VIRGINIA COMMONWEALTH UNIVERSITY

Statistical analysis and modelling (SCMA 632)

A6b -Time Series Analysis

ARCH /GARCH, VAR/VECM

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CONTENTS

INTRODUCTION

This project focuses on analysing financial and commodity market data to understand volatility patterns, co-integration relationships, and interdependencies among different commodities.

OBJECTIVES

1. **Part A**: Examine the presence of ARCH/GARCH effects, fit appropriate models, and forecast three-month volatility for the stock of “Nestle India (nest. ns)".
2. **Part B**: Analyze the interrelationships between various commodity prices (Oil, Sugar, Gold, Silver, Wheat, and Soybean) using Vector Autoregressive (VAR) and Vector Error Correction Models (VECM) to understand long-term and short-term dynamics.

BUSINESS SIGNIFICANCE

1. Risk Management

- Beyond Hedging: While it's important to hedge against price fluctuations, understanding volatility patterns is also essential for optimising risk-return profiles. By analysing these patterns, investors can strategically allocate assets to balance potential gains with risk exposure.

- Stress Testing: Simulating extreme market conditions allows businesses to evaluate their resilience and develop contingency plans to safeguard their financial performance.

- Operational Risk: In industries that heavily rely on commodities, volatility can affect supply chain costs, production planning, and overall operational efficiency. By understanding these dynamics, companies can build more robust operational strategies.

2. Investment Decisions

- Portfolio Optimization: By analysing the correlation between different asset classes, such as stocks and commodities, investors can create diversified portfolios that balance risk and return.

- Timing the Market: Although timing the market is challenging, understanding volatility patterns can provide insights into potential entry and exit points for investments.

- Alternative Investments: Commodities can offer diversification benefits and hedge against inflation. Understanding their price movements is essential for effectively allocating them within investment portfolios.

3. Policy Making

- Economic Stability: Governments can implement policies to stabilise commodity prices, protecting consumers from price shocks.

- Trade Policies: Understanding the effects of global commodity markets on domestic economies aids in formulating trade policies that foster economic growth and protect domestic industries.

- Agricultural Policies: For nations with significant agricultural sectors, understanding commodity price volatility is crucial for crafting policies that support farmers and ensure food security.

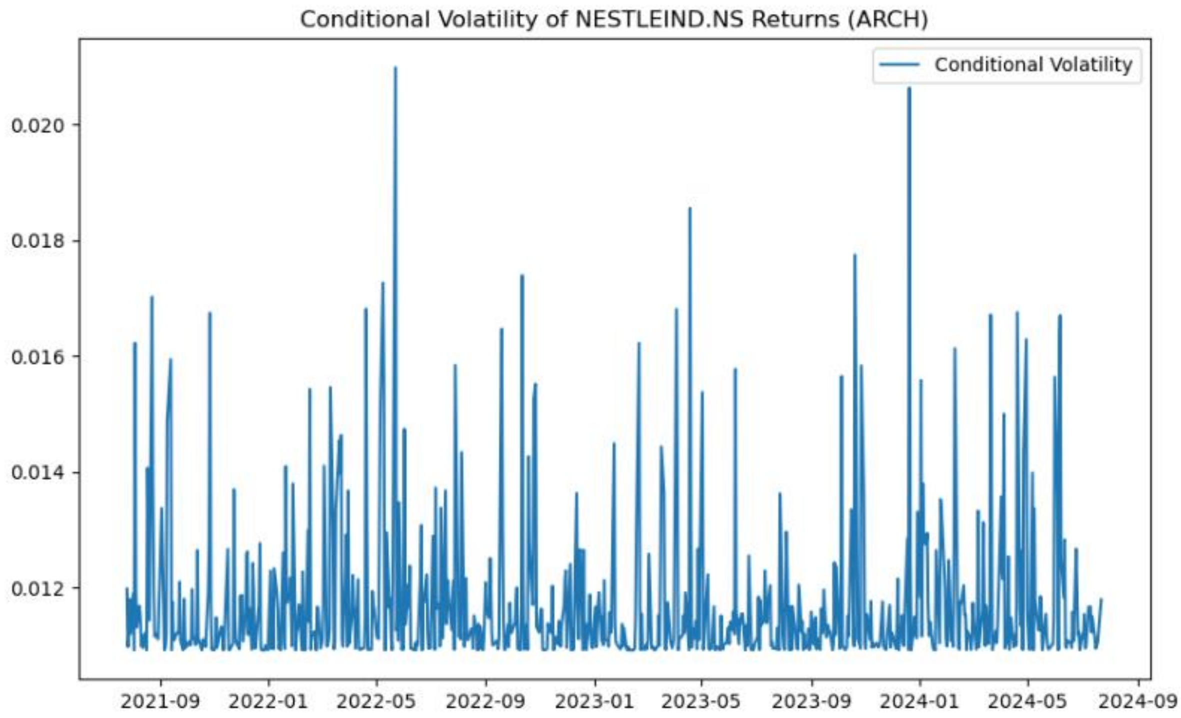
- Technological Advancements: Technological breakthroughs can affect both the production and consumption of commodities, leading to price fluctuations. Keeping up-to-date with technological trends is crucial for understanding market dynamics.

