

Timetable development

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Introduction

Definition (Timetabling (Schedule building)). Given a set of transit lines L and their associated frequencies $f : L \mapsto \mathbb{R}$, find the exact dispatching time of each trip of each line and their arrival and departure time at every stop they visit so as to optimize a given objective.

- ▶ Timetable should be responsive to demand patterns
 - An agency's assumption about passengers adjusting to timetable instead of planners adjusting the timetable to passenger demand could become a major source of unreliable service.
- ▶ Common objectives of timetabling are:
 - Even distribution of passenger loads across all trips of any transit line
 - Even headway
 - Maximizing the number of transfer synchronization events
 - Minimizing the passenger wait time
- ▶ Irregular time headway of consecutive trips is justified in case of
 - varying demand patterns
 - transfer time synchronization

Examples

பேருந்துநிலைய கால அட்டவணை																			
வெளிநிலையம் Perundurai					வெளிநிலையம் Chennai					வெளிநிலையம் Chennai					வெளிநிலையம் Chennai				
12.5-4.40	12.0-9.35	PTS-200	PTS-635	59.100	ATC-12.45	C5-10.30	SRL-2.40	C5A-8.20	7-1.30	P4-7.20	LNT-12.00	B2-11.10	B12-10.20	B12-10.40	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-4.55	SSM-9.50	SIS-2.40	51-6.42	20.2.10	ATC-12.45	C5-10.30	C5-2.55	C4-8.45	7-1.40	7-6.20	P5-12.20	8E-12.00	B12-10.40	B12-10.40	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
31-5.10	12-10.05	12.0-2.20	PTS-650	39-2.30	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TSP-5.20	P7/12-10.15	31-2.55	12.0-7.05	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
PTS-5.40	SIS-10.25	PTS-2.40	12.0-7.15	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.6-5.50	12.0-10.30	12.0-2.45	TSP-7.25	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12-6.00	31-10.55	12.0-2.50	PTS-7.55	39-4.45	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.25	PTS-10.40	12.0-2.50	12.0-7.40	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.35	PTS-10.50	12.0-3.15	SSM-7.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
PTS-6.40	12.0-11.00	TSP-3.22	12.0-7.45	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-6.55	12.0-11.15	12.0-3.30	12-8.35	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-7.15	12.0-11.25	P7/12-3.35	PTS-8.40	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12.0-7.22	TSP-11.30	SSM-3.50	12.0-8.35	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TSP-7.30	12.0-11.45	12.0-3.55	PTS-9.35	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
31-7.40	SSM-11.50	12.0-4.15	12-10.00	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
SSM-7.45	P7/12-11.55	PTS-4.15	12.0-10.00	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
12-8.05	12-12.05	SIS-4.25	39-7.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.20	31-12.15	PTS-4.35	P7/12-12.10	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.30	SIS-12.25	12.0-4.40	VRS-6.40	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.40	12.0-12.35	12.0-5.10	39A-6.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-8.45	PTS-12.40	12.0-5.15	39-7.30	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.00	12.0-13.00	12.0-5.30	39-7.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.10	12.0-13.15	SSM-5.50	39A-7.20	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.15	12.0-13.30	12.0-5.50	39-7.50	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.25	P7/12-14.00	12-6.15	20.3.10	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.30	SSM-14.5	SSM-6.25	39-12.00	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20
TS-9.35	12-1.50	12-6.30	39-12.10	39-7.10	சென்னை-15	C5-10.40	C5SS-5.00	PSB-9.05	7A-1.55	K1-9.10	P5-1.00	17-12.50	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20	B12-11.20

Figure: Perundurai to Chennimalai bus timings ¹

¹<http://kkthecommonman.blogspot.com/2017/09/perundurai-to-chennimalai-bus-timings.html>

