>.

- ▶ also referred to as "recovery time"
- ▶ typically used as rest time for transit vehicle operator between trips.
- ▶ other purpose is to ensure an on-time departure for the next trip.

**Definition (Span of service).** The length of time, from the beginning of the first trip to the end of the last trip, during which service operates on the street.

- ▶ can be expressed for a route or for the system as a whole.

**Definition (Cycle time or round trip cycle time ).** Sum of the round-trip running time plus layover time.

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<sup>3</sup>One endpoint of a route where trips usually begin or end

## Definitions (TCRP 135)

**Definition (Service pattern).** The unique sequence of stops associated with each type of trip on a route.

- ▶ if all trips operate from one end to the other on a common path the route has one service pattern.
- ▶ branches, deviations, or short turns introduce additional service patterns.

**Definition (Deadhead).** The time and distance that a bus needs to travel in places where it will not pick up passengers.

- ▶ typically required to get buses to and from their garage,
- ▶ or need to travel from one route or point to another during their scheduled work day.

**Definition (Pull-out and Pull-in time).** The time the vehicle spends traveling from the garage to the route and vice-versa are called pull-out and pull-in time respectively .

- ▶ pull-out and pull-in times are included in vehicle hours, but not in revenue hours.
- ▶ both are components of deadhead miles.

## Definitions (TCRP 135)

**Definition (Block).** A vehicle (or train) assignment that includes the series of trips operated by each vehicle from the time it pulls out to the time it pulls in. A complete block includes a pull-out trip from the garage followed by one or (usually) more revenue trips and concluding with a pull-in trip back to the garage.

Building a simple schedule (In-class exercise 11)



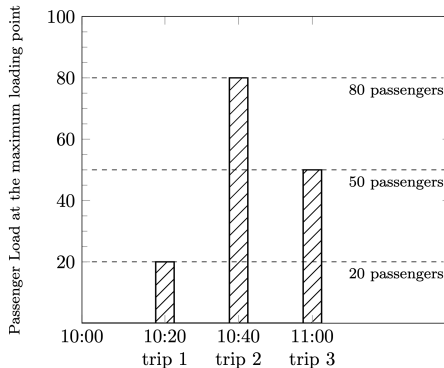
## Steps

- Step 1.** Calculate round trip cycle time including the layover time.  
In our case,  $8 + 14 + 11 + 11 + 14 + 8 = 66$  minutes.
- Step 2.** Figure out the layover time.
- Possibly from the union contract.
  - Past practice may guide the layover time (e.g., 10% of running time or six minutes per round trip, whichever is greater)
  - In our case, 10% of 66  $\approx 7$  minutes.
  - Therefore, overall round trip cycle time =  $66 + 7 = 73$  minutes.
  - # of buses required =  $\frac{73 \times 2}{60} = 2.43 \approx 3$  (as we need to provide frequency of 2 buses/hr)
  - 3 buses will provide extra layover time, i.e.,  $\frac{3 \times 60}{2} - 66 = 24$  minutes.
- Step 3.** Prepare the basic schedule pattern.
- Step 4.** Decide when to start the service.
- Step 5.** Populate the schedule.
- Step 6.** Fill in the intermediate times.

## Even load method

This method tries to even the load of all trips of the line at maximum loading stop.

Consider a transit service of 3 buses/hr from 10:00 AM - 11:00 AM with average passenger load at the maximum loading point for each one its trip shown below:



**Figure:** Passenger load at max loading point<sup>4</sup>

<sup>4</sup>Figure taken from Gkiotsalitis, PT optimization

## Even load method

Assume uniform arrival rate, then the passenger arrival during three 20-minute intervals are:

- ▶ 10:00-10:20 AM we have  $20/20 = 1$  passenger/min
- ▶ 10:20-10:40 AM we have  $80/20 = 4$  pax/min
- ▶ 10:40-11:00 AM we have  $50/20 = 2.5$  pax/min

We attempt to even the load and would like to have  $\frac{20+80+50}{3} = 50$  pax for each trip.

