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Problem Set 3
Tutorial

In the spirit of Numerical Recipes in C, write a tutorial on this reasoning technique, such that an interested robotics student could apply it on a scenario of their choosing. Be precise about the inputs and outputs and how the algorithm works, but convey intuitions over mathematics.

Background of our robot problem:
Captain Fridge is responsible for managing food options for the owner that are stored in the fridge. The robot takes orders from the master for food items, gives the master alternate options if the asked item is currently not available or running low on supply and fetches final items from the fridge to the master. Over time it learns master's preference for food items and preferred combinations of food items, drinks and dessert and so on. It also learns and remembers preference of food items based on time of the day - so which food and food combinations are preferred for breakfast, lunch and dinner. While giving options to master, Captain remembers the preference of items in combination with each other.

For any particular meal, the captain knows the most preferred combination of items, however, due to lack of supplies, a highly preferred item may not be available. In this case, captain presents to its master, a set of choices that are guaranteed to have the minimum level of compromise and inversely, the maximum level of satisfaction for the master.

Since this problem entails finding a set of acceptable options that may not be the most preferred (in case all constraints are not met) but to maximize the aggregate satisfaction level for the chosen combination, we picked fuzzy constraint satisfaction technique as a reasoning technique for our project.

Fuzzy Reasoning Technique for Constraint Satisfaction Problem*:*

Constraint satisfaction problems (CSPs) are defined as a set of objects whose state must satisfy a number of constraints or limitations[1].

Lots of problems in real-life cannot be expected to have a perfect solution because the problems are over-constrained and do not lead to a feasible solution. In such a case, we relax the restrictions so that we can at least find a solution as close as possible to the actual solution. These relaxable constraints are termed as soft constraints and problem that employ such relaxation are called soft CSP.[2]

In general, in a soft CSP not all the given constraints need to be satisfied — either because all of them cannot be met, theoretically, or in practice it would require too much time to find a

[1] http://en.wikipedia.org/wiki/Constraint_satisfaction_problem
[2] http://www.constraintsolving.com/tutorials/soft-constraints-tutorial

solution, or simply, the real problem to be modeled is such that soft constraints are adequate and should be introduced, e.g. to express possible alternative requirements. In the case of a fuzzy constraint, different tuples satisfy the given constraint to a different degree.

Problems in AI such as planning and scheduling are modeled as constraint satisfaction problem. Fuzzy constraint satisfaction based approach finds the best combination of values that satisfy the requirements not in a binary truth values of yes or no but in terms of degree of satisfaction for a given category choices where the preferences are known only among a few of the choices at a time. This also entails creating a method to combine the preferences in some manner to get an aggregate degree of satisfaction.

In fact several real life problems are inconsistent and do not have a solution. These problems can be modeled using fuzzy logic. In real life when we our best choices are inconsistent, we compromise with choices that are no longer feasible or available to us with the next best choice - it may not satisfy us 100% but among the set of alternatives, we can decide to what level of satisfaction the other alternatives give us at that time.

Fuzzy constraint satisfaction problems have been found useful in modeling various realistic problems that have flexible, priority or conditional types of constraints. FCSP are an optimization type of problem where the goal is to maximize the aggregate level of satisfaction of the goal. Since optimization problems are computationally expensive to solve, several heuristics have been applied to FCSP to handle the complexity.

Background:

What do you mean by Fuzzy?

- Fuzzy logic is a superset of conventional(Boolean) logic that has been extended to handle the concept of partial truth- truth values between "completely true" and "completely false". The importance of fuzzy logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature.[3]

    i. In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
    ii. In fuzzy logic everything is a matter of degree.
    iii. Any logical system can be fuzzified
    iv. In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently , fuzzy constraint on a collection of variables
    v. Inference is viewed as a process of propagation of elastic constraints.

[3] http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol4/sbaa/report.fuzzysets.html

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

Representation of a Fuzzy Constraint Satisfaction Problem:

A FCSP is a tuple P = (X, D, C) defined by:

- A set of n variables X = {X1, ..., Xn}. These are the decision variables in our problem whose assignment we are solving.

- A set of n definition domains D = {D1, ..., Dn}, Di being the definition domain of variable Xi. Each variable can have any number of possible values allowed in its corresponding domain.

- A set of m fuzzy constraints C = {C1, ..., Cm} where each constraint C is defined as a pair (V, R):
  i. V (X) is a set for variable X for which the constraint C fuzzy restricts the values these variables can simultaneously take.

  ii. R is a fuzzy relation that represents the combinations of values that the constraint C authorizes for these variables which corresponds to the constraint satisfaction ranging from [0,1] which each assignment of these values to the variables.

Level of satisfaction of a Fuzzy Constraint

The solution of a Fuzzy constraint satisfaction problem is also fuzzy, for any instantiation, the degree of satisfaction of the final goal indicates how well the constraints are satisfied jointly. There are several way to define the degree of joint satisfaction of individual constraints using fuzzy set theory methods.

An individual constraint can be considered a membership function of a fuzzy set, where the way we define joint satisfaction of more than one constraint is the problem of defining membership function for intersection of fuzzy sets.

There are several ways to define **joint satisfaction degree** and we cover three of them here:

1. Based on conjunctive combination principle: where the joint satisfaction is the minimum satisfaction of the individual constraints.

2. Based on productive combination principle: where the joint satisfaction is the minimum satisfactions of the individual constraints.

3. Based on average combination principle where the joint satisfaction is the average satisfaction from all individual constraint.

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

3. Describe the solution approach at a high level. Illustrate how the reasoning process works on a simple example.

Lets study the FCSP technique for a simple example for illustration of the technique.

Suppose we are hosting a party and have a lunch menu to plan for a three-course meal with assorted drink. We have 3 types of appetizer, 3 types of main dish 2 types of dessert and 3 types of drink options available as shown below.

The fuzzy constraints relate 3 decision variables at a time and indicate the preferences for the values in each variable against each of the other variable.

Lunch menu (3 decision variables):

1. Appetizer in the fridge (A)
    - Frozen spring rolls (a1)
    - Frozen samosa (b1)

    A belongs to D1= [a1, b1]

2. Main dish (M)
    - Fried rice (a2)
    - Chicken curry (b2)
    - Pasta (c2)

    M belongs to D2= [a2, b2, c2]

3. Drink (D)
    - Wine (a3)
    - Coke (b3)

    T belongs to D3= [a3, b3]

Constrains (binary):

C1:

Satisfaction	A	M
1	a1 (spring rolls)	a2 (fried rice)
0.4	a1 (spring rolls)	b2 (curry)

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

0.2	a1 (spring rolls)	c2 (pasta)
0.8	b1 (samosa)	c2 (pasta)
0.5	b1 (samosa)	b2 (curry)

C2:

Satisfaction	A	D
1	a1 (spring rolls)	b3 (coke)
0.7	a1 (spring rolls)	a3 (wine)
1	b1 (samosa)	a3 (wine)
0.1	b1 (samosa)	b3 (coke)

C3:

Satisfaction	M	D
1	a2 (fried rice)	a3 (wine)
0.7	a2 (fried rice)	b3 (coke)
1	b2 (curry)	a3 (wine)
0.4	b2 (curry)	b3 (coke)
1	c2 (pasta)	b3 (coke)
0.6	c2 (pasta)	a3 (wine)

Fuzzy Solution:

It can be seen that this problem has no perfect solution - i.e. no solution with joint satisfaction level of 1. So, we need to find the next best solution which is as close to the perfect solution as possible.

