Colorado CSCI 5454: Algorithms
Homework 4

Instructor: Bo Waggoner
Due: September 26, 2019 at 11:59pm
Turn in electronically via Gradescope.

Remember to list people you worked with and any outside sources used.

Problem 1 (6 points)

Consider the initial idea we discussed for a max-flow algorithm: Find a path from s to t where the flow $f(u, v)$ is less than the capacity $c(u, v)$ for every edge (u, v) along the path, and increase the flow on these edges. Repeat until there is no such path.

Part a (4 points) Give an example showing how this algorithm can fail to find the max flow. Describe the algorithm's solution as well as the correct max flow solution. (Try to make the example as small and concise as possible.)

Part b (2 points) Briefly explain why a correct max-flow algorithm can avoid getting stuck on your example in the same way.

Problem 2 (6 points)

Part a (4 points) A set of paths from s to t are *edge-disjoint* if they have no edges in common. Given an unweighted, directed graph G and vertices s, t, give an algorithm to count the largest possible set of edge-disjoint paths from s to t. Briefly argue correctness and running time. *Hint: use max flow.*

Part b (2 points) Now, with the same input, give an algorithm for finding the minimum number of edges that, if removed from the graph, will make t unreachable from s. Briefly explain correctness and running time.

Problem 3 (6 points)

Part a (4 points) Give an algorithm to determine if an unweighted, undirected graph $G = (V, E)$ is bipartite. Argue correctness and running time.
Hint 1: Use the graph search techniques we learned in lectures 2 and 3!
Hint 2: You may use the fact that if a graph has a cycle with an odd number of vertices, then it is not bipartite. For a bonus point, prove this fact.

Part b (2 points) Modify your algorithm to return the partition of the vertices, if the graph does turn out to be bipartite.