Examples

► Southbound To Farragut Square

Monday thru Friday — De Lunes a viernes





Route Number	Chevy Chase Circle NW	Connecticut & Nebraska Aves. NW	Connecticut Ave. & Veazey Terr. NW (Van Ness-UDC)	Connecticut Ave. & Porter St. NW (Cleveland Park)	Calvert St. & Connecticut Ave. NW (Woodley Park)	Connecticut Ave. & Leroy Pl. NW (Florida Ave.)	17th St. (E) & I St NW (FARRAGUT N&W)
							
AM Service — Servicio matutino							
L2	5:05	5:09	5:12	5:14	5:19	5:24	5:29
L2	5:35	5:39	5:42	5:44	5:49	5:54	5:59
L2	5:55	6:00	6:04	6:07	6:13	6:19	6:25
L2	6:10	6:15	6:19	6:22	6:28	6:34	6:40
L2	6:24	6:29	6:33	6:36	6:42	6:48	6:54
L2	6:36	6:41	6:45	6:48	6:54	7:00	7:06
L2	6:48	6:53	6:57	7:00	7:06	7:12	7:18
L2	7:00	7:05	7:09	7:12	7:18	7:24	7:30
L2	7:10	7:16	7:21	7:24	7:31	7:39	7:47
L2	7:20	7:26	7:31	7:34	7:41	7:49	7:57
L2	7:30	7:36	7:41	7:44	7:51	7:59	8:07
L2	7:40	7:47	7:53	7:57	8:04	8:12	8:21
L2	7:50	7:57	8:03	8:07	8:14	8:22	8:31
L2	8:00	8:07	8:13	8:17	8:24	8:32	8:41
L2	8:10	8:17	8:23	8:27	8:34	8:42	8:51
L2	8:20	8:27	8:33	8:37	8:44	8:52	9:01

Figure: L2 Connecticut Avenue Line in Washington D.C.²

²https://www.wmata.com/schedules/timetables/upload/L2_231217.pdf

Definitions (TCRP 135)

Definition (Layover time). The time between the scheduled arrival and departure of a vehicle at a transit terminal³.

- ▶ also referred to as "recovery time"
- ▶ typically used as rest time for transit vehicle operator between trips.
- ▶ other purpose is to ensure an on-time departure for the next trip.

Definition (Span of service). The length of time, from the beginning of the first trip to the end of the last trip, during which service operates on the street.

- ▶ can be expressed for a route or for the system as a whole.

Definition (Cycle time or round trip cycle time). Sum of the round-trip running time plus layover time.

³One endpoint of a route where trips usually begin or end

Definitions (TCRP 135)

Definition (Service pattern). The unique sequence of stops associated with each type of trip on a route.

- ▶ if all trips operate from one end to the other on a common path the route has one service pattern.
- ▶ branches, deviations, or short turns introduce additional service patterns.

Definition (Deadhead). The time and distance that a bus needs to travel in places where it will not pick up passengers.

- ▶ typically required to get buses to and from their garage,
- ▶ or need to travel from one route or point to another during their scheduled work day.

Definition (Pull-out and Pull-in time). The time the vehicle spends traveling from the garage to the route and vice-versa are called pull-out and pull-in time respectively .

- ▶ pull-out and pull-in times are included in vehicle hours, but not in revenue hours.
- ▶ both are components of deadhead miles.

Definitions (TCRP 135)

Definition (Block). A vehicle (or train) assignment that includes the series of trips operated by each vehicle from the time it pulls out to the time it pulls in. A complete block includes a pull-out trip from the garage followed by one or (usually) more revenue trips and concluding with a pull-in trip back to the garage.