Now, depending upon how joint satisfaction is defined, the assignment for each decision variable is made to maximize the joint satisfaction of the full assignment. If lets say, the joint satisfaction is a minimizing function then:

Joint satisfaction of [a1,a2,b3] = min ((a1,a2) , (a2,b3))
= min (1.0,0.7)
= 0.7 is the best solution with maximum joint satisfaction value.

Pallavi Mishra
Pramod Kandel
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16.412 – Cognitive Robotics

Highlight the important features, particularly if they are interesting or novel.

As seen in the simple demonstrative example above, the final solution depends largely on the type of joint satisfaction approach that we chose for the problem. In the example above, we took a minimum approach for joint satisfaction while combining constraints because we wanted a 'safe' or 'pessimistic' approach (we are planning for a party and want to make sure the most constrained preference is satisfied in the menu).

However, the choice of type of joint satisfaction function depends on the nature of the problem. The minimum based function is a 'rough' approach since it does not differentiate between two equally bad choices that may have better degree of satisfaction of other constraints.

The average joint satisfaction function does not differentiate between instantiations that fully violate at least one of the constraints while the product joint satisfaction function takes that into account by the virtue of zero multiplication. The definition of joint constraint satisfaction function can be modified to take into account the importance of individual constraints.

In the following sections we delve deeper into the features of our implementation that define 'good enough' solution by defining a threshold value of joint satisfaction of instantiations above which solutions are acceptable. This threshold can be defined in absolute or relative measure of the maximum possible joint satisfaction value known for the solution.

We also discuss about the algorithm that we have implemented to search through solution space (instantiation tree).

Promised capability

In general, all constraint satisfaction problems can be modeled as fuzzy constraint satisfaction problem by carefully assigning satisfaction levels to constraint related variables that denote the level of compromise required if that constraint was not met.

For example, in the simple temporal constraint satisfaction problem from one of our thought exercises, we were given the temporal constraint for starting for

- Leaving MIT (A),
- Reaching North End (B),
- Finishing lunch at North End and leaving for movie (C)
- Arriving at Boston Commons (D)
- Watch movie starting at a fixed time (E)

This problem can be modeled in form of a Fuzzy CSP and solved using our APIs. There are two decision points in the problem for choice of mode of transport for A and C. These two decision points are the decision variables for the problem. Each variable has a domain that represents the choices available for every variable assignment. In this case, the two variables have the domain {walk, mbta, hubway, uber}.

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

The constraints are represented in form of set of relationships between assignment values for the variables. The relationship represents the level of satisfaction of the constraint for the assigned values of each variable. In this case, the level of satisfaction is the inverse of the cost function associated with the two choices:

□□	□□	□□□+□□□	□1+□2	□□(□□)+□□(□□)	Cost Saved (11 - x)	Satisfaction Value (normalized cost)
walk	walk	65	35	0	11	1
walk	Hubway	55	25	3	8	0.727272727
walk	MBTA	″	″	2	9	0.818181818
walk	Uber	″	″	5	6	0.545454545
Hubway	walk	45	15	3	8	0.727272727
Hubway	Hubway	35	5	6	5	0.454545455
Hubway	MBTA	″	″	5	6	0.545454545
Hubway	Uber	″	″	8	3	0.272727273
MBTA	walk	50	20	2	9	0.818181818
MBTA	Hubway	40	10	5	6	0.545454545
MBTA	MBTA	″	″	4	7	0.636363636
MBTA	Uber	″	″	7	4	0.363636364
Uber	walk	35	5	6	5	0.454545455
Uber	Hubway	25	−5	9	2	0.181818182
Uber	MBTA	″	″	8	3	0.272727273
Uber	Uber	″	″	11	0	0

Once we map the values for each variable in relation to the other variables, we can solve this problem as an optimization problem to find an assignment for each variable that has the maximum satisfaction value for the final solution.

Pallavi Mishra
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16.412 – Cognitive Robotics

Generalizing Fuzzy CSPs:

In fuzzy CSPs, a satisfaction level of 1 means fully compatible choices of variables. This is equivalent to the absence of a constraint. Inversely, the satisfaction level of 0 denotes complete incompatibility. To convert a Constraint Satisfaction Problem into a Fuzzy CSP, instead of zero, a value between 0 and 1 is assigned to the relationship to depict the level of compromise required while choosing the corresponding values for the decision variables.

In general, a fuzzy CSP can have several number of decision variable and several number of acceptable values is their corresponding domains.

Let's say if the n is the number of decision variables and m is the number of values in their domains (for simplicity of illustration of these concepts we are taking a uniform sized domain for all variables, in our implementation, we allow variables to take any domain size)

Problem Size depends on:

a. n - number of variables
b. m - size of their domains

Problem size can be measured as the product of the domain sizes for each variable, i.e. the number of combinations of values that could be generated as potential solutions.
In general, the problem size = m ^ n

Problem Complexity depends on:

If all the constraints present in the problem is represented as a graph where each variable is a node and the existence of an edge depicts the existence of a constraint between two nodes, then the connectivity of the FCSP is defined as the fraction of various binary combination of variables that have a non unitary satisfaction value.

As an example, if the constraints are binary, ie they express relationship between two variables at a time, and if the number of decision variables are say, N then there are: NC2 possible constraints.

There is no limitation on the number of variables that can be a part of constraints. A constraint may involve as many variables as the problem requires. Our implementation allows the user to choose the arity of constraint as required by the problem.

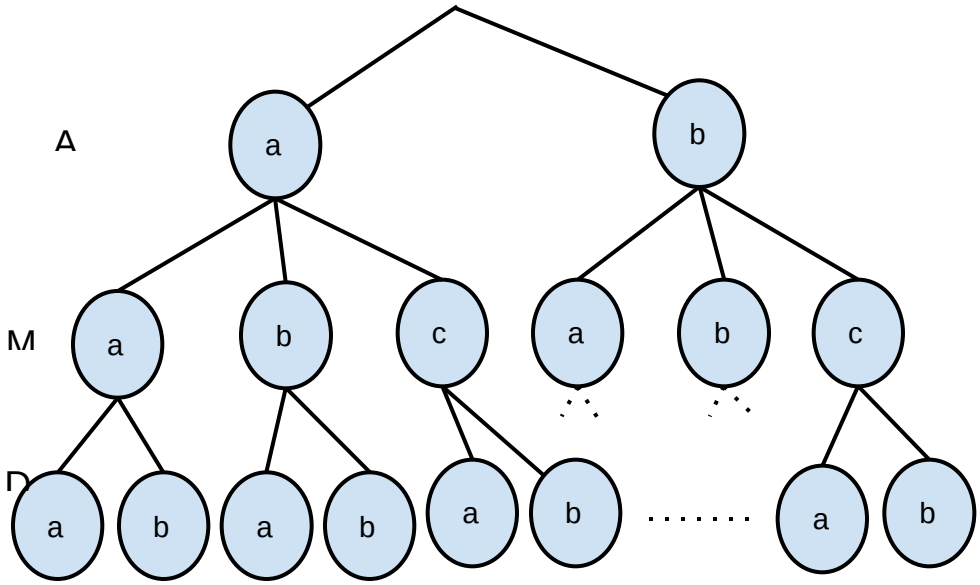
The detailed discussion for solution approaches and features in technical detail for solving FCSPs are described below. Most of the discussion is based on papers [1] and [2]:

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

Figure 0 - Search tree for the lunch problem, used by backtracking and branch-and-bound algorithms

Backtracking:

One of the basic solution approaches for solving general CSPs is backtracking. Backtracking is a tree search (usually depth-first search) over the domains of all the variables of a CSP. A tree in CSPs is a tree of domains, with each level denoting a variable, with nodes being the values in



its domain. For example, fig 0, shows the tree for our example lunch problem. The variables can be ordered arbitrarily in the tree, and that determines the order in which variables are assigned during the search.

