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Aquila Optimizer: A novel meta-heuristic optimization algorithm

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A R T I C L E I N F O

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A B S T R A C T

This paper proposes a novel population-based optimization method, called Aquila Optimizer (AO), which is inspired by the Aquila's behaviors in nature during the process of catching the prey. Hence, the optimization procedures of the proposed AO algorithm are represented in four methods; selecting the search space by high soar with the vertical stoop, exploring within a diverge search space by contour flight with short glide attack, exploiting within a converge search space by low flight with slow descent attack, and swooping by walk and grab prey. To validate the new optimizer's ability to find the optimal solution for different optimization problems, a set of experimental series is conducted. For example, during the first experiment, AO is applied to find the solution of well-known 23 functions. The second and third experimental series aims to evaluate the AO's performance to find solutions for more complex problems such as thirty CEC2017 test functions and ten CEC2019 test functions, respectively. Finally, a set of seven real-world engineering problems are used. From the experimental results of AO that compared with well-known meta-heuristic methods, the superiority of the developed AO algorithm is observed. Matlab codes of AO are available at https://www.mathworks.com/matlabcentral/fileexchange/89381-aquila-optimizer-a-meta-heuristic-optimization-algorithm and Java codes are available at https://www.mathworks.com/matlabcentral/fileexchange/89386-aquila-optimizer-a-meta-heuristic-optimization-algorithm.

1. Introduction

An optimization process refers to find the optimal values for specific parameters of a system to fulfill the system design at the lowest cost (Hajipour, Kheirkhah, Tavana, & Absi, 2015). Generally, real-world applications and problems in artificial intelligence and machine learning have a discrete, unconstrained, or discrete nature (Hajipour, Mehdizadeh, & Tavakkoli-Moghaddam, 2014). Accordingly, employing traditional mathematical based programming methods is hard to find the optimal solutions (Wu, 2016). Therefore, in all science fields, we can find optimization problems. In recent decades we witnessed various optimization algorithms have been proposed to improve different systems performance and reduce computation cost. The traditional optimization methods suffer from certain shortcomings and limitations, for example, converging to local optima and unknown search space. Besides, they have only a single-based solution (Hashim, Houssein, Mabrouk, Al-Atabany, & Mirjalili, 2019). To solve these shortcomings, in recent years, many new optimization methods have been proposed. They have been applied to solve various problems.

Meta-heuristic (MH) algorithms have received wide attention and have been employed to solve various optimization problems. In general, we can categorize MH into four categories, swarm intelligence (SI) algorithms, Evolutionary Algorithms (EA), and Physics-based algorithms (PhA), and Human-based algorithms. SI algorithms include a group of algorithms inspired by swarms and animals' social behaviors, such as (Dorigo, Maniezzo, & Colorni, 1996; Eberhart & Kennedy, 1995;

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Karaboga & Basturk, 2007). The EA algorithms are proposed by simulating biological evolutionary behaviors, for example, mutation, crossover, and selections, such as (Holland, 1992). PhA algorithms are inspired by physical laws, such as (Mirjalili, Mirjalili, & Hatamlou, 2016; Rashedi, Nezamabadi-Pour, & Saryazdi, 2009). Human-based methods are inspired by some human behavior, such as (Kumar, Kulkarni, & Satapathy, 2018).

Generally, all of these MH algorithms have standard features, such as the searching process, which in general has two phases, the first one called diversification (exploration), and the second phase called intensification (exploitation) (Abualigah, Diabat, & Geem, 2020; Salcedo-Sanz, 2016). In the first phase, the MH algorithm generates random operators to explore different search space regions. In the second phase, the optimization method tries to find the optimal solution from the search space. Therefore, an efficient MH optimization algorithm has to balance the exploration and exploitation phase tendencies to avoid trapping at local optima.

We intend to introduce a more productive and effective algorithm; this paper introduces a novel natural-inspired based meta-heuristic optimization algorithm, called Aquila Optimizer (AO). The proposed AO algorithm simulates the Aquila's behavior during hunting in which showing the actions of each step of the hunt. A set of twenty-three classical, thirty CEC2017, and ten CEC2019 test functions is used to verify the robustness and effectiveness of the proposed AO rigorously. Moreover, seven engineering design problems are used to investigate the proposed AO's effectiveness further in solving real-world problems.