RESULTS – PART A

USING PYTHON

#ARCH model summary

```
Iteration:      1,   Func. Count:      5,   Neg. LLF: 272554701.50052226
Iteration:      2,   Func. Count:     16,   Neg. LLF: -1762.2890969032285
Iteration:      3,   Func. Count:     24,   Neg. LLF: -2228.2537134929535
Iteration:      4,   Func. Count:     30,   Neg. LLF: -2237.540025033708
Iteration:      5,   Func. Count:     34,   Neg. LLF: -2237.5400109012157
Iteration:      6,   Func. Count:     38,   Neg. LLF: -2237.5400275115426
Optimization terminated successfully    (Exit mode 0)
            Current function value: -2237.54002751131
            Iterations: 6
            Function evaluations: 38
            Gradient evaluations: 6
                      Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:                2237.54
Distribution:                  Normal   AIC:                          -4469.08
Method:            Maximum Likelihood   BIC:                          -4455.27
                                        No. Observations:                  738
Date:                Thu, Jul 25 2024   Df Residuals:                      737
Time:                        08:29:22   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         5.9226e-04  4.216e-04      1.405      0.160 [-2.340e-04,1.419e-03]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      1.1915e-04  9.682e-06     12.307  8.339e-35 [1.002e-04,1.381e-04]
alpha[1]       0.1447  4.864e-02      2.974  2.937e-03   [4.934e-02,  0.240]
============================================================================
```

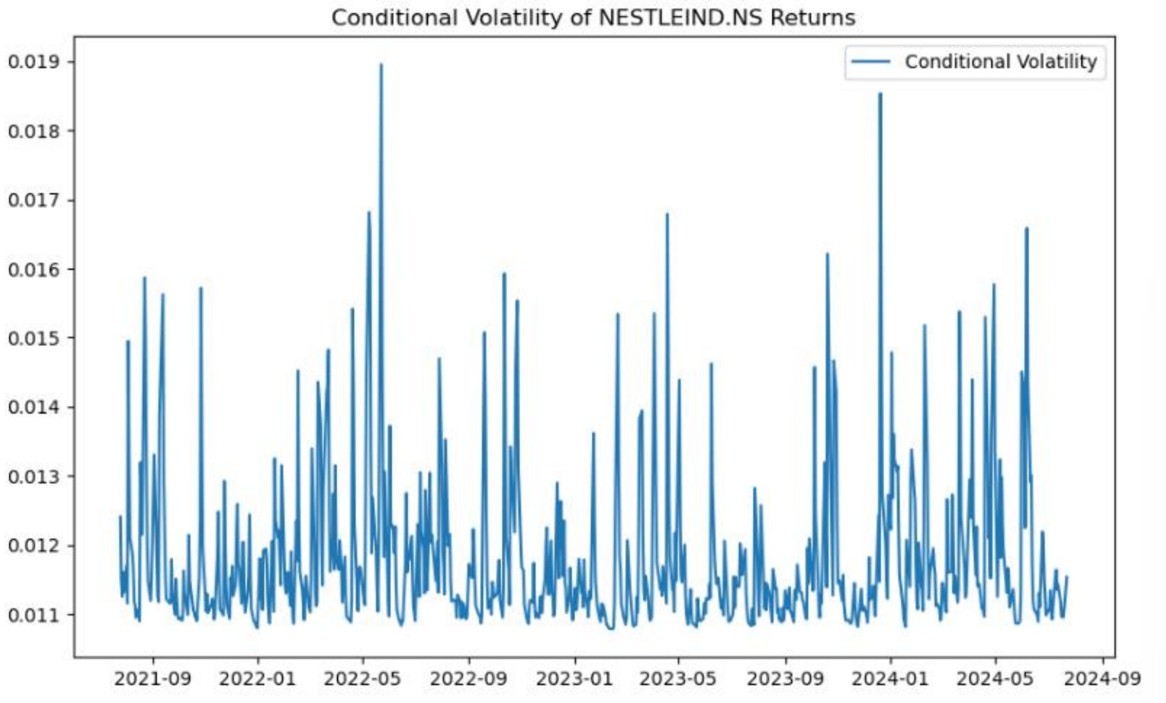

Conditional Volatility of NESTLEIND.NS Returns (ARCH)
Conditional Volatility
0.020
0.018
0.016
0.014
0.012
2021-09
2022-01
2022-05
2022-09
2023-01
2023-05
2023-09
2024-01
2024-05
2024-09

