Colorado CSCI 5454: Algorithms
Homework 1

Instructor: Bo Waggoner
Due: by beginning of class, September 3, 2019
Turn in electronically via Gradescope. LaTeX(LaTeX) is preferred, but not required.

Problem 1 (6 points)

Let $f(n) = \sum_{j=1}^{n} j^{10}$.

Part a (2 points) Show that $f(n) \in O(n^{11})$. *Hint: for each term in the sum, $j \leq n$.*

Part b (2 points) Show that $f(n) \in \Omega(n^{11})$. *Hint: only consider the second half of the terms.* We conclude that $f(n) \in \Theta(n^{11})$.

Part c (2 points) Argue that $f(n) \in o(n^{12})$.

Problem 2 (4 points)

In the course notes, it is proven that the Deferred Acceptance algorithm with n doctors and n hospitals (there referred to as men and women) always terminates within $O(n^2)$ rounds, where there is a single proposal in each round. Describe a family of instances where the algorithm takes $\Omega(n^2)$ rounds to terminate.

For any n, you should explain how to construct an input of size n and why the Deferred Acceptance algorithm requires $\Omega(n^2)$ rounds to terminate. You may specify exactly how the algorithm chooses the next proposal in each round in order to make the running time as bad as possible.

Problem 3 (6 points)

We are given an undirected, unweighted graph $G = (V, E)$ and a particular edge $e = (u, v)$. We must give an algorithm to determine whether G has any cycle that contains e. For this problem, we assume all paths and cycles are *simple*, i.e. contain no repeated edges.

Part a (2 points) Prove that G has a cycle containing e if and only if, when we remove e from the graph, there exists a path from u to v. *Hint: First write down the precise definitions of path and cycle.*

Part b (4 points) Design a linear-time algorithm (i.e. running time $O(|V| + |E|)$) to determine where G has any cycle containing a given edge e. You should argue both correctness and running time.