During each assignment of a variable, the consistency of the assignment is checked with all the previously assigned variables. If the assignment is not consistent with previous assignments, the algorithm backtracks and looks for other values of the variable. In the end, backtracking either returns a consistent solution instance, if found, or continues to find other solutions.[2]

Backtracking can be extended to fuzzy problems as well, with custom definitions of "consistency" of two variables. In fuzzy CSP, whether a solution instance is consistent depends on the degree of joint satisfaction sought for the solution, so-called alpha. It may be computationally difficult to check if any two variables are consistent with one another given the

alpha for the entire solution, therefore the consistency check can be done after finding an entire solution instance.

Advantages and disadvantages of backtracking:

Backtracking, in worst case, searches over all tree space. There is no intelligence on which variables to check first or which values to assign first during the algorithm. Also, the pruning doesn't occur in the middle of the search, and the algorithm reaches at the leaf stage until it knows to backtrack.

On the other hand, it is the guaranteed way of finding either alpha solution or the best solution possible. It is simple to understand.

Implementation:

We have implemented a backtracking algorithm for finding alpha solutions, which are the instances of solutions satisfying the problem constraints with joint degree greater than or equal to alpha. We have implemented a version that checks alpha-consistency of a solution instance not at every variable assignment, but once a candidate solution instance is found. It is done so because in fuzzy problem setting, there is not as straight-forward way to check consistency of a partial soution as in the crisp CSPs. Once the solution instance is found, if its joint satisfaction degree is less than alpha, the algorithm backtracks in the tree to try to find other consistent solutions.

Pseudocode:

```
get_alpha_solution_backtracking(alpha):
   #stack keeps track of current vertex to explore and path until that vertex
   stack = [values in first level]
   vertex, path = stack.pop()
   for next in tree[vertex]:
      if tree[next] == []: #current vertex is leaf
         if_path_consistent: #joint_sat >= alpha:
            return solution # if want all solutions, do “yield” instead of return
      else:
         stack.add(next, path+[next])
```

Running backtracking on the example problem:

In our example, following are the steps for backtracking:

1. Assign A= a1
2. Assign M = a2
3. Assign D = a3

4. Check consistency of (a1, a2, a3). In our example, if we take “productive” as our joint satisfaction metric for consistency, joint_sat = sat(a1, a2) x sat(a1, a3) x sat (a2, a3) = 1x0.7x1 = 0.7. Because joint_sat > 0, this assignment is consistent.
5. Return (a1, a2, a3)
6. If choose to continue search, now backtrack and assign D = b3. Joint satisfaction of (a1, a2, b3) = 1 x 1 x 0.7 = 0.7
7. Now backtrack and assign M = b2. Continue similarly

Branch and bound:

The next natural approach to avoid searching over a lot of space is to try to prune the search tree early on in the tree if some paths are known to yield provably worse solutions than something that we know we can definitely get. One of the well known ways to do that is branch and bound algorithm. The idea is to have some kind of upper bound and/or a lower bound on the satisfaction of the solution, and prune parts of tree that provably result in worse solutions, i.e. whose upper bound of the satisfaction is less than the current lower bound.

Advantages and disadvantages:

In general, it doesn’t search over all tree space. However, it still has to finish searching all tree to find the best solution. More importantly, it doesn’t control the order of variables to assign, therefore may need to go over all possibilities. Another advantage/disadvantage comes along with the type of upper bound chosen. If it is difficult to calculate upper bound at each node, it may still take a long time.

Implementation:

In our implementation, we first find any feasible solution with one run of backtracking search, and have as the lower bound, its joint satisfaction degree. Then, we run branch and bound search. The lower bound keeps on increasing as we find better and better solutions. At any point in the search, if we know that assigning a value to a variable cannot yield a solution better than the lower bound, we do not visit that variable and onwards. We have implemented a branch and bound solution with an upper bound for a variable assignment called “appropriateness” described in [1] (also described later on in “heuristics search” topic), as well as the upper bound called “joint partial satisfaction” that we came up with (also described later on in “heuristics search”). However, choice of upper bound/lower bound is not fixed in branch and bound.

Pseudocode:

```
get_best_solution_branch_n_bound():
    #stack keeps track of current vertex to explore and path until that vertex
    stack = [values in first level]
    vertex, path = stack.pop()
    feasible_solution = get_a_feasible_solution_backtracking()
    lower_bound = joint_satisfaction(feasible_solution)
```

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

```
for next in tree[vertex]:
    partial_assignment = path + [next]
    upper_bound = get_upper_bound(partial_assignment)
    if upper_bound < lower_bound:
        continue
    if tree[next] == []: # vertex is leaf
        joint_sat = get_joint_satisfaction(path)
        if joint_sat > lower_bound:
            lower_bound = joint_sat #update current lower bound
    else:
        stack.add(next, path+[next])
```

Run on example problem:

1. Get a feasible solution from backtracking. Let's say it returns (a1, b2, b3) with joint_sat = 1 x 0.4 x 0.4 = 0.16. So, lower_bound = 0.16
2. Assign A = a1.
3. Try M = a2. Get upper bound on resulting solutions with partial assignment (A=a1, M=a2). With "appropriateness" measure, upper_bound = 1.0
4. Assign M = a2 because upper_bound > lower_bound. If that was not the case, algorithm would backtrack and assign M = b2
5. Try D = a3. Get upper bound on solution (a1, a2, a3) which is a tight bound, exactly equal to joint satisfaction, i.e. 0.7
6. Update lower bound to 0.7, and continue search, now checking for (a1, a2, b3)

Branch and bound can be implemented to find the best solution, as in the pseudocode and the example run above, or to find an alpha solution. In case of alpha solution, the lower bound is alpha to start with, and as soon as we find the first alpha solution, we return the solution. So, the lower bound is fixed in this case.

Heuristic search:

One disadvantage of backtracking and branch and bound are that their performance is dependent on how the variables are ordered and how the assignments for each variable are ordered in the search tree. In the worst case, both of them have to search over the entire problem space, and look at all possible solutions.

Heuristic search is an approach to guide the selection of the variables in such a way that we find a good enough solution, if not the best solution, without having to search over all possibilities exhaustively. Heuristic searches are also used in the general CSPs, and the basic idea is to assign the most constrained variables first with the values that are least constraining. For example, if a person only highly prefers wine for drinks over coke or mango lassi or any other values, and for variables other than drinks, he is more flexible, then the variable "drinks" is

assigned first, and it is assigned with the least constraining value, which is wine. Progressively, this approach is applied to other variables.

Implementation:

The implementation of our heuristic search is based on the approaches described in [1]. The paper uses the metrics “appropriateness” and “difficulty” to guide the variable assignments.

Appropriateness of a value of a variable:

Taking directly from the paper, "the appropriateness of a value $v \in D_i$ for a variable x_i is evaluated on the basis of the degree of the best possible joint satisfaction of the constraints referring to x_i." To find best joint satisfaction, we should fix $x_i = D_i$ and find the max of joint satisfaction over all possible solution instances with x_i fixed.