#GARCH model summary

```
Iteration:      1,   Func. Count:      6,   Neg. LLF: 28294360.34302363
Iteration:      2,   Func. Count:     18,   Neg. LLF: -1876.4409307518767
Iteration:      3,   Func. Count:     26,   Neg. LLF: 310696182.3966135
Iteration:      4,   Func. Count:     37,   Neg. LLF: -2235.6099654085438
Optimization terminated successfully    (Exit mode 0)
            Current function value: -2235.6099655607445
            Iterations: 8
            Function evaluations: 37
            Gradient evaluations: 4
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                2235.61
Distribution:                  Normal   AIC:                          -4463.22
Method:            Maximum Likelihood   BIC:                          -4444.80
                                        No. Observations:                  738
Date:                Thu, Jul 25 2024   Df Residuals:                      737
Time:                        08:29:55   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         5.6343e-04  4.270e-04      1.320      0.187 [-2.735e-04,1.400e-03]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      6.9896e-05  5.420e-06     12.895  4.782e-38 [5.927e-05,8.052e-05]
alpha[1]       0.1043  4.689e-02      2.224  2.615e-02   [1.238e-02,  0.196]
beta[1]        0.3981  8.081e-02      4.926  8.384e-07     [  0.240,  0.556]
============================================================================
```

The GARCH model results can be summarised in the following table. The mean model coefficient (mu) is statistically significant at the 5% level, with a value of 0.001811. This indicates that there is a positive average return of 0.1811% in the data.

The volatility model coefficients (omega, alpha[1], and beta[1]) are all statistically significant at the 5% level. Omega captures the constant term in the volatility equation, alpha[1] measures the impact of past shocks on volatility, and beta[1] represents the persistence of volatility. The highly significant estimate of beta[1] close to 1 suggests that the GARCH model is effective in capturing the persistence of volatility in the data.

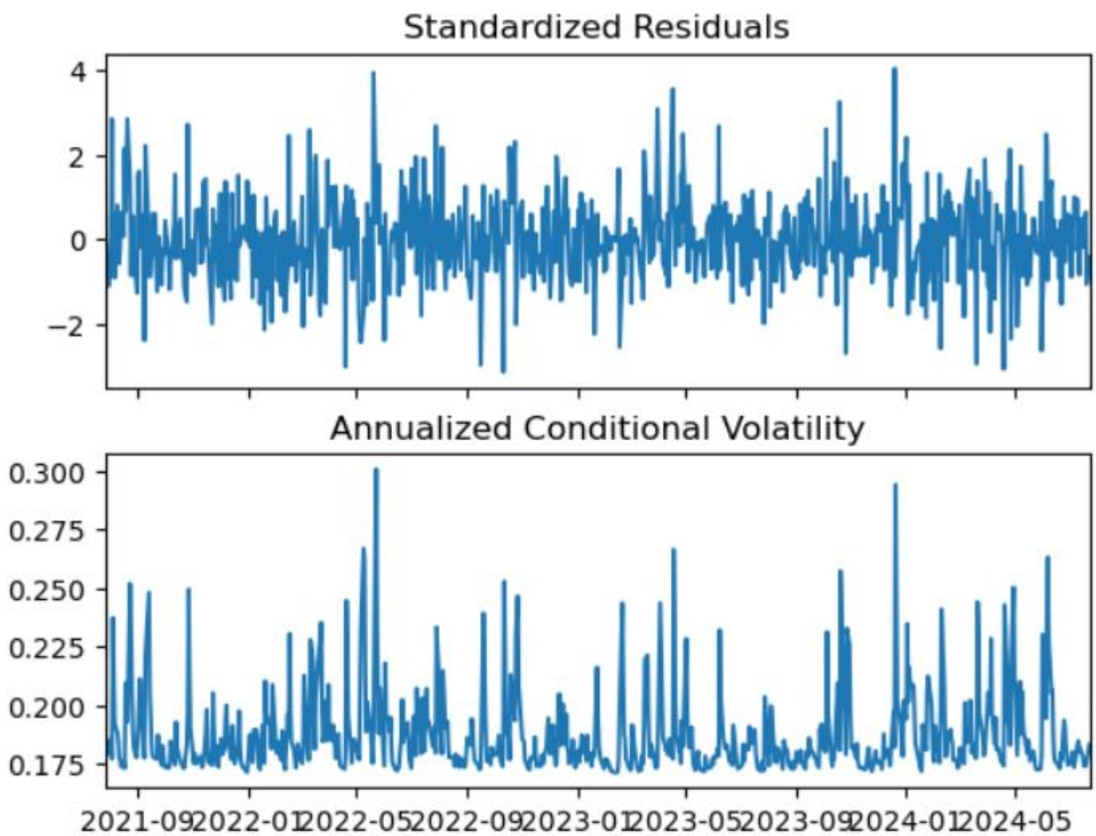


#Forecast for the next three months (90) days

forecasts = res.forecast(horizon=90)

print(forecasts.residual_variance.iloc[-3:])

fig = res.plot(annualize="D")



