Transit demand estimation

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Ridership

Usually measured using unlinked passenger trips or pass-km

Significance

- give us an estimate of current and future transit needs
- important input for any service design
- help us select the best alternative among several alternatives at planning stage
- help us assess the effect of changes to the service, infrastructure, fares, etc.

Factors affecting transit ridership

Internal

- Fare
- Travel time (walking, waiting, transferring, in-vehicle)
- Service frequency
- Service coverage
- Stop location
- Route structure
- Transfers
- Comfort and convenience
- Information
- Crowding and reliability

External

- Socio-economic factors (e.g., age, gender, income, auto ownership)
- Fuel prices
- Employment opportunities
- Land-use
- Safety
- Security
- Competition from other modes

Remark. Factors affecting passengers QoS also affect transit ridership. Better QoS help increase ridership.

Forecasting techniques

- Expert judgment
- Rules of thumbs
- Surveys (e.g., stated preference)
- Elasticity
- Regression model
- Time series econometric model
  - Moving averages
  - Exponential smoothing
  - Double exponential smoothing (Holt's method)
- Trip distribution
- Discrete choice models
- Four step travel demand model

Remark. can vary based on scale (spatial, temporal) and market segments

But ...

- Take forecasts with a pinch of salt
- Forecasts are usually wrong
- Aggregated forecasts are more accurate
- A good forecast is more than a single number
- Longer the forecast horizon, the less accurate the forecast will be

Remark. George Box said "All models are wrong but some are useful"

Demand function and elasticity

Theoretically, demand D can be expressed as a function of various attributes (explanatory variables) $x_1, \cdots, x_m$, i.e.,

$$D = f(x_1, \cdots, x_m) \tag{1}$$

Definition (Elasticity). Percentage change in the demand wrt 1 % change in any attribute.
If $D = f(x_i)$, then

$$\epsilon_{D,x_i} = \frac{\frac{\Delta D}{D_0}}{\frac{\Delta x_i}{x_{i0}}} \tag{2}$$

Example(s). Simpson-Curtin rule: 3% fare increase reduces ridership by 1%
If $\Delta x_i \to 0$, then $\epsilon_{D,x_i} = \frac{\partial D}{\partial x_i} \times \frac{x_{i0}}{D_0}$

Elasticity

- $\epsilon_{D,x_i} < 0$ means demand curve is downward slopping, i.e., increase in x leads to decrease in the demand
- $\epsilon_{D,x_i} > 0$ means demand curve is upward slopping, i.e., increase in x leads to increase in the demand
- $\epsilon_{D,x_i} = 0$ means perfectly inelastic demand. This happens when there is no substitute for the current service.
- $|\epsilon_{D,x_i}| > 1$ means demand is elastic
- $|\epsilon_{D,x_i}| < 1$ means demand is inelastic
- Fare induces an inelastic demand.

Remark. In a competitive environment, a change in the attribute of one service may affect the demand of another service. Such changes are captured using cross elasticity.

Regression modeling

1. State the problem
2. Model specification
   - An equation linking response and explanatory variables
   - Probability distribution of response variables
3. Parameter estimation
4. Check model adequacy
5. Inference

Travel Demand Forecasting

We divide the geographical region into Transportation Analysis Zones (TAZs).

1. **Trip Generation** : Whether/when to travel? Estimates the number of trips from/to each zone.
2. **Trip Distribution** : Where to travel (which destination)? Estimates the other end of trips (OD trip matrix).
3. **Mode Choice** : How to travel (which mode)? Estimates the share of each mode from OD trips.
4. **Traffic Assignment** : How to travel (which route). Estimates traffic flow in transportation network.

Trip distribution (OD estimation) methods

- No. of trips going from zone to another
- Expressed in the form of origin-destination passenger flow matrix
- Techniques
  - Growth factor method
  - Gravity method
  - Optimization
    - Entropy maximization
    - Maximum likelihood
    - Generalized least squares
  - Bayesian inference
  - Clustering
  - Trip chaining

Growth factor method

Three types

1. Uniform
2. Singly-constrained
3. Doubly-constrained

Uniform

$$d^{rs}_{\text{next year}} = \gamma d^{rs}_{\text{this year}}, \forall (r,s) \in R \times S \tag{3}$$

where, γ is growth factor.

Issues

- Need to know the base demand which is not available for a new service
- All O-D pairs multiplied by the same growth factor. However, some areas can be developed more than others.

