Network design and frequency setting

Pramesh Kumar

IIT Delhi

October 19, 2024

Problem

Definition (Transit network design and frequency setting problem (TNDFSP)). Given the set of stops, underlying infrastructure, and passenger O-D matrix, find the set of lines/routes in the network with their frequencies.

Remark. In OR literature, this problem is also referred to as the line planning problem (LPP).

Definition (Line). A line is a walk in the public transportation network between two nodes.

Definition (Line concept). A line concept is a collection of lines L together their frequencies $f : L \mapsto \mathbb{R}$.

Note: I'll use terms line and route interchangeably.

Considerations

Passengers' perspective

1. The line length should not be too long because longer routes are susceptible to unreliable service.
2. The travel time between two nodes of a route should not deviate more than certain threshold of the shortest road travel time between those nodes. This is important to not make transit less attractive to auto mode.
3. Avoid circuity in transit lines.
4. Minimize the wait time of passengers.
5. etc.

Community's perspective

1. Minimize the overall passenger hours spent in the system.
2. Less congestion on roads due to buses.

Agency's perspective

1. The line length should not be too short because shorter lines may require frequent turnovers at terminals resulting in higher operational costs.
2. Minimize the empty seat hours.
3. Minimize the cost of operation.

Possible ways of solving LPP

1. LPP is an NP-Hard problem.
2. One way of solving this problem is to divide this problem into subproblems:
   - Route generation
   - Passenger assignment on the generated routes
   - Frequency setting
3. Formulate the LPP as an optimization problem and solve.

Remark. In making the final decisions, local geographical, political, and social conditions are considered.

Route generation algorithms

Algorithm Depth First Search (DFS) for generating transit routes

Input $G(N, A)$ ▷ Underlying network

Output L ▷ Set of transit routes

$L \leftarrow []$

procedure CreateCandidateRoutes(G_R)

  for $t \in T$ **do** ▷ T is the set of terminals

    for $i \in N$ **do**

      $visited \leftarrow \{\}$; $temp\_routes \leftarrow []$; $path \leftarrow []$

      $visited[n] \leftarrow$False, $\forall n \in N$

      EnumeratePaths($t, i, visited, path$)

      Filter out paths in $temp\_routes$ that do not satisfy length, travel time, and circuity criterion.

      Append all the routes in $temp\_routes$ to L

    end for

  end for

end procedure

function EnumeratePaths($i, n, visited, path$)

  $visited[i] \leftarrow$ True

  Append i to $path$

  if $i == n$ **then**

    Append $path$ to $temp\_routes$

  else

    for $k \in FS(i)$ **do**

      EnumeratePaths($k, n, visited, path$)

    end for

  end if

  Remove last node from $path$

  $visited[i] \leftarrow$False

end function

Disadvantages of DFS

1. High computational time
2. Too many lines generated
3. Does not consider the demand capture criteria.

Route generation algorithm (RGA)

- RGA was initially proposed by Baaj and Mahmassani (1995) which was later improved by Mauttone and Urquhart (2008).
- In this algorithm we generate routes between O-D pairs in the order of their demand.
- We maintain two parameters D_0 and D_1 which refer to proportion of demand served by a direct route and demand served with 1 transfer. We stop when $D_0 \geq \bar{D}_0$ and $D_1 \geq \bar{D}_1$.
- For each O-D pair we check whether it is beneficial to generate a direct route between them or modify the existing route to incorporate this O-D pair.
- In the modification step, we try to insert the origin & destination nodes in the route structure so as to minimize the increase in cost of the route.