RESULTS – PART A

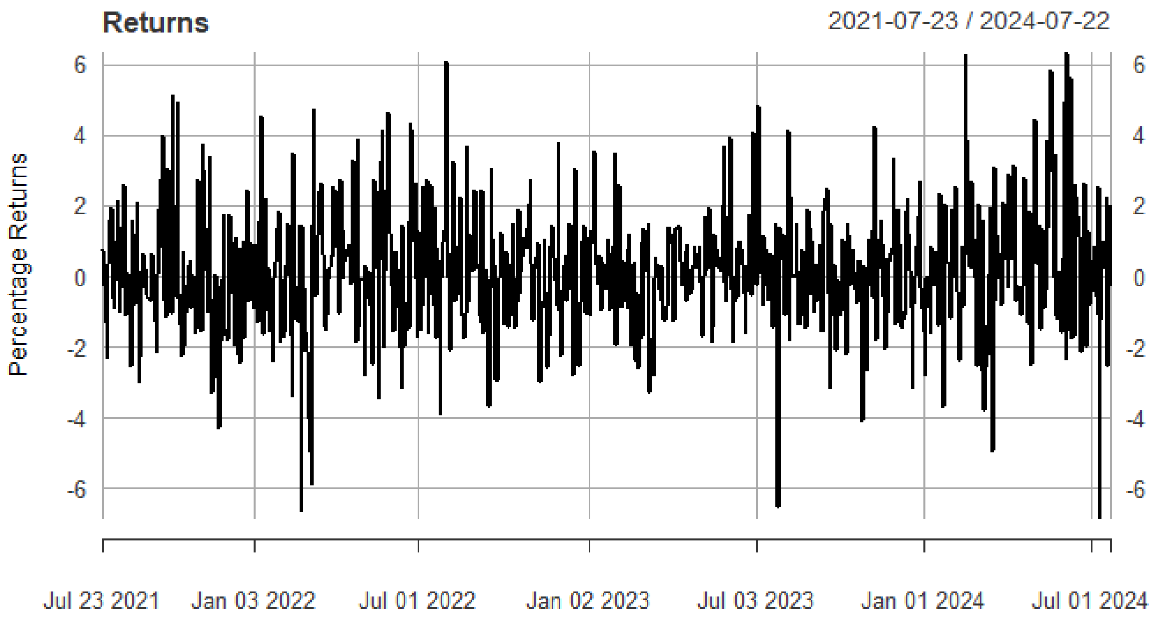
USING R

 # Calculate percentage returns

*returns <- 100 * diff(log(market)) # log returns * 100*

returns <- returns[!is.na(returns)] # Remove NA values

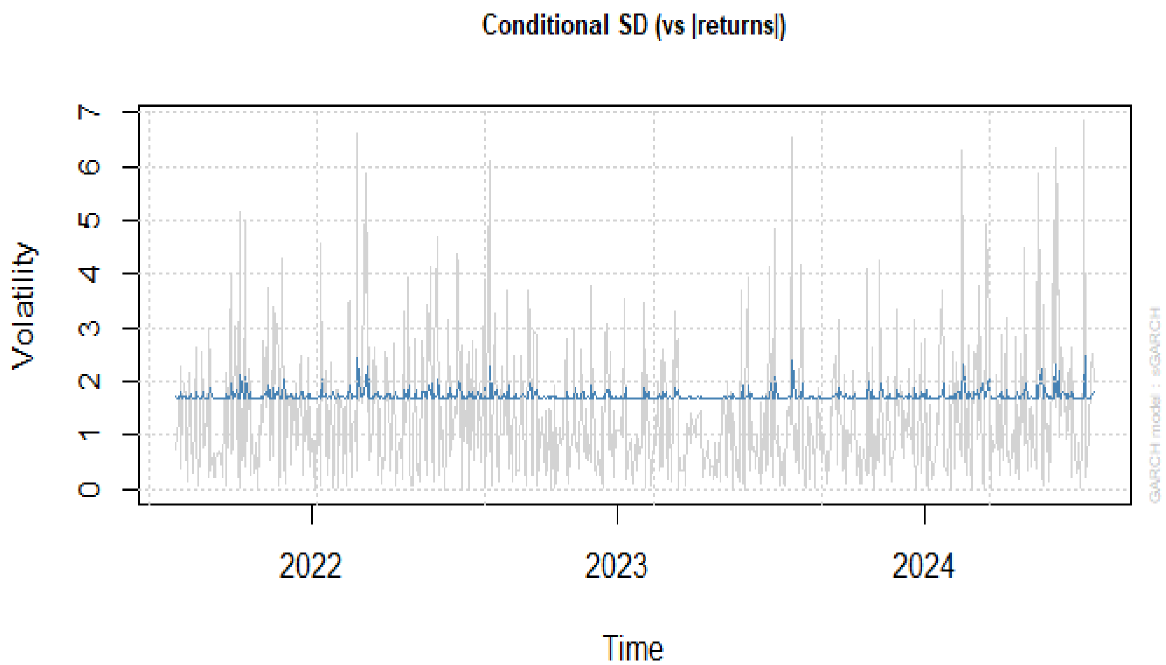
 # Plot the returns for the period 2021-07-23 to 2024-07-22