Singly-constrained

- Origin-specific growth rate $d^{rs}_{\text{next year}} = \gamma_r d^{rs}_{\text{this year}}, r \in R$
- Destination-specific growth rate $d^{rs}_{\text{next year}} = \gamma_s d^{rs}_{\text{this year}}, s \in S$
- but not both

Doubly-constrained

$$d^{rs}_{\text{next year}} = 0.5 \times (\gamma_r + \gamma_s) d^{rs}_{\text{this year}}, \forall (r, s) \in R \times S$$

If $O_r \neq \sum_{s \in S} d^{rs}, \forall r \in R$ and $D_s \neq \sum_{r \in R} d^{rs}, \forall s \in S$, then we balance.

Gravity model

- a widely-used, successful, aggregate model
- interaction between two locations:
  - increases with the amount of activity at each location
  - declines with increasing distance, time, and cost of travel between them
- general formula:

$$d^{rs} = \gamma_r \gamma_s O_r D_s f(c_{rs}) \tag{4}$$

- e.g., when the impedance is travel cost:

$$d^{rs} = \gamma_r \gamma_s \frac{O_r D_s}{c_{rs}} \tag{5}$$

  If If $O_r \neq \sum_{s \in S} d^{rs}, \forall r \in R$ and $D_s \neq \sum_{r \in R} d^{rs}, \forall s \in S$, then we balance.

Issues:

- Trip distribution and travel impedance are interdependent. Results of trip distribution should be used to update travel impedance.
- Does not take into account behavioral consideration. More sophisticated destination choice models that take into account user behavior in decision making should be used.

Iterative proportional fitting (IPF)

1. Obtain the trips originated O_r (row sums) and destined D_s (column sums)
2. Obtain a seed matrix $\{\hat{d}^{rs}\}_{(r,s)\in R\times S}$
3. Repeat the following steps:
   - $\hat{d}^{rs}_{k+1} = \frac{O_r}{\sum_{s\in S} d^{rs}} \hat{d}^{rs}_k$, where k is the iteration number.
   - $\hat{d}^{rs}_{k+2} = \frac{D_s}{\sum_{r\in R} d^{rs}} \hat{d}^{rs}_{k+1}$
4. Repeat until $\frac{O_r}{\sum_{s\in S} d^{rs}}$ and $\frac{D_s}{\sum_{r\in R} d^{rs}} \approx 1$.

Issues:

- Non-structural zeros problem due to which a zero entry remains zero in every iteration.
- Quality of seed matrix should be good.

Entropy maximization

Notations

- Z: set of zones
- R: set of origins
- S: set of destinations
- d^{rs}: passenger trips from r to s

From trip generation we know,

1. Trip generation
$$O_r = \sum_{s \in S} d^{rs}, \forall r \in R \tag{6}$$
2. Trip attraction
$$D_s = \sum_{r \in R} d^{rs}, \forall s \in S \tag{7}$$

We want to fill the following matrix

$$
\begin{array}{c|c|c|c|c|c|c|}
 & s_1 & s_2 & . & . & . & s_n \\
\hline
r_1 & d^{r_1 s_1} & & & & & \\
\hline
r_2 & & . & & & & \\
\hline
. & & & . & & & \\
\hline
. & & & & . & & \\
\hline
r_n & & & & & & d^{r_n s_n} \\
\hline
\end{array}
$$

There are $k = |R||S|$ entries in the OD matrix and total demand is $Z = \sum_{r \in R} \sum_{s \in S} d^{rs}$. Assuming that it is equally likely to travel on one of the k entries of the matrix. The probability that $\{d^{rs}\}_{r \in R, s \in S}$ travelers will be traveling on individual O-D pairs is given by the multinomial probability distribution

$$
\frac{Z!}{d^{r_1 s_1}!\ d^{r_2 s_2}!\ \cdots d^{k}!} \left(\frac{1}{k}\right)^{d^{r_1 s_1}} \left(\frac{1}{k}\right)^{d^{r_1 s_2}} \cdots \left(\frac{1}{k}\right)^{d^{k}}
$$

$$
= \frac{Z!}{d^{r_1 s_1}! d^{r_1 s_2}! \ldots d^{k}!} \left(\frac{1}{k}\right)^{Z}
$$