In our example, to find appropriateness of value a2 for the variable M, we follow these steps:

1) Find all possible instances with M=a2. These instances are: (a1, a2, a3), (a1, a2, b3), (b1,a2, a3), (b1, a2, b3)
2) Eliminate all constraints except the ones referring to the variable M, i.e. eliminate C2.
3) For C1 and C3, the best joint satisfaction is by (a1, a2, a3) is , with joint_sat = 1.0.
4) Therefore, appropriateness for M=a2 is 1.0

In a similar way, we can find the appropriateness of a value assignments with other values fixed. For example, fixing A=a1, the appropriateness of M=a2 is found by getting best joint satisfaction of all constraints containing the variables A and M, i.e. C1. The best joint satisfaction is 1, which is the only joint satisfaction in this case because our example has binary constraints with 2 variables, and a pair of variables has only 1 corresponding constraint.

Difficulty of a variable:

Difficulty of a variable is simply the sum of appropriateness of each of its values. In our example, difficulty of variable M = appropriateness(a2) + appropriateness(b2) = 1.0 + 0.5 = 1.5

Partial joint satisfaction metric:

This is a metric we came up with, while discussing on the upper bound of a partial instantiation during branch and bound search. The paper [1] suggests using “appropriateness” as an upper bound for a partial instantiation while assigning a variable, but we thought we can find a more efficient metric. The metric is similar to appropriateness, but while calculation, we don’t need to enumerate all possible instantiations for the purpose of finding the best joint satisfaction.

This is a step-by-step way of calculating this metric, for given partial assignment (x1 =v1,x2= v2,x3= v3, …):

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

1) If length of partial assignment is less than the number of variables per constraint, partial_joint_sat = 1.0 (in case of binary FCSP, metric = 1.0 when only one variable is assigned).
2) If length >= number of variables per constraint, find all the constraints with the combinations of those variables.
3) Then, take values of satisfaction of each of the constraint, and find the joint satisfaction of those values. For example, if joint satisfaction approach is "productive", multiply all those values. This is the metric

Now, given all these metrics, let's discuss about the heuristic search algorithm.

The steps of the algorithm are:

a. For each variable left to be assigned (given a list of already assigned variables) find the difficulty. We get a list of difficulty of all unassigned variables given already assigned ones.
b. Choose the variable with the least difficulty to assign a value
c. Within that chosen variable, find the value that has the highest appropriateness
d. Assign that value to the variable
e. Go to step a., fixing the current chosen variable

<u>Running heuristic search in our example:</u>

a. Find difficulty and appropriateness for each variable and values (notation: $a_x(v)$ = appropriateness of value v for value v, d_x = difficulty of variable x):
   $a_A(a1)=1$, $a_A(b1)=0.8$, and thus $d_A=1.8$;
   $a_M(a2)=1$, $a_M(b2)=0.5$, $a_M(c2)=0.8$, and thus $d_M=2.3$;
   $a_D(a3)=1$, $a_D(b3)=1$, and thus $d_D=2$.

b. Assign the least difficult (= highly constrained) variable A, with least constraining assignment a1. So, A = a1.
c. Given A=a1, find difficulty of variables B and C with appropriateness of respective values (some appropriateness values decrease as a result of A=a1):
   $a_M(a2)=1$, $a_M(b2)=0.4$, $a_M(c2)=0.2$, and thus $d_M=1.6$;
   $a_D(a3)=1$, $a_D(b3)=0.7$, and thus $d_D=1.7$.

d. Assign M=a2.
e. Now, given A = a1 and M = a2, find appropriateness of values of D.
   $a_D(a3)=0.7$, $a_D(b3)=0.7$
f. Therefore, we can choose any value for D. Say we choose D = a3.
g. Final assignment is (a1, a2, a3)

Pallavi Mishra
Pramod Kandel
Manushaqe Muco
16.412 – Cognitive Robotics

Benchmarking results

Benchmark requirements

We measured the performance of Backtrack, Branch and Bound and Heuristics Search algorithms against problems differing in size and complexity as referenced from [3]. Section A discusses our benchmarking result against problem size and complexity. Section B discusses comparison of our algorithms.

Performance Metrics for comparison:

- Number of constraint table lookups (constraint checks):
  - Every time the algorithm queries the constraint table for the satisfaction degree of an instantiation, it incurs a cost of lookup. This metric serves a good measure for algorithm performance.
- Search nodes:
  - Every time an algorithm assigns a value to a variable, it is equivalent of expanding the search node in the search tree except for the last instantiation. This serves a good measure of performance for tree based search algorithms like backtracking and branch and bound.
- Runtime
  - Running time for the particular algorithm on a relative scale.

Problem Characterization (For varying problem size and complexity):

1. Problem Size:

Problem size can be measured as the product of the domain sizes for each variable, i.e. the number of combinations of values that could be generated as potential solutions.

1. n - number of variables
2. m - size of domains (fixed for now)

Problem size = m ^ n
This metric corresponds to the size of the search tree created for tree-based algorithms.

2. Problem Complexity:

We characterized problem complexity using two features of the constraint satisfaction problem:

1. Connectivity of the constraints:

The connectivity of the constraint graph that represents the problem is the proportion of variables that are related by a constraint. We vary this parameter in steps of 20% from not connected to fully connected. For the sake of simplicity, we only use binary constraints.
If a graph is 50% connected this implies that half of the variables in the problem have constraints defined between them.
Since there is no constraint between the unconnected variables, we assign the satisfaction value of 1 for all the constraints between the unconnected variables for the problem.

A fully connected problem is therefore the most complex to find the solution assignments as each pair of variables have to be satisfied with the instantiation of each of the other variables.

2. Tightness of the constraints:

Tightness for a given problem is the proportion of assignment of variables disallowed by each constraint.

Suppose the size of a domain (m) = 6 then number of tuples in a binary constraint = m*m = 36
(a1,b1), (a1,b2), … (a2,b1),(a2,b2) … (a6,b6) = 36 values in C1

If tightness == 0.1 => 0.1*36 = 4 tuples are missing.
If tightness == 0.9 => 0.9*36 = 32 tuples are missing.

The characteristic of tightness relates to the difficulty of finding solution to a given FCSP since the solution space becomes narrower as tightness increases.

Section A – Performance of APIs with Problem Size and Complexity.