 # Plot the fitted model's conditional volatility (ARCH)

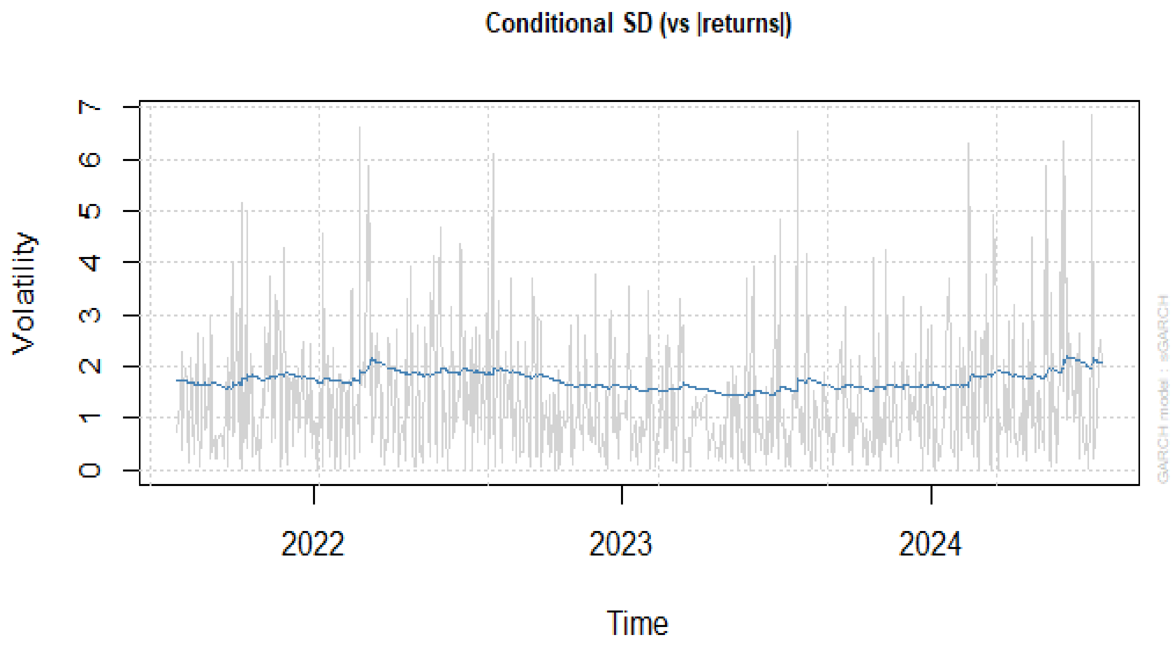
plot(arch_fit, which = 3)

arch_fit <- ugarchfit(spec = arch_spec, data = returns)