With uniform arrival time, we want to shift the first trip from 10:20 AM to 10:20 +  $x$  AM so that we get extra 30 pax to have a total of 50 pax on this trip.

i.e.,  $4 \times x = 30 \implies x = \frac{30}{4} = 7.5$ . This means first trip should depart at 10 : 27 : 30.

Repeat this procedure for other trips.

## Even load method

One can solve the above problem graphically. Create a cumulative passenger load plot. Draw a horizon line from the desired passenger load point and find the point where it intersects with the load curve. Use that point to find the time on the x-axis.

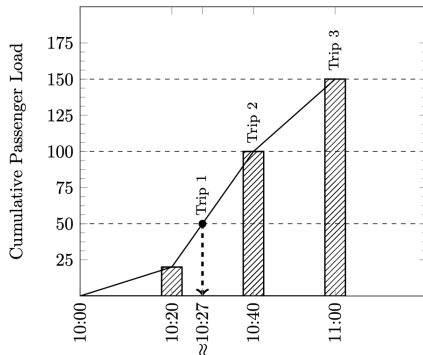


Figure: Even load method<sup>5</sup>

<sup>5</sup>Figure taken from Gkiotsalitis, PT optimization

# Optimization model by Wang et al. (2017)

## Sets

- ▶  $S$ : Set of ordered stops of a transit route
- ▶  $P$ : Set of passengers waiting to board the route
- ▶  $K$ : Set of ordered trips served by the route during the day
- ▶  $Z$ : Set of peak/off-peak time intervals with homogeneous travel times

## Parameters

- ▶  $D_f$ : Departure time of the first bus
- ▶  $D_l$ : Departure time of the last bus
- ▶  $c_{sk}$ : Avg. travel time from stop  $s$  to  $s + 1$  during  $k^{\text{th}}$  time interval
- ▶  $B_p$ : Stop at which the  $p^{\text{th}}$  passenger is waiting
- ▶  $A_p$ : Arrival time of  $p^{\text{th}}$  passenger

## Decision variables

- ▶  $x_k$ : Dispatching time of the  $k^{\text{th}}$  trip
- ▶  $V_{sk}$ : Arrival time of the  $k^{\text{th}}$  trip at stop  $k$
- ▶  $\tau_p$ : Vehicle trip onto which the  $p^{\text{th}}$  passenger boards
- ▶  $w_p$ : Waiting time of the  $p^{\text{th}}$  passenger

## Optimization model by Wang et al. (2017)

$$\underset{\mathbf{x}, \mathbf{V}, \tau, \mathbf{w}}{\text{minimize}} \quad \sum_{p \in P} w_p \quad (1a)$$

$$\text{subject to} \quad x_1 = D_f \quad (1b)$$

$$x_{|K|} = D_l \quad (1c)$$

$$x_k \leq x_{k+1}, \forall k \in K \setminus \{|K|\} \quad (1d)$$

$$V_{1k} = x_k, \forall k \in K \quad (1e)$$

$$V_{sk} = V_{s-1,k} + c_{sz}(: V_{s-1,k} \in z), \forall s \in S \setminus \{1\}, \forall k \in K \quad (1f)$$

$$\tau_p = \min_k \{V_{B_p k} : V_{B_p k} > A_p\}, \forall p \in P \quad (1g)$$

$$w_p = V_{B_p, \tau_p} - A_p, \forall p \in P \quad (1h)$$

$$x_k \in \mathbb{Z}, \forall k \in K \quad (1i)$$

$$\tau_p \in K, \forall p \in P \quad (1j)$$

$$w_p \in \mathbb{R}, \forall p \in P \quad (1k)$$

$$V_{sk} \in \mathbb{R}, \forall s \in S, \forall k \in K \quad (1l)$$

## Suggested reading

- ▶ Ceder. Public Transit Planning and Operations, Chapters 4 and 5.
- ▶ Gkiotsalitis, Konstantinos. Public transport optimization, Chapter 9.
- ▶ TCRP Report 135
- ▶ Y. Wang, D. Zhang, L. Hu, Y. Yang and L. H. Lee, "A data-driven and optimal bus scheduling model with time-dependent traffic and demand", IEEE Trans. Intell. Transp. Syst., vol. 18, no. 9, pp. 2443-2452, Sep. 2017.

Thank you!