A.1 Analysis of increase in problem size

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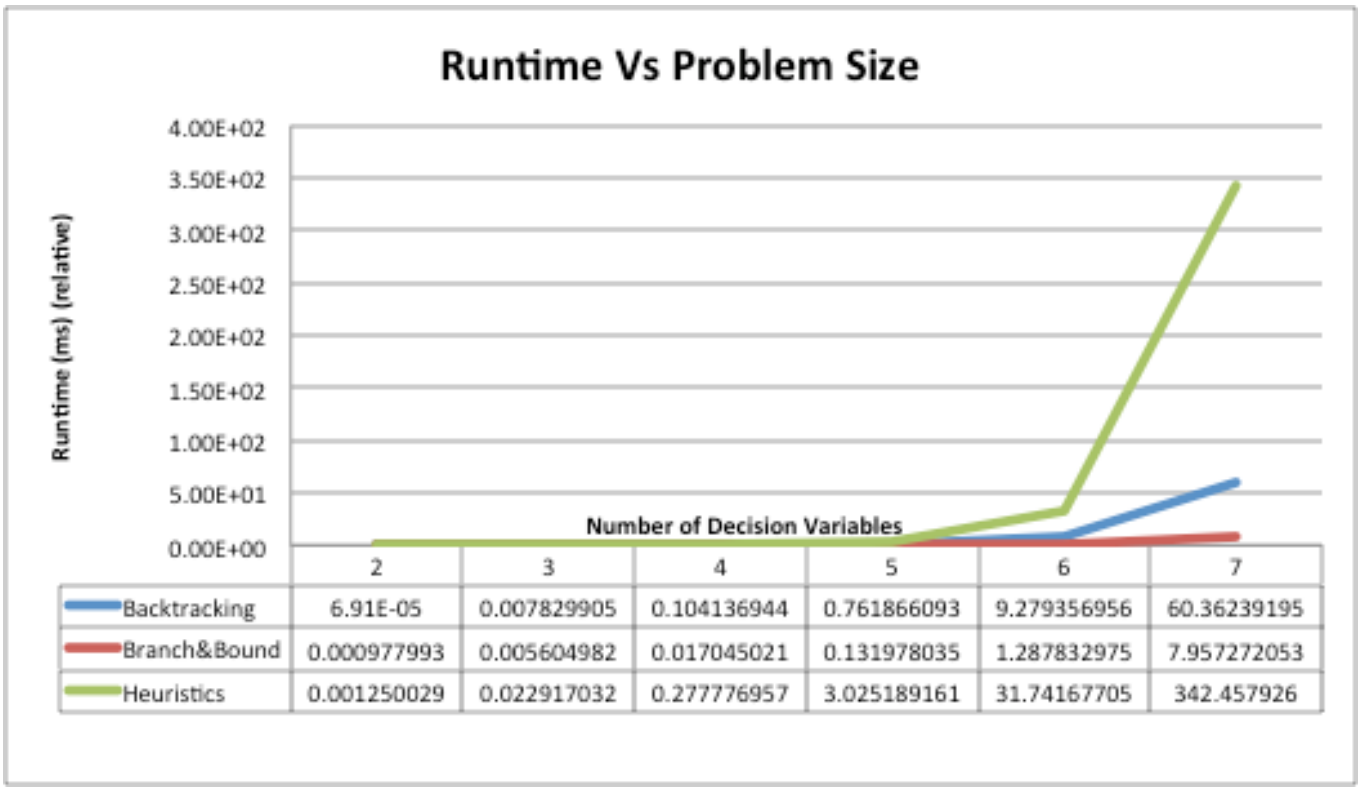

	2	3	4	5	6	7
Backtracking	6.91E-05	0.007829905	0.104136944	0.761866093	9.279356956	60.36239195
Branch&Bound	0.000977993	0.005604982	0.017045021	0.131978035	1.287832975	7.957272053
Heuristics	0.001250029	0.022917032	0.277776957	3.025189161	31.74167705	342.457926

Figure 1 - Benchmarks results for performance of all algorithms relative to problem size.

As seen in Figure 1, the runtimes for Backtracking and Branch&Bound algorithms are more resilient with increasing problem size. We increase the problem size by increasing the number of decision variables and therefore increasing the search space of the problem space of the problem. Since heuristics search front-loads the work of calculation of appropriateness and difficulty in order to pick the variables in the correct order, it has the worst scaling effect with increasing problem size. On the other hand, tree based algorithms do better compared to heuristics search as only the required number of nodes are expanded to find the best solution. Branch&Bound scales the best because of the pruning advantage whereby it chops off a section of the search-space when the joint satisfaction of the partial solution belonging to that sub tree falls below the lower bound.

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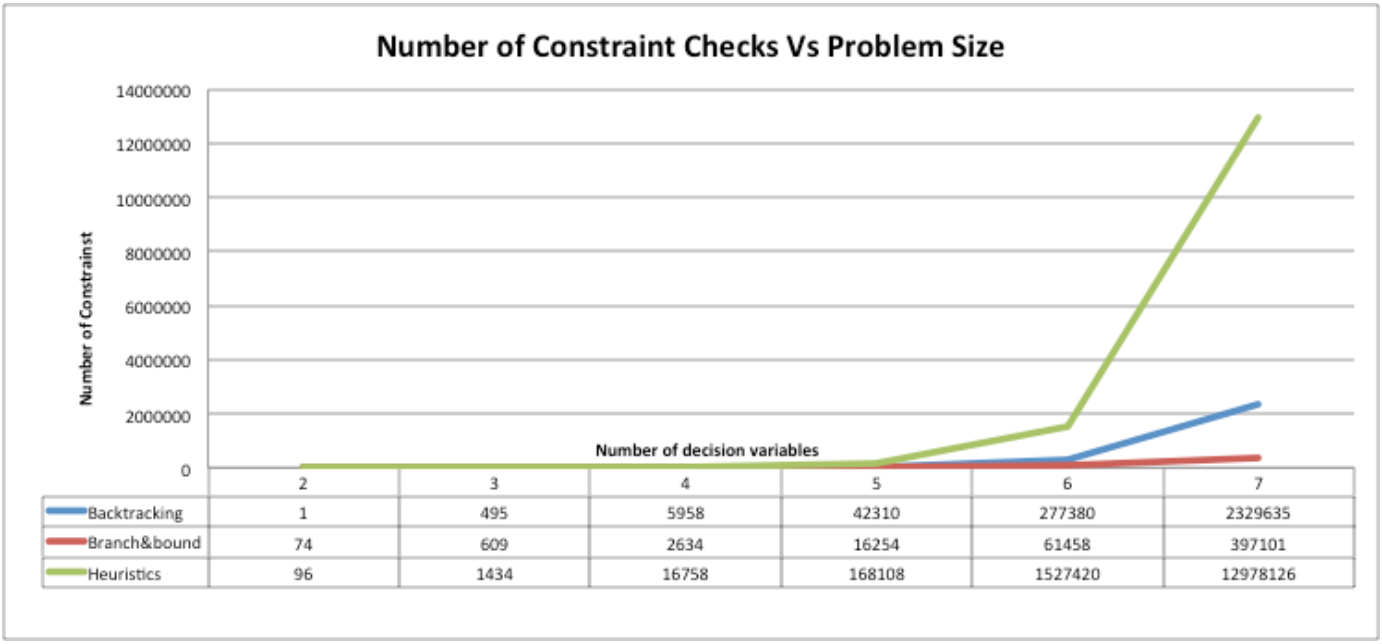

Figure 2 - Performance comparison of the APIs with increasing problem size

The number of lookups in the constraint table is directly proportional to the performance of the algorithms as seen when the Figure 1 and 2 are compared. The relative graphs of all algorithms are the same in both the cases. Since the number of lookups increase with increasing search-space of the problem, algorithms that search the entire solution space like heuristics search have the maximum increase in constraint table lookups.

Figure 3 shows the relationship between increase in problem size and the number of variable assignments among all the algorithms implemented. It is not surprising to see that the number of variable assignments for heuristics search is the same as the number of variables in the problem as all computation for variable ordering and value ordering is done as part of preprocessing the heuristics value. On the other hand, the tree based algorithms of backtracking and branch and bound show a linear increase in variable assignments as the search tree size increases. Clearly, backtracking is less optimal than branch and bound due to the advantage of pruning in the later case.