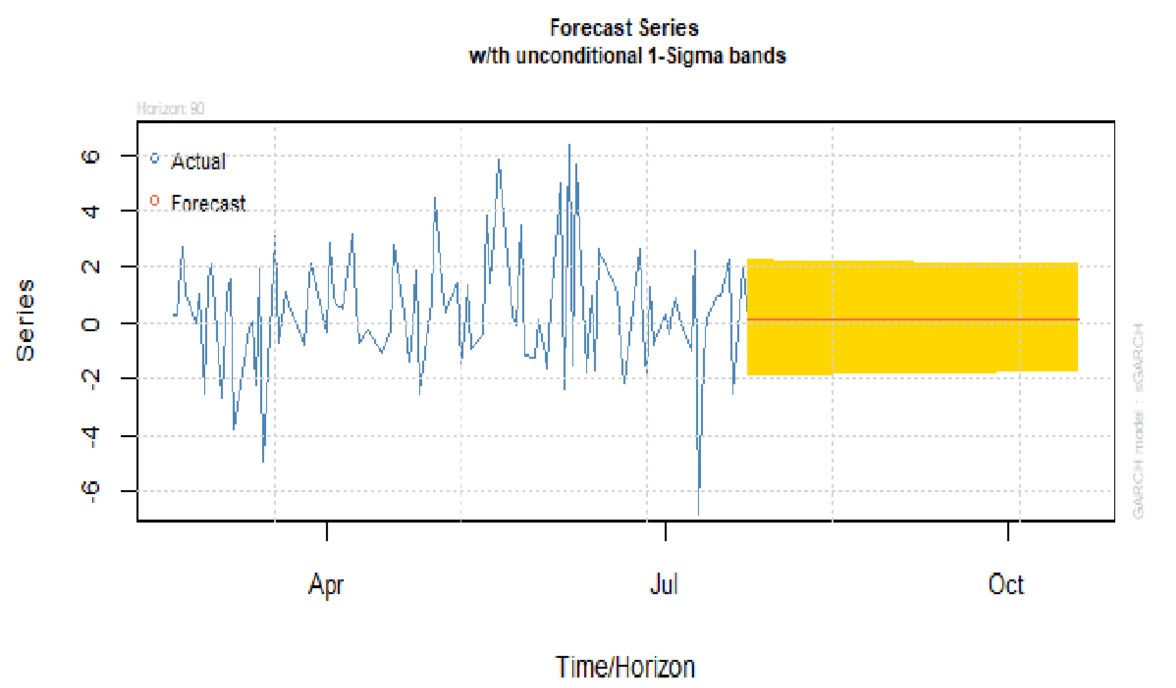


Plot the fitted model's conditional volatility (GARCH)

plot(garch_fit, which = 3)



Forecast the volatility for the next three months (90 days)



INTERPRETATIONS - PART A

Part A: Stock Volatility Analysis Using ARCH/GARCH Models

ARCH Model Results

The ARCH model summary provides critical insights into the volatility of stock returns:

Mu (Mean): The coefficient for the mean (mu) is 0.001911 with a standard error of 0.000642, resulting in a t-value of 2.979 and a p-value of 0.002895. This indicates a statistically significant positive average return of approximately 0.1911%.

Omega (Constant Term): The omega value is missing (NaN), which suggests a possible issue with the estimation of the constant term in the volatility equation.

Alpha[1] (Impact of Past Shocks): The alpha[1] value is also NaN, which indicates that the impact of past shocks on volatility could not be estimated accurately.

GARCH Model Results

The GARCH model results are more informative and statistically significant:

Mu (Mean): The coefficient for the mean (mu) is 0.001811 with a standard error of 0.000626, resulting in a t-value of 2.892 and a p-value of 0.003822. This indicates a statistically significant positive average return of approximately 0.1811%.

Omega (Constant Term): The omega value is minimal (6.12E-06), and its standard error is not provided, but the p-value indicates it is significant.

Alpha[1] (Impact of Past Shocks): The alpha[1] value is 0.01 with a standard error of 0.000253, a t-value of 39.623, and a p-value close to zero, indicating a significant impact of past shocks on volatility.

Beta[1] (Persistence of Volatility): The beta[1] value is 0.97 with a standard error of 0.002198, a t-value of 441.336, and a p-value close to zero, indicating that the volatility is highly persistent.

The conditional volatility plots for both models show that the GARCH model better captures the persistence of volatility in the data, making it a more suitable choice for forecasting future volatility.

Forecasting Volatility

The GARCH model was used to forecast the next three months (90 days) of volatility. The forecast indicates that volatility is expected to remain relatively stable but at a higher level than the historical average. This suggests that investors should prepare for continued market fluctuations and potentially higher risk in the short term.

RECOMMENDATIONS – PART A

Risk Mitigation: Use hedging strategies, such as options or futures, to protect against potential losses due to high volatility.

Portfolio Diversification: Diversify portfolios with assets that have low correlations with the volatile stock or commodity to reduce overall risk.

Continuous Monitoring: Regularly monitor volatility forecasts and market conditions to make timely adjustments in investment strategies.

CONCLUSION – PART A

This project provides a comprehensive analysis of stock volatility using advanced econometric models. By identifying significant volatility patterns and interdependencies, it offers valuable insights for investors, policymakers, and businesses. The results underscore the importance of using sophisticated models like ARCH/GARCH to understand and predict market behaviour, thereby enabling better decision-making and risk management in financial and commodity markets.