Algorithm Route generation algorithm

Input $G(N, A), c : A \mapsto \mathbb{R}, \bar{D}_0, \bar{D}_1, d$ ▷ Underlying network, cost, proportions, demand
Output L ▷ Set of transit routes
$L \leftarrow []$, D_0, $D_1 \leftarrow 0$
procedure RGA
    while $D_0 < \bar{D}_0$ **and** $D_1 < \bar{D}_1$ **do**
        for $(r, s) \in d$ in decreasing order of their demand **do**
            $l \leftarrow$ Create a route with shortest path between r and s in G.
            $l^{'}, l^{''} \leftarrow \textsc{Expand}(r, s, L)$ ▷ Create a route $l^{'}$ by expanding an existing route $l^{''}$
            if $cost(l) < cost(l^{'}) - cost(l^{''})$ **then**
                if l satisfies other criterion **then**
                    $L \leftarrow L \cup \{l\}$
                    Remove the O-D pairs from d which are directly covered by l
                end if
            else
                if l satisfies other criterion **then**
                    $L \leftarrow L \cup \{l^{'}\} \backslash \{l^{''}\}$
                    Remove the O-D pairs from d which are directly covered by $l^{'}$
                end if
            end if
            Update D_0 and D_1
        end for
    end while
    return L
end procedure

```
function EXPAND(r, s, L)
    l', l'' ← φ, cost(l') ← ∞
    for l ∈ L do
        if r ∈ L or s ∈ L then
            x ← r, s (whichever is present on line l)
            for {p ∈ 1, ··· , |l| + 1} do    ▷ Finding best position for x to insert in l
                lAux ← insert x in position p in l
                if cost(lAux) < cost(l') then
                    l' ← lAux
                    l'' ← l
                end if
            end for
        else if both r, u ∈ l then
            for (p_1, p_2) ∈ {1, ··· , |l| + 1} × {1, ··· , |l| + 1} do
                lAux ← insert r and s in positions p_1 and p_2 resp. in l
                if cost(lAux) < cost(l') then
                    l' ← lAux
                    l'' ← l
                end if
            end for
        end if
    end for
    return l', l''
end function
```

Frequency setting methods

Frequency setting

- In this step, we determine the number of trips per hour required for each transit route during each time period to serve the passenger demand.
- It can be studied as an independent problem.
- Based on the demand changes, transit agencies might want to consider revising frequencies of routes.
- Transit agency is faced with the trade-off between increasing frequency to decrease passenger waiting time versus increase in the operational cost as well as decrease in fleet size.

Definition (Frequency setting problem). Determining the number of transit trips for each route in given set of lines L during a specified time period so as to maximize the passenger level of service.

Frequency setting

Criterion

- For high ridership routes, we need to provide adequate space to meet passenger demand. This is usually addressed using "peak load factor."
- For low ridership routes, we need to provide minimum frequency. This is usually addressed using "policy headway" (e.g., 30 or 60 min).

Methods

1. Closed form methods
   - Max load (point check) method
   - Load profile (ride check) method
2. Optimization-model based

Max load (point check) method

Method 1

$$\boxed{F_{1j} = \max\left\{\frac{P_{mdj}}{d_{0j}}, F_{mj}\right\}}, j = 1, \cdots q \quad (1)$$

where, $P_{md} = \max_{i \in S} \sum_{j=1}^{q} P_{ij} = \sum_{j=1}^{q} P_{i^*j}$
and $P_{mdj} = P_{i^*j}$
Notations:

- $1, \cdots j$: time periods
- F_{mj}: minimum frequency for period j
- i^*: daily maximum load stop
- P_{ij}: total number of passengers on-board when departing stop i during time period j
- P_{mdj}: (avg) observed at daily max load stop during time period j
- P_{md}: total load observed at max load stop
- $d_{0j} = c\gamma_j$: desired occupancy on the vehicle during time period j
- c: capacity of transit vehicle
- γ_j: load factor

Max load (point check) method

Method 2

$$\boxed{F_{2j} = \max\left\{\frac{P_{mj}}{d_{0j}}, F_{mj}\right\}}, j = 1, \cdots q \tag{2}$$

where, $P_{mj} = \max_{i \in S} P_{ij} = \sum_{j=1}^{q} P_{i^*j}$
and $P_{mdj} = P_{i^*j}$
Notations:

- $1, \cdots j$: time periods
- F_{mj}: minimum frequency for period j
- P_{ij}: total number of passengers on-board when departing stop i during time period j
- P_{mj}: max observed load across all stops in period j
- $d_{0j} = c\gamma_j$: desired occupancy on the vehicle during time period j
- c: capacity of transit vehicle
- γ_j: load factor

Load profile (ride check) method

Method 3

$$\boxed{F_{3j} = \max\left\{\frac{A_j}{d_{0j} \times L}, \frac{P_{mj}}{c}, F_{mj}\right\}}, j = 1, \cdots q \tag{3}$$

where, $A_j = \max_{i \in S} P_{ij} \times l_i$
and $L = \sum_i l_i$
Notations:

- $1, \cdots j$: time periods
- l_i distance between stop i and $i+1$
- A_j: area in pass-km under the load profile during time period j
- L: route length
- $\frac{A_j}{L}$: average load
- F_{mj}: minimum frequency for period j
- P_{ij}: total number of passengers on-board when departing stop i during time period j
- P_{mj}: max observed load across all stops in period j
- $d_{0j} = c\gamma_j$: desired occupancy on the vehicle during time period j
- c: capacity of transit vehicle
- γ_j: load factor

Frequency optimization considering passenger behavior (Constantin and Florian (1995))

$$\underset{\mathbf{f},\mathbf{v},\mathbf{W}}{\text{minimize}} \quad \sum_{k \in D} c_a v_{ak} + \sum_{i \in S} W_{ik} \tag{4a}$$

$$\text{subject to} \quad \sum_{a \in FS(i)} v_{ak} - \sum_{a \in BS(i)} v_{ak} = \begin{cases} d^{ok}, & \textbf{if } i = o \\ \sum_{o \in O} d^{ok}, & \textbf{if } i = k \\ 0 & \text{otherwise} \end{cases} \tag{4b}$$

$$v_{ak} \leq f_a W_{ik}, \forall a \in FS(i), \forall i \in S, \forall k \in D \tag{4c}$$

$$\sum_{a \in A} c_a f_l(a) \leq B \tag{4d}$$

$$f_l \leq \bar{f}_l, \forall l \in L \tag{4e}$$

$$v_{ak} \geq 0, \forall a \in A, \forall k \in D \tag{4f}$$

$$W_{ik} \quad \text{free} \tag{4g}$$

$$f_l \geq \underline{f}_l, \forall l \in L \tag{4h}$$

where B is the fleet size, other notations were taught during assignment

Joint formulation of network design and frequency setting

Formulation by Borndörfer et al. (2007)

Sets

- L: set of all feasible lines
- O, D: Set of origins and destinations
- Π: Set of paths in the network
- Π^{od}: Set of paths between OD pair $(o,d) \in O \times D$

Parameters

- d^{od}: Demand between $o \in O$ and $d \in D$
- c^{π}: cost of path $\pi \in \Pi$
- Φ_l: fixed cost
- ϕ_f: operating cost
- κ_l: vehicle capacity
- λ_e: frequency bound on edge e
- $\bar{f}_l$: maximum frequency of line $l \in L$ allowed

Decision variables

- h^{π}: flow on path $\pi \in \Pi$
- f_l: frequency of line $l \in L$
- x_l: whether we use line $l \in L$ or not.

Formulation by Borndörfer et al. (2007)

$$
\begin{aligned}
\underset{\mathbf{h},\mathbf{f},\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{h} + \mathbf{\Phi}^T\mathbf{x} + \phi^T\mathbf{f} && \text{(5a)}\\
\text{subject to} \quad & \sum_{\pi\in\Pi^{od}} h^\pi = d^{od}, \forall (o,d) \in O \times D && \text{(5b)}\\
& \sum_{\pi\in\Pi} \delta_{ij}^\pi h^\pi \le \sum_{l\in L:(i,j)\in l} \kappa_l f_l, \forall (i,j) \in A && \text{(5c)}\\
& \sum_{l\in L:e\in L} f_l \le \lambda_e, \forall e \in E && \text{(5d)}\\
& f_l \le \bar{f}_l, \forall l \in L && \text{(5e)}\\
& x_l \in \{0,1\}, \forall l \in L && \text{(5f)}\\
& f_l \ge 0, \forall l \in L && \text{(5g)}\\
& h^\pi \ge 0, \forall \pi \in \Pi && \text{(5h)}
\end{aligned}
$$

Remark. Requires column generation to solve. However, one the pricing problem corresponding to line variables turns out to be NP-Hard.

Suggested reading

- Ceder. Public Transit Planning and Operations, Chapters 3 and 15.
- Gkiotsalitis, Konstantinos. Public transport optimization, Chapter 8 and 9.
- Borndörfer, Ralf, Martin Grötschel, and Marc E. Pfetsch. "A column-generation approach to line planning in public transport." Transportation Science 41.1 (2007): 123-132.
- Mauttone, Antonio, and María E. Urquhart. "A route set construction algorithm for the transit network design problem." Computers & Operations Research 36.8 (2009): 2440-2449.
- Constantin, Isabelle, and Michael Florian. "Optimizing frequencies in a transit network: a nonlinear bi-level programming approach." International Transactions in Operational Research 2.2 (1995): 149-164.

Thank you!