Developing schedule

Stop 1 (Terminal)	Stop 2	Stop 3	Stop 4	...	Stop n (Terminal)
t_0	$t_0 + t_{12}$	$t_0 + t_{12} + t_{23}$	$t_0 + t_{12} + t_{23} + t_{34}$...	$t_0 + \sum_{i=1}^{n-1} t_{i,i+1}$
$t_0 + h$	$t_0 + h + t_{12}$	$t_0 + h + t_{12} + t_{23}$	$t_0 + h + t_{12} + t_{23} + t_{34}$...	$t_0 + h + \sum_{i=1}^{n-1} t_{i,i+1}$
$t_0 + 2h$	$t_0 + 2h + t_{12}$	$t_0 + 2h + t_{12} + t_{23}$	$t_0 + 2h + t_{12} + t_{23} + t_{34}$...	$t_0 + 2h + \sum_{i=1}^{n-1} t_{i,i+1}$
$t_0 + 3h$	$t_0 + 3h + t_{12}$	$t_0 + 3h + t_{12} + t_{23}$	$t_0 + 3h + t_{12} + t_{23} + t_{34}$...	$t_0 + 3h + \sum_{i=1}^{n-1} t_{i,i+1}$
...
$t_0 + k \times h$	$t_0 + k \times h + t_{12}$	$t_0 + k \times h + t_{12} + t_{23}$	$t_0 + k \times h + t_{12} + t_{23} + t_{34}$...	$t_0 + k \times h + \sum_{i=1}^{n-1} t_{i,i+1}$

Figure: Example schedule⁴

- ▶ t_0 : dispatch time of the first vehicle from the terminal
- ▶ t_{ij} : time between stop i and stop j along the route
- ▶ h : headway
- ▶ n : number of stops
- ▶ k : # of trips

⁴Source: Fundamentals of Transportation Wiki

String diagram

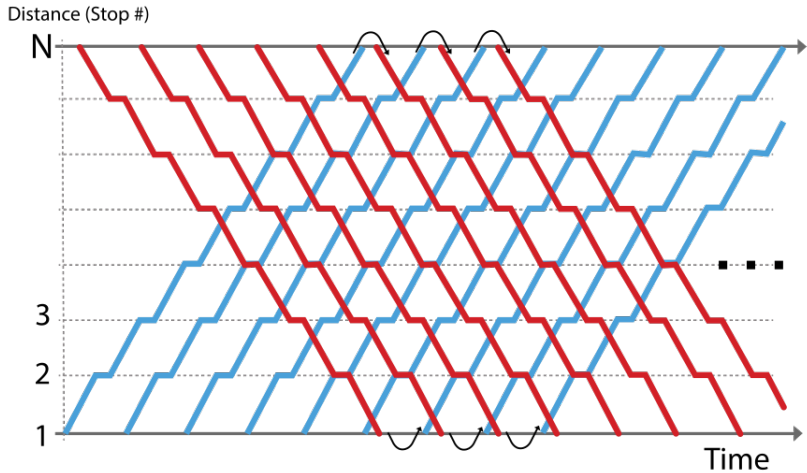


Figure: String diagram⁵

⁵Source: Fundamentals of Transportation Wiki

Building a simple schedule

Steps

- Step 1.** Calculate round trip cycle time including the layover time.
In our case, $8 + 14 + 11 + 11 + 14 + 8 = 66$ minutes.
- Step 2.** Figure out the layover time.
- Possibly from the union contract.
 - Past practice may guide the layover time (e.g., 10% of running time or six minutes per round trip, whichever is greater)
 - In our case, 10% of 66 ≈ 7 minutes.
 - Therefore, overall round trip cycle time = $66 + 7 = 73$ minutes.
 - # of buses required = $\frac{73 \times 2}{60} = 2.43 \approx 3$ (as we need to provide frequency of 2 buses/hr)
 - 3 buses will provide extra layover time, i.e., $\frac{3 \times 60}{2} - 66 = 24$ minutes.
- Step 3.** Prepare the basic schedule pattern.
- Step 4.** Decide when to start the service.
- Step 5.** Populate the schedule.
- Step 6.** Fill in the intermediate times.

Final output should look like this...