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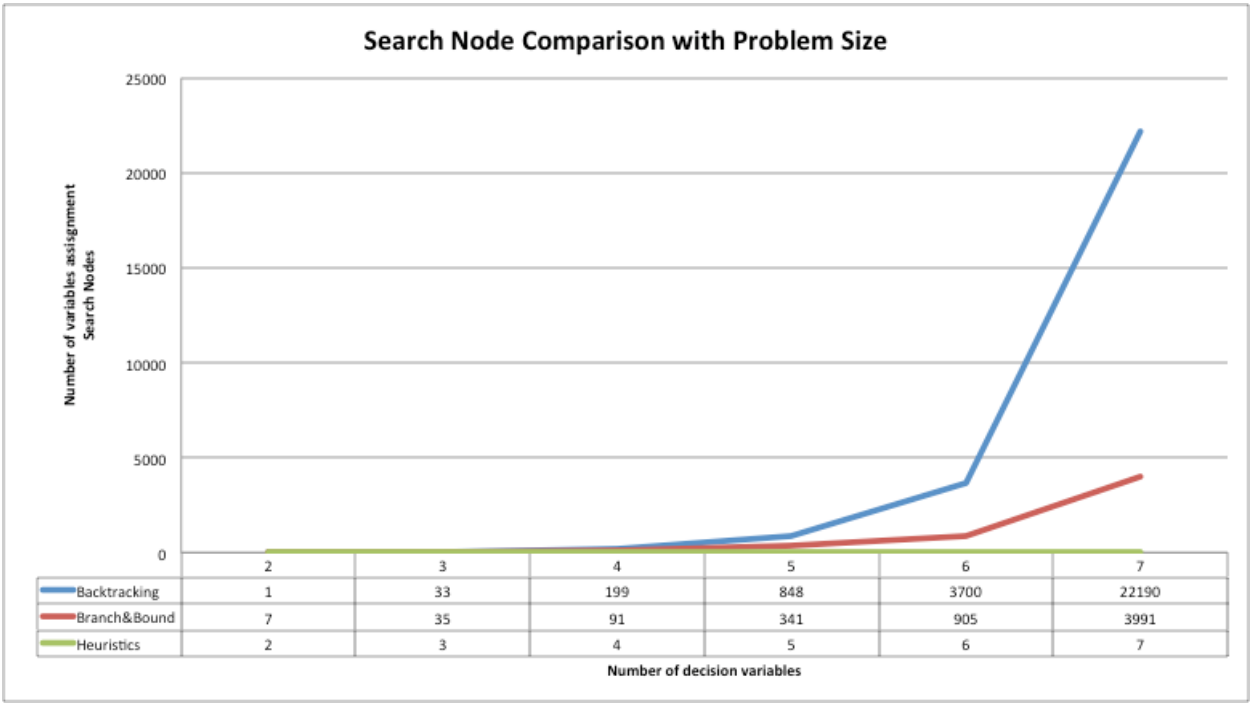

	2	3	4	5	6	7
Backtracking	1	33	199	848	3700	22190
Branch&Bound	7	35	91	341	905	3991
Heuristics	2	3	4	5	6	7

Figure 3 - Comparison of search nodes with increasing problem size

A.2 Analysis of increase in problem complexity

As discussed in the beginning of the benchmarking section, we have found two suitable parameters for characterizing complexity of the problem. Connectivity is the connectivity of the graph representation of the constraint satisfaction graph of the problem. The more connected the graph is – the more relationships exists between the decision variables. In general, the branch and bound implementation scales well with increasing complexity of the problem and pruning of unnecessary variable checks lead to better results as complexity increase with a constant problem size. Heuristics search as expected has no relationship with connectivity of the problem as it checks for variable ordering by checking all possible relationships in the problem before it assigns a variable. Backtracking performs well with increasing complexity since the search for greater than alpha solution leads to termination of the search before the entire search tree is visited.

We also measure the performance of the implementation with respect to the ‘tightness’ of the problem. The tighter a problem is the narrower is the possible solution space compared to the entire solution space. This property is exploited by branch and bound as well as backtracking with alpha-best solution, however, heuristics search remains unaffected by the tightness except for a bump seen in Figure 5 – where the tradeoff for frontloading computation overshoots the variable ordering based optimal partial instantiation and in this case, it has to backtrack several times to recalculate the ordering. The increase in number of constraint checks with a constant problem size is therefore seen high around the 50% tightness ratio.

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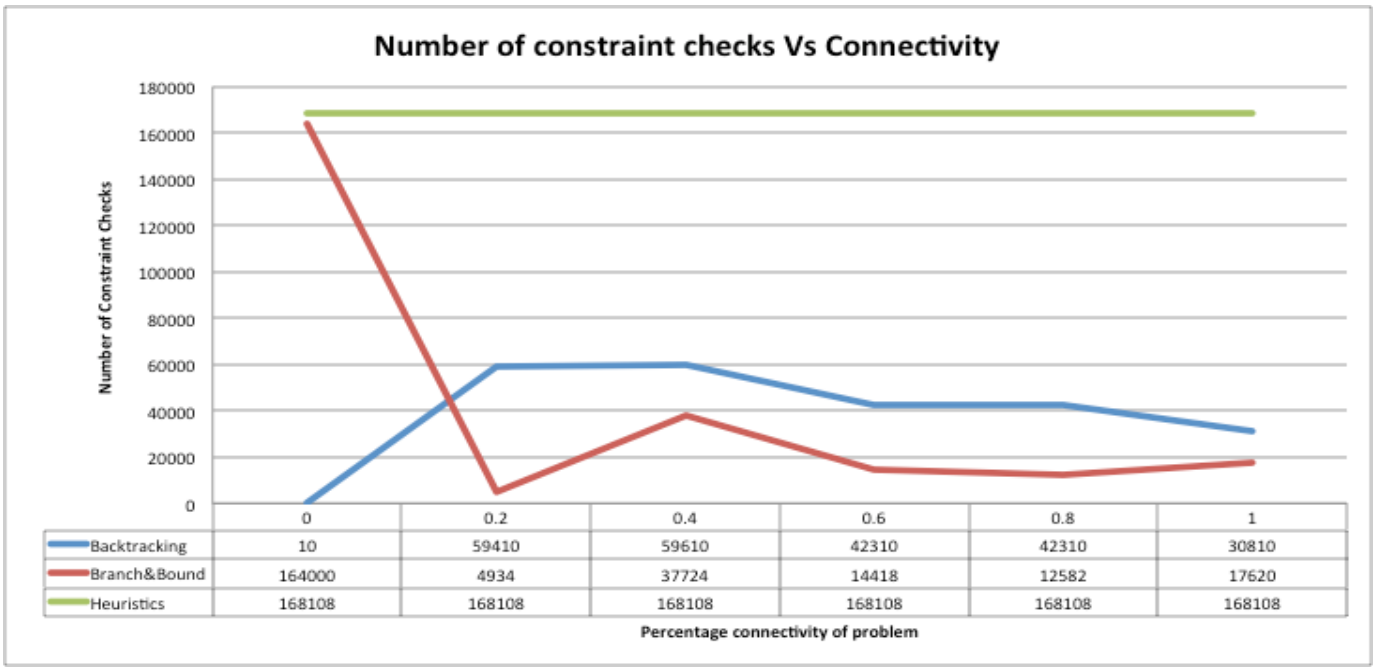
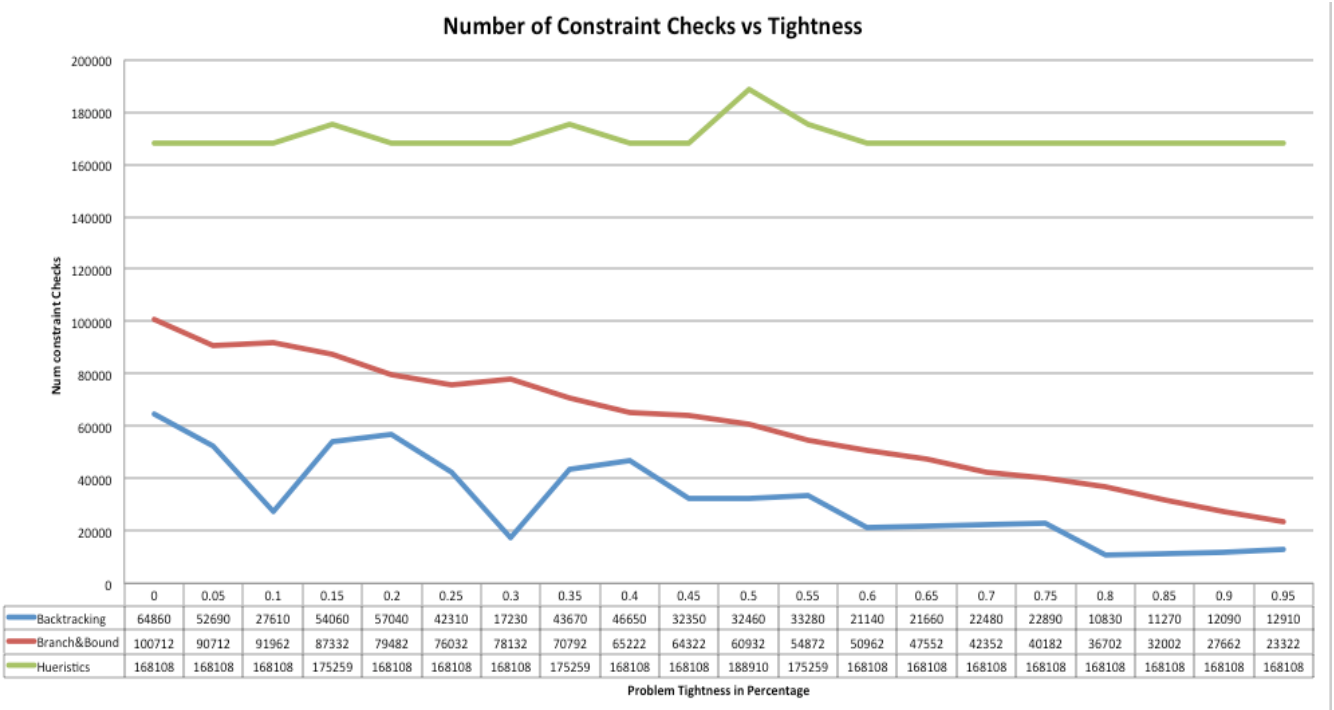