The rest of this study is as follows: Section 2 describes the introduced meta-heuristic optimization algorithms' backgrounds. Section 3 illustrates the structure of the proposed AO. In Section 4, we evaluate the proposed AO with different optimization problems. Also, the comparisons with the state-of-the-art methods are presented. Finally, Section 5 concludes the study.

2. Related work

This section presents a light review of some selected MH algorithms that belong to the four mentioned types and highlight some of their applications in recent years.

2.1. Swarm Intelligence (SI) algorithms

Swarms inspire (SI) methods and animal behaviors in nature (Abualigah, 2020a). Scholars and researchers have proposed different SI algorithms. Here we highlight some of these algorithms as follows. One of the most popular SI algorithms is the Particle Swarm Optimization (PSO), which is inspired by swarm particles' natural behaviors, in which each particle represents a candidate solution. Then each particle can be updated according to the global best position and its local position (Eberhart & Kennedy, 1995). In the previous decades, the PSO has been employed to solve different problems, for example, to solve global optimization problems, document classification, image segmentation, feature selection (Shang, Zhou, & Liu, 2016), data clustering, and also other industrial and engineering problems and applications (Elsheikh & Abd Elaziz, 2019). Some ant species foraging behaviors inspire the Ant Colony Optimization (ACO) (Dorigo et al., 1996). In nature, ants deposit pheromone on the ground to mark the optimal path that needs to be followed by the colony members (Dorigo, Birattari, & Stutzle, 2006). It has received wide attention, and it has been applied in different optimization tasks. For example, data mining, vehicle routing problems, classification, feature selection, time series prediction, and others (Abualigah & Diabat, 2021).

Artificial Bee Colony (ABC) is inspired by the behaviors of the honey bee colony. It has three collections, employed bees to search for food resources, onlooker bees to choose the food sources, and scout bees to randomly search for food sources (Karaboga & Basturk, 2007). The ABC has been used in many optimization applications, such as image segmentation, global optimization, wireless sensor network, job shop scheduling problems, and others. Firefly Algorithm (FA) is inspired by flashing light of fireflies in oceans (Yang, 2009). It also received wide attentions and has been adopted in different applications, such as image processing, feature selection, and other optimization problems. In addition, there are many SI algorithms that proposed in literature, and they showed good performances in various optimization tasks, such as Cuckoo Search (CS) Algorithm (Gandomi, Yang, & Alavi, 2013), Whale Optimization Algorithm WOA (Mirjalili & Lewis, 2016), Salp Swarm Algorithm (SSA) (Mirjalili et al., 2017), Moth Flame Optimization (MFO) (Mirjalili, 2015a), Marine Predators Algorithm (MPA) (Faramarzi, Heidarinejad, Mirjalili, & Gandomi, 2020), Lion Optimization Algorithm (LOA) (Yazdani & Jolai, 2016), Grasshopper Optimization Algorithm (Saremi, Mirjalili, & Lewis, 2017), Emperor Penguin Optimizer (Dhiman & Kumar, 2018), and Squirrel Search Algorithm (Jain, Singh, & Rani, 2019).

2.2. Evolutionary Algorithms (EA)

Several EA algorithms have been proposed in the literature to solve optimization problems based on biological evolution's natural behaviors. Here we present several examples of the EA algorithms. The Genetic Algorithm (GA) is the most popular EA algorithm. It was developed by Holland Holland (1992), which inspired by the Darwinian theory of evolution. It has received wide attention and has been employed in many applications. For example, face recognition, feature selection, network anomaly detection, scheduling problems, and many other engineering applications and problems (Bharathi, Rekha, & Vijayakumar, 2017). Differential evolution (DE) is presented by Storn and Price (1997). Also, it has been applied in various optimization tasks, such as, image classification, global optimization, text classification, parallel machine scheduling, and others. Other popular EA-based MH algorithms confirmed their performance in various optimization tasks, including Biogeography-Based Optimizer (BBO) (Simon, 2008), and Invasive Tumor Growth (Tang, Dong, Jiang, Li, & Huang, 2015).