Eastbound				Westbound						
A	B	C	D		D	C	B	A		Next Trip
06:00	06:08	06:22	06:33		06:15	06:26	06:40	06:48		07:00
06:30	06:38	06:52	07:03		06:45	06:56	07:10	07:18		07:30
07:00	07:08	07:22	07:33		07:15	07:26	07:40	07:48		08:00
07:30	07:38	07:52	08:03		07:45	07:56	08:10	08:18		08:30
08:00	08:08	08:22	08:33		08:15	08:26	08:40	08:48		09:00
08:30	08:38	08:52	09:03		08:45	08:56	09:10	09:18		09:30
09:00	09:08	09:22	09:33		09:15	09:26	09:40	09:48		10:00
09:30	09:38	09:52	10:03		09:45	09:56	10:10	10:18		10:30
10:00	10:08	10:22	10:33		10:15	10:26	10:40	10:48		11:00
10:30	10:38	10:52	11:03		10:45	10:56	11:10	11:18		11:30
11:00	11:08	11:22	11:33		11:15	11:26	11:40	11:48		12:00
11:30	11:38	11:52	12:03		11:45	11:56	12:10	12:18		12:30
12:00	12:08	12:22	12:33		12:15	12:26	12:40	12:48		13:00
12:30	12:38	12:52	13:03		12:45	12:56	13:10	13:18		13:30
13:00	13:08	13:22	13:33		13:15	13:26	13:40	13:48		14:00
13:30	13:38	13:52	14:03		13:45	13:56	14:10	14:18		14:30
14:00	14:08	14:22	14:33		14:15	14:26	14:40	14:48		15:00
14:30	14:38	14:52	15:03		14:45	14:56	15:10	15:18		15:30
15:00	15:08	15:22	15:33		15:15	15:26	15:40	15:48		16:00
15:30	15:38	15:52	16:03		15:45	15:56	16:10	16:18		16:30
16:00	16:08	16:22	16:33		16:15	16:26	16:40	16:48		17:00
16:30	16:38	16:52	17:03		16:45	16:56	17:10	17:18		17:30
17:00	17:08	17:22	17:33		17:15	17:26	17:40	17:48		18:00
17:30	17:38	17:52	18:03		17:45	17:56	18:10	18:18		18:30
18:00	18:08	18:22	18:33		18:15	18:26	18:40	18:48		19:00
18:30	18:38	18:52	19:03		18:45	18:56	19:10	19:18		
19:00	19:08	19:22	19:33		19:15	19:26	19:40	19:48		

Figure: Schedule⁶

⁶Source: TCRP135

Even load method

This method tries to even the load of all trips of the line at maximum loading stop.

Consider a transit service of 3 buses/hr from 10:00 AM - 11:00 AM with average passenger load at the maximum loading point for each one its trip shown below:

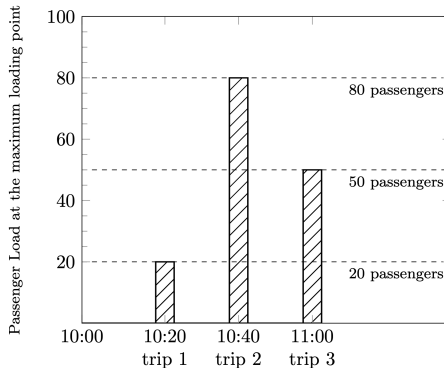


Figure: Passenger load at max loading point⁷

⁷Figure taken from Gkiotsalitis, PT optimization

Even load method

Assume uniform arrival rate, then the passenger arrival during three 20-minute intervals are:

- ▶ 10:00-10:20 AM we have $20/20 = 1$ passenger/min
- ▶ 10:20-10:40 AM we have $80/20 = 4$ pax/min
- ▶ 10:40-11:00 AM we have $50/20 = 2.5$ pax/min

We attempt to even the load and would like to have $\frac{20+80+50}{3} = 50$ pax for each trip.