	0	0.2	0.4	0.6	0.8	1
Backtracking	10	59410	59610	42310	42310	30810
Branch&Bound	164000	4934	37724	14418	12582	17620
Heuristics	168108	168108	168108	168108	168108	168108

Figure 4 - Performance comparison with connectivity of the constraint graph



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Backtracking	64860	52690	27610	54060	57040	42310	17230	43670	46650	32350	32460	33280	21140	21660	22480	22890	10830	11270	12090	12910
Branch&Bound	100712	90712	91962	87332	79482	76032	78132	70792	65222	64322	60932	54872	50962	47552	42352	40182	36702	32002	27662	23322
Hueristics	168108	168108	168108	175259	168108	168108	168108	175259	168108	168108	188910	175259	168108	168108	168108	168108	168108	168108	168108	168108

Figure 5 - Performance comparison with narrowing solution space (tightness) within a problem of constant size

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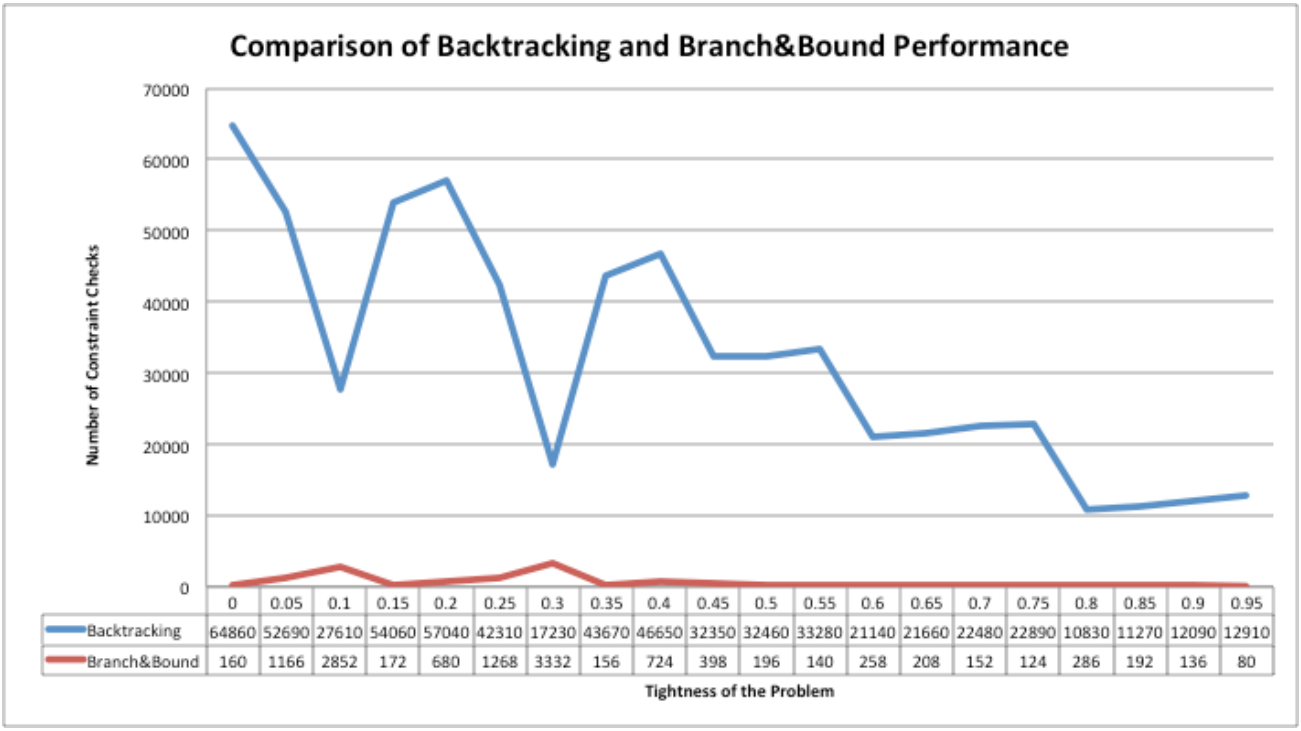
Section B – Comparison of Performance of implemented Algorithms



	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Backtracking	64860	52690	27610	54060	57040	42310	17230	43670	46650	32350	32460	33280	21140	21660	22480	22890	10830	11270	12090	12910
Branch&Bound	160	1166	2852	172	680	1268	3332	156	724	398	196	140	258	208	152	124	286	192	136	80

Figure 5 – Comparison of backtracking and branch&bound with problem complexity

We compare two similar algorithms we have implemented that use tree based search to find the best solution or the best alpha solution. The performance of branch&bound is significantly better than backtracking due to pruning sub-optimal sub trees and reducing the number of lookups. However, backtracking searches for the entire tree in the worst case. As the narrowness of the solution space within the problem space increases, branch and bound is able to utilize the fact to prune the infeasible parts of the tree effectively increasing performance whereas backtracking lacks that feature and still may end up searching the entire tree for best alpha solution.

We compare the features of heuristic search with branch and bound in the Figure 7 shown below. As seen before, pruning in branch and bound decreases the search space effectively – heuristics search however, is not able to utilize the narrow search space of the problem to its advantage and still front-loads all the computation regardless of the nature of the problem.