To maximize this, we take the logarithm

$$= \log Z! - \sum_{(r,s)\in R\times S} \log d^{rs}! - Z \log k$$

$$= Z \log Z - Z - \sum_{(r,s)\in R\times S} \left(d^{rs} \log d^{rs} - d^{rs}\right) - Z \log k^{1}$$

$$= \sum_{(r,s)\in R\times S} d^{rs} \log \left(\sum_{(r,s)\in R\times S} d^{rs}\right) - \sum_{(r,s)\in R\times S} d^{rs} \log d^{rs} - \left(\sum_{(r,s)\in R\times S} d^{rs}\right) \log k$$

$$= - \sum_{(r,s)\in R\times S} \frac{d^{rs}}{\sum_{(r,s)\in R\times S} d^{rs}} \left(\log \frac{d^{rs}}{\sum_{(r,s)\in R\times S} d^{rs}}\right) - \log k$$

$$= - \sum_{(r,s)\in R\times S} p^{rs} \log p^{rs} - \log k$$

where, p^{rs} is the probability of traveling between $(r, s) \in R \times S$

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[1] Stirling's approximation $\log x! \approx x \log x - x$

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We get the following optimization problem

$$
\begin{aligned}
\underset{\mathbf{p}}{\text{maximize}} \quad & -\sum_{(r,s)\in R\times S} \left(p^{rs}\log p^{rs}\right) && \text{(8a)}\\
\text{subject to} \quad & \sum_{s\in S} p^{rs} = \frac{O_r}{Z}, \forall r \in R && \text{(8b)}\\
& \sum_{r\in R} p^{rs} = \frac{D_s}{Z}, \forall s \in S && \text{(8c)}\\
& p^{rs} \geq 0, \forall r \in R, \forall s \in S && \text{(8d)}
\end{aligned}
$$

Maximum likelihood estimation

- We assume that trips in OD pairs are i.i.d. random variables.
- Assuming Poisson distribution for the OD pairs, the probability of observing certain number of trips in that OD pair

$$\mathbb{P}(\hat{d}^{rs}) = \frac{(d^{rs})^{\hat{d}^{rs}}}{\hat{d}^{rs}!} e^{-d^{rs}}$$

  where, d^{rs} is the estimated number of trips and $\hat{d}^{rs}$ is the trips in the seed matrix.
- The likelihood function is given by

$$L = \Pi_{(r,s) \in R \times S} \frac{(d^{rs})^{\hat{d}^{rs}}}{\hat{d}^{rs}!} e^{-d^{rs}}$$

We get the following optimization problem

$$\underset{\mathbf{d}}{\text{maximize}} \quad \log L \tag{9a}$$

$$\text{subject to} \quad \sum_{s \in S} d^{rs} = O_r, \forall r \in R \tag{9b}$$

$$\sum_{r \in R} d^{rs} = D_s, \forall s \in S \tag{9c}$$

$$d^{rs} \geq 0, \forall r \in R, \forall s \in S \tag{9d}$$

Generalized least squares

Let us express the conservation constraints (using on-off counts from APC data or link counts from ETM data) as $A\mathbf{d} = \mathbf{b}$, where, $\mathbf{d} = \{d^{rs}\}_{(r,s)\in R\times S}$.

$$\underset{\mathbf{d}}{\text{minimize}} \quad (A\mathbf{d} - \mathbf{b})^T W^{-1} (A\mathbf{d} - \mathbf{b}) + (\mathbf{d} - \hat{\mathbf{d}})^T V^{-1} (\mathbf{d} - \hat{\mathbf{d}}) \tag{10a}$$

- W, V are weighting matrices (typically diagonal).
- The second term is referred to as a regulariser. Regularisation make sure that the estimated OD is not significantly deviating from the seed OD matrix.

Bayesian estimation

- Bayes' theorem gives the posterior of unknown parameters (trips in the OD matrix) θ given an observed measurement Y (link or on-off counts) as proportional to the likelihood of the observation and prior probability of the unknowns $\mathbb{P}(\theta)$

$$\mathbb{P}(\theta|Y) \propto \mathbb{P}(Y|\theta)\mathbb{P}(\theta)$$

- Estimates can be obtained as those giving the maximum a posteriori (MAP) density.

Trip chaining

- AFC systems can be of two types:
  - Open: Only passengers' boarding/alighting location is recorded (usually transit systems with fixed fare)
  - Closed: Passengers' both boarding and alighting locations are recorded (usually transit systems with distance-based fare)
- In case of open system, alighting locations in the AFC data are inferred based on rule-based heuristics by making use of schedule, AVL and/or APC data.
  - Rule-based trajectories are usually based on walking time threshold, waiting time threshold, and space-time constraints.
- In both cases, transfers also need to be inferred in order to get entire trajectory.

Clustering

- When boarding stop is not available in the AFC data, then clustering can be used to assign the boarding GPS locations to various transit stops.
- Clustering methods
  - K-means clustering
  - DBSCAN
  - others

Thank you!