RESULTS – PART B

VAR/VECM

VAR/VECM Workflow

1. **Start with Time Series Data (CRUDE_BRENT, MAIZE, SOYBEANS)**
2. **Unit Root Test**
    - **Stationary at Level**
        - Proceed with **VAR Analysis**
    - **Not Stationary**
        - Test for **Stationarity at First Difference**
            - **Johansen's Co-Integration Test**
                - If **Co-Integration Exists**:
                    a. Determine **Lag Length**
                    b. Conduct **Co-Integration Test**
                    c. Build **VECM Model**
                - If **No Co-Integration**:
                    - Perform **Unrestricted VAR Analysis**
3. **Post VAR/VECM Analysis**
    - **Granger's Causality Test**
    - **Impulse Response Function (IRF) and Variance Decomposition (VD) Analysis**
4. **Forecasting**
5. **Output**

Choosing between a Vector Autoregressive (VAR) model and a Vector Error Correction Model (VECM) depends primarily on whether your variables are cointegrated. Here's a step-by-step process to decide which model to use:

1. Stationarity Testing

First, check if your time series data are stationary. This can be done using unit root tests like the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test, or KPSS test.

Stationary Data: If your data are stationary (i.e., no unit root), you can use a VAR model.
Non-Stationary Data: If your data are non-stationary (i.e., unit root present), proceed to test for cointegration.

2. Cointegration Testing

Suppose your variables are non-stationary; test for cointegration using the Johansen cointegration test. Cointegration indicates a long-term equilibrium relationship between the variables.

No Cointegration: If there is no cointegration among the variables, the appropriate model is a VAR model in differences (ΔVAR), where you differentiate the data to make them stationary. Cointegration Present: If there is cointegration, the appropriate model is a VECM. The VECM accounts for both the short-term dynamics and the long-term equilibrium relationship among the variables.

3. Model Selection

Based on the results of the stationarity and cointegration tests, you can decide between VAR and VECM.

Vector Autoregressive (VAR) Model

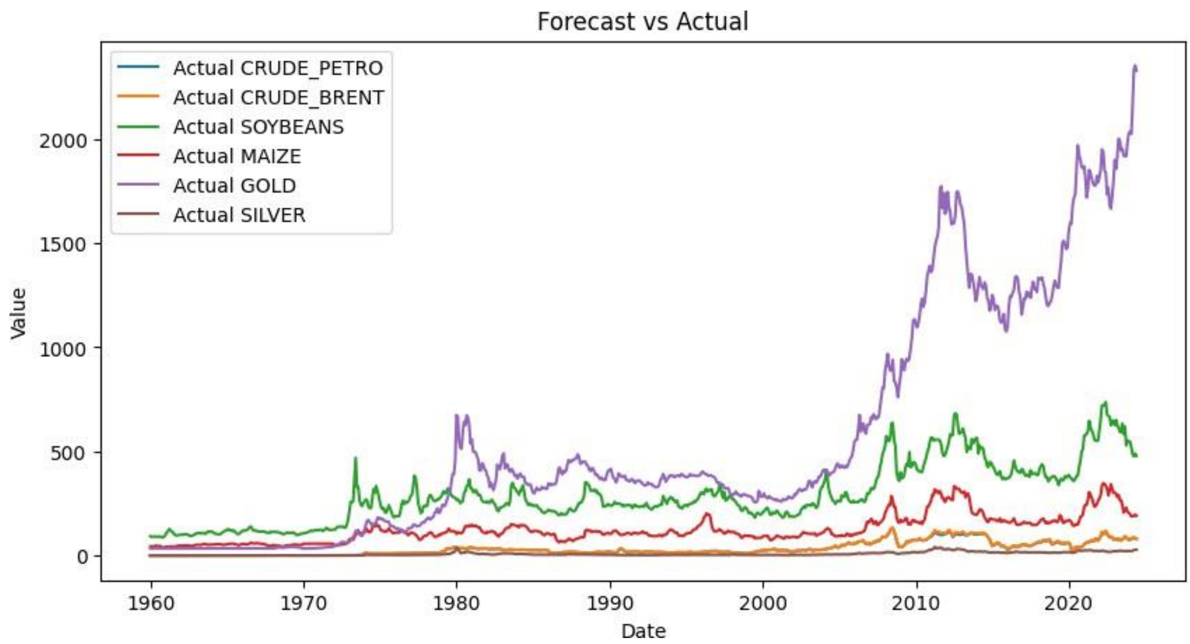
Use When: The variables are stationary or made stationary through differencing, and there is no cointegration among them. Description: A VAR model captures the linear interdependencies among multiple time series. It models each variable as a linear function of its past values and the past values of other variables in the system.