With uniform arrival time, we want to shift the first trip from 10:20 AM to 10:20 + x AM so that we get extra 30 pax to have a total of 50 pax on this trip.

i.e., $4 \times x = 30 \implies x = \frac{30}{4} = 7.5$. This means first trip should depart at 10 : 27 : 30.

Repeat this procedure for other trips.

Even load method

One can solve the above problem graphically. Create a cumulative passenger load plot. Draw a horizon line from the desired passenger load point and find the point where it intersects with the load curve. Use that point to find the time on the x-axis.

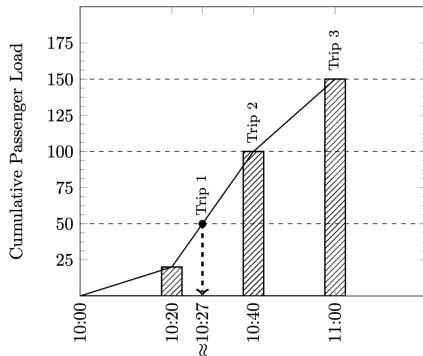


Figure: Even load method⁸

⁸Figure taken from Gkiotsalitis, PT optimization

Optimization model by Wang et al. (2017)

Sets

- ▶ S : Set of ordered stops of a transit route
- ▶ P : Set of passengers waiting to board the route
- ▶ K : Set of ordered trips served by the route during the day
- ▶ Z : Set of peak/off-peak time intervals with homogeneous travel times

Parameters

- ▶ D_f : Departure time of the first bus
- ▶ D_l : Departure time of the last bus
- ▶ c_{sk} : Avg. travel time from stop s to $s + 1$ during k^{th} time interval
- ▶ B_p : Stop at which the p^{th} passenger is waiting
- ▶ A_p : Arrival time of p^{th} passenger

Decision variables

- ▶ x_k : Dispatching time of the k^{th} trip
- ▶ V_{sk} : Arrival time of the k^{th} trip at stop k
- ▶ τ_p : Vehicle trip onto which the p^{th} passenger boards
- ▶ w_p : Waiting time of the p^{th} passenger

Optimization model by Wang et al. (2017)

$$\underset{\mathbf{x}, \mathbf{V}, \tau, \mathbf{w}}{\text{minimize}} \quad \sum_{p \in P} w_p \quad (1a)$$

$$\text{subject to} \quad x_1 = D_f \quad (1b)$$

$$x_{|K|} = D_l \quad (1c)$$

$$x_k \leq x_{k+1}, \forall k \in K \setminus \{|K|\} \quad (1d)$$

$$V_{1k} = x_k, \forall k \in K \quad (1e)$$

$$V_{sk} = V_{s-1,k} + c_{sz}(\cdot : V_{s-1,k} \in z), \forall s \in S \setminus \{1\}, \forall k \in K \quad (1f)$$

$$\tau_p = \min_k \{V_{B_p k} : V_{B_p k} > A_p\}, \forall p \in P \quad (1g)$$

$$w_p = V_{B_p, \tau_p} - A_p, \forall p \in P \quad (1h)$$

$$x_k \in \mathbb{Z}, \forall k \in K \quad (1i)$$

$$\tau_p \in K, \forall p \in P \quad (1j)$$

$$w_p \in \mathbb{R}, \forall p \in P \quad (1k)$$

$$V_{sk} \in \mathbb{R}, \forall s \in S, \forall k \in K \quad (1l)$$

Suggested reading

- ▶ Ceder. Public Transit Planning and Operations, Chapters 4 and 5.
- ▶ Gkiotsalitis, Konstantinos. Public transport optimization, Chapter 9.
- ▶ TCRP Report 135
- ▶ Y. Wang, D. Zhang, L. Hu, Y. Yang and L. H. Lee, "A data-driven and optimal bus scheduling model with time-dependent traffic and demand", IEEE Trans. Intell. Transp. Syst., vol. 18, no. 9, pp. 2443-2452, Sep. 2017.

Thank you!