In figure 8, we analyze the characteristics of various algorithms by comparing the number of variable assignments done or the number of search nodes expanded during the execution of the algorithm. As seen in the graph, heuristics search is the most efficient in this case as it orders the variable before solving the problem and it is able to get the correct order based on the computation for appropriateness for every value.

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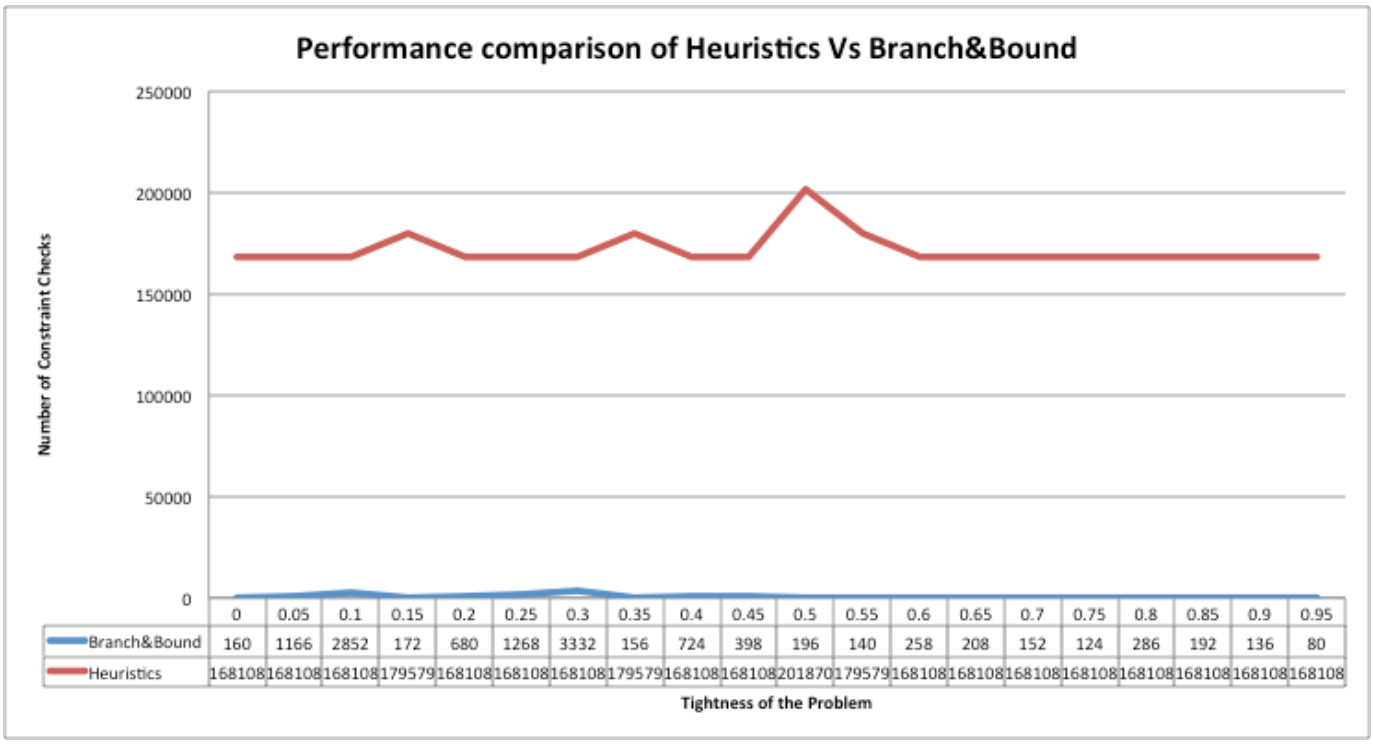
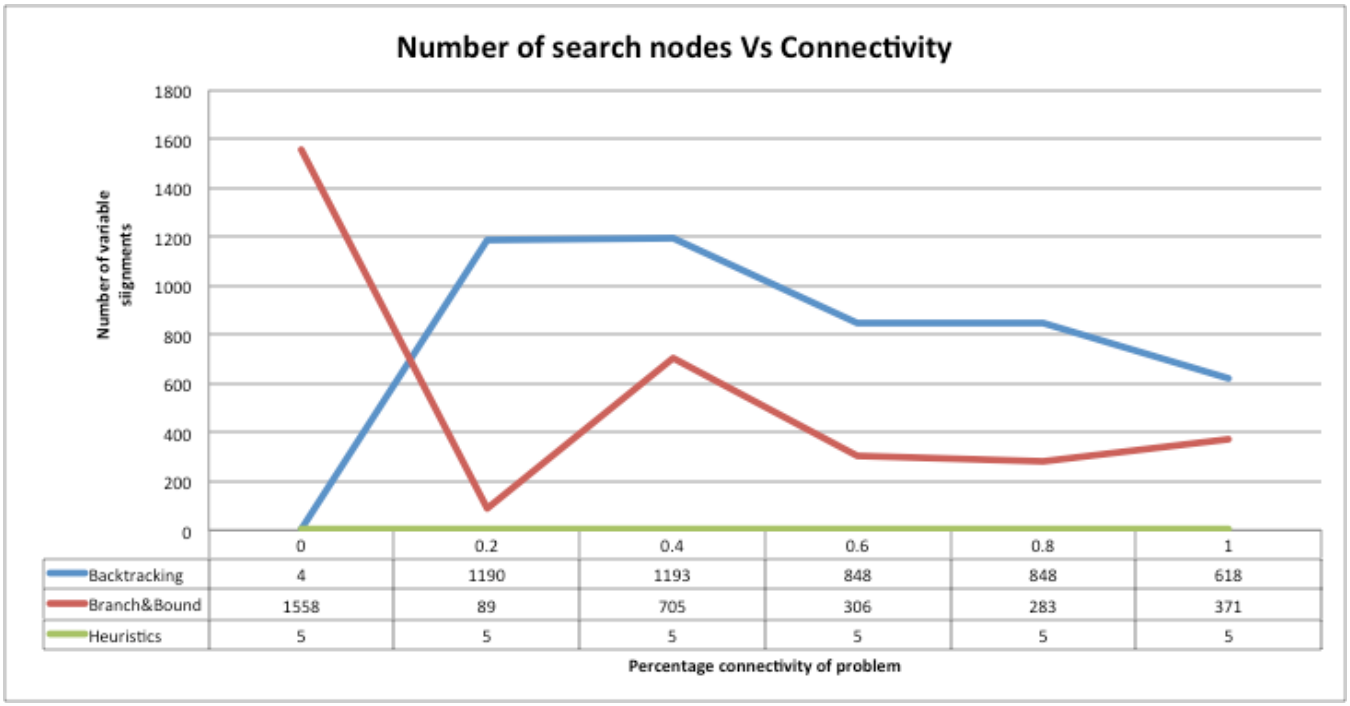


	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
Branch&Bound	160	1166	2852	172	680	1268	3332	156	724	398	196	140	258	208	152	124	286	192	136	80
Heuristics	168108	168108	168108	179579	168108	168108	168108	179579	168108	168108	201870	179579	168108	168108	168108	168108	168108	168108	168108	168108

Figure 6 - Analyzing the behavior of heuristics search and branch&bound with problem complexity



	0	0.2	0.4	0.6	0.8	1
Backtracking	4	1190	1193	848	848	618
Branch&Bound	1558	89	705	306	283	371
Heuristics	5	5	5	5	5	5

Figure 7 - Analyzing characteristics of algorithm based on problem connectivity

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