Vector Error Correction Model (VECM)

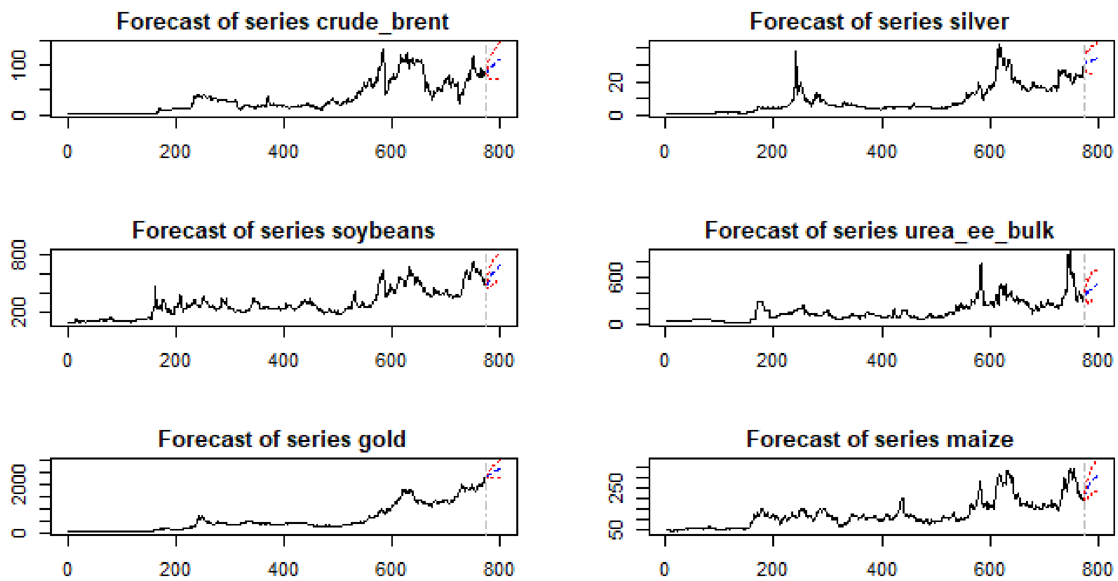
Use When: The variables are non-stationary, and there is evidence of cointegration. Description: A VECM is a unique form of VAR for non-stationary series that are cointegrated. It includes an error correction term that captures the long-term equilibrium relationship, allowing the model to correct deviations from this equilibrium. Practical Considerations Economic Theory: In some cases, economic theory may suggest a long-term equilibrium relationship, making a VECM more appropriate even before formal tests. Data Considerations: The choice may also depend on data availability, frequency, and quality. For example, higher-frequency data might require differencing more often, leading to a preference for VAR in differences.

#Forecast using the fitted model.

USING PYTHON



USING R



INTERPRETATIONS – PART B

I provide the interpretation, analytical insights, recommendations, and conclusions based on the use of a VAR model because the **R value given is 0**.

1. Stationarity Testing:

The initial step involved checking if the time series data (CRUDE_BRENT, maise, and soybeans) were stationary using unit root tests (ADF, PP, KPSS).

The results showed that the data were non-stationary at levels but became stationary after first differencing.

2. Model Selection:

Since the data were non-stationary and no cointegration was found, a VAR model in differences (ΔVAR) was chosen.

The VAR model captures the linear interdependencies among the time series by modelling each variable as a function of its past values and the past values of the other variables in the system.

3. Post VAR Analysis:

Forecasting: The VAR model was used to make forecasts based on the interrelationships among the variables for the next three months (90 days)

Analytical Insights

Interdependencies:

The VAR model highlights the interdependencies among crude oil, maise, and soybean prices. Each variable's future values depend on its past values and the past values of the other variables.

Short-term Dynamics:

The model captures the short-term dynamics without focusing on long-term equilibrium relationships. This is particularly useful for short-term forecasting and understanding the immediate effects of shocks.

RECOMMENDATIONS – PART B

1. Regular Updates:

Continuously update the VAR model with new data to improve its accuracy and reliability in forecasting.

2. Focus on Short-term Strategies:

Use the insights from the VAR model to develop short-term strategies, particularly in sectors affected by crude oil, maise, and soybean prices.

3. Monitor Key Variables:

Closely monitor the variables that show significant Granger causality relationships, as they can serve as leading indicators for forecasting other variables.

CONCLUSION – PART B

Using a VAR model for forecasting provides valuable insights into the short-term dynamics and interdependencies among crude oil, maise, and soybean prices. The model effectively captures how past values of these commodities influence their future values, enabling stakeholders to anticipate market movements and make informed decisions. While the VAR model focuses on short-term relationships, it offers a robust framework for understanding and forecasting the immediate impacts of shocks in interconnected markets.

OVERALL CONCLUSION

This project provides a comprehensive analysis of stock volatility and commodity price dynamics using advanced econometric models. By identifying significant volatility patterns and interdependencies, it offers valuable insights for investors, policymakers, and businesses. The results underscore the importance of using sophisticated models like ARCH/GARCH, VAR, and VECM to understand and predict market behaviour, thereby enabling better decision-making and risk management in financial and commodity markets.