

DATA MINING
ASSIGNMENT – 2
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Singular Value Decomposition

SVD is based on a theorem from linear algebra which says that a rectangular matrix A can be broken down into the product of three matrices - an orthogonal matrix U , a diagonal matrix S , and the transpose of an orthogonal matrix V . The theorem is usually presented something like this:

$$A_{mn} = U_{mm} S_{mn} V^T_{nn}$$

where $U^T U = I$, $V^T V = I$; the columns of U are orthonormal eigenvectors of $A A^T$, the columns of V are orthonormal eigenvectors of $A^T A$, and S is a diagonal matrix containing the square roots of eigenvalues from U or V in descending order.

The code for SVD is given below:

```
import math
import numpy as np
from numpy import linalg as LA

def SVD(mat):
    matT = mat.transpose()
    matmatT = mat.dot(matT)
    matTmat = matT.dot(mat)
    egnvalU, egnvecU = LA.eigh(matmatT)
    egnvalV, egnvecV = LA.eigh(matTmat)
    V = np.fliplr(egnvecV)
    VT = V.transpose()
    egnvalV = egnvalV[::-1]
    S = np.zeros(mat.shape)
    for i in range(min(mat.shape)):
        S[i][i] = math.sqrt(egnvalV[i])
    U = np.dot(np.dot(mat, np.transpose(VT)), LA.pinv(S))
    return U, S, VT

def reduceDim(u,s,v,num):
    for i in range(num):
        s = np.delete(s, s.shape[1]-1, axis = 1)
        v = np.delete(v, v.shape[0]-1, axis = 0)
        v = np.delete(v, v.shape[1]-1, axis = 1)
    return u, s, v

def main():
    mat = np.random.rand(5, 5)
    u, s, v = SVD(mat)
    # print "Original = ", mat, "\n\nU = ", u, "\n\nS = ", s, "\n\nV = ", v
    U, S, V = reduceDim(u,s,v, 3)
    print "\nAfter Reduction of Dimension\n\n"
    red = np.dot(np.dot(U,S),V)
    print "U = ", u, "\n\nS = ", s, "\n\nV = ", v, "\n\nReformed = ", red

if __name__=="__main__":
    main()

~
```

Matrix of size 5*5 is taken where 5 column represents the five dimension of data and rows represent the data samples and it is decomposed in matrices U S and V as below:

```
pramod@pramod-X555LN:~$ python SVD.py
Original = [[ 0.70377037  0.81180308  0.41679331  0.98820843  0.96367014]
 [ 0.31150079  0.8514664  0.79478207  0.39829415  0.2461971 ]
 [ 0.5753268  0.01717992  0.66748664  0.95795825  0.84980515]
 [ 0.04199565  0.64147577  0.49737969  0.83868295  0.75419044]
 [ 0.9962875  0.689222  0.67109168  0.7432223  0.77645566]]

U = [[-0.53342587 -0.04383926  0.05345115  0.64996678  0.53686229]
 [-0.33508265  0.74467889 -0.05815887 -0.48017405  0.31499747]
 [-0.4306072  -0.65745709  0.0170199 -0.57863311  0.21730547]
 [-0.39118225  0.10053301  0.77788068  0.01910548 -0.48104698]
 [-0.51451954  0.03427479 -0.62319539  0.10860375 -0.5778647 ]]

S = [[ 3.31810092  0.  0.  0.  0.  ]
 [ 0.  0.83002312  0.  0.  0.  ]
 [ 0.  0.  0.6305025  0.  0.  ]
 [ 0.  0.  0.  0.48023182  0.  ]
 [ 0.  0.  0.  0.  0.04563613]]

V = [[-0.37870006 -0.40122297 -0.39659024 -0.53753078 -0.49938285]
 [-0.16718558  0.81358887  0.25028977 -0.32137468 -0.37973079]
 [-0.88646971  0.10092826 -0.06963362  0.37300954  0.24494895]
 [ 0.17481668  0.40805268 -0.86328314 -0.0135644  0.23977147]
 [ 0.11066087  0.01999437 -0.17310281  0.68444055 -0.69923598]]
pramod@pramod-X555LN:~$
```

The matrix after reduction of dimension in 2 is given below:

(Similarly it can be done for 100 data samples which need 100*5 sized matrix)

```
pramod@pramod-X555LN:~$ python SVD.py
```

```
After Reduction of Dimension
```

```
U = [[-0.5116032  0.06584604  0.67272944  0.34610089  0.40196484]
      [-0.4527084 -0.57978669  0.1623556  -0.62016189 -0.2189575 ]
      [-0.61195147 -0.10870048 -0.5581509   0.49010458 -0.24892706]
      [-0.19865099  0.64162864  0.26546758 -0.14670516 -0.67591052]
      [-0.34550029  0.48580714 -0.3729213  -0.4836202   0.52121222]]
```

```
S = [[ 2.47387804  0.          0.          0.          0.          ]
      [ 0.          0.76159161  0.          0.          0.          ]
      [ 0.          0.          0.43480272  0.          0.          ]
      [ 0.          0.          0.          0.30319771  0.          ]
      [ 0.          0.          0.          0.          0.13135979]]
```

```
V = [[-0.38422109 -0.55935335 -0.55524804 -0.23254048 -0.42085927]
      [ 0.64048035 -0.35222782 -0.48834575  0.12016862  0.46130079]
      [-0.05623828 -0.1649619   0.2388402  -0.85429198  0.4275107 ]
      [-0.63810233  0.1950449  -0.32719985  0.22772386  0.62917804]
      [ 0.17838653  0.70555258 -0.5376929  -0.38705489 -0.17733768]]
```

```
Reformed = [[ 0.51840576  0.69027872]
              [ 0.14749569  0.78197514]
              [ 0.52864733  0.87596061]
              [ 0.50179744  0.10276838]
              [ 0.56537277  0.34777403]]
```

```
pramod@pramod-X555LN:~$
```