2.3. Physics-based algorithms (PhA)

PhA algorithms depend on the physical law to propose solutions to optimization tasks. Among PhA algorithms, we list some efficient PhA based optimization algorithms as follows. Big Bang-Big Crunch (BBBC) is one of the popular MH algorithms inspired by the evolution of the universe (Erol & Eksin, 2006). It has been utilized by researchers in different fields, such as data clustering, global optimization, classification problems, different engineering designs, and others. Gravitational Search Algorithm (GSA) is inspired by the law of gravity and mass interactions (Rashedi et al., 2009). Also, it received wide attention and has been used to improve and solve various applications and problems. For example, image segmentation, feature selection, global optimization, engineering designs, and others. Multi-verse Optimizer (MVO) (Mirjalili et al., 2016) is inspired by the multi-verses theory in physics. In recent years, MVO has been employed to solve several problems, such as global optimization problems, time series forecasting, image segmentation, feature selection, and others. Also, there are other PhA based MH algorithms, such as Central Force Optimization (CFO) (Formato, 2007), and Henry gas solubility optimization (HGSO) (Hashim et al., 2019).

2.4. Human-based algorithms

By simulating some natural human behaviors, researchers proposed several MH algorithms for solving optimization problems. Here, we highlight some of these methods as follows. Teaching based learning algorithm (TBLA) (Rao, Savsani, & Vakharia, 2012) is inspired by the influence of a teacher on the output of learners in the class. It has been applied to solve various problems such as constrained optimization problems (Rao & Patel, 2013) and various problems such as (Ho, Dao, Le

Table 1
A summary of popular MH algorithms.

Type	Algorithm	Ref.	Inspired by
SI	Particle Swarm Optimization (PSO)	(Eberhart & Kennedy, 1995)	The natural behaviors of swarm particles
	Ant Colony Optimization (ACO)	(Dorigo et al., 1996)	Ants deposit pheromone on the ground
	Artificial Bee Colony (ABC)	(Karaboga & Basturk, 2007)	The behaviors of the honey bees colony
	Firefly Algorithm (FA)	(Yang, 2009)	Flashing light of fireflies in oceans
	Cuckoo Search (CS)	(Gandomi et al., 2013)	The behavior of cuckoo breeding parasitism
	Whale Optimization Algorithm (WOA)	(Mirjalili & Lewis, 2016)	The behavior of humpback whales
	Salp Swarm Algorithm (SSA)	(Mirjalili et al., 2017)	The behavior of salps navigating in oceans
	Moth Flame Optimization (MFO)	(Mirjalili, 2015a)	The moths navigation method in nature
	Lion Optimization Algorithm (LOA)	(Yazdani & Jolai, 2016)	Lifestyle and cooperation of lions
	Marine Predators Algorithm (MPA),	(Faramarzi, Heidarinejad, Mirjalili, et al., 2020)	predators foraging strategy in oceans
	Squirrel Search Algorithm (SSA)	(Jain et al., 2019)	The behavior of southern flying squirrels
	Grasshopper Optimization Algorithm (GOA)	(Saremi et al., 2017)	The behavior of grasshopper swarms
	Golden Eagle Optimizer (GEO)	(Mohammadi-Balani et al., 2021)	The behavior of golden eagles in tuning speed
EA	Genetic Algorithm (GA)	(Holland, 1992)	Darwinian theory of evolution
	Differential evolution (DE)	(Storn & Price, 1997)	the natural phenomenon of evolution
	Biogeography-Based Optimizer (BBO)	(Simon, 2008)	Biogeography related to species migration
	Invasive Tumor Growth (ITG)	(Tang et al., 2015)	Kidney process.
	Tree Growth Algorithm (TGA)	(Cheraghalipour et al., 2018)	Competition of trees for acquiring foods and light
	Arithmetic Optimization Algorithm (AOA)	(Abualigah et al., 2021)	The distribution behavior of the main arithmetic operators
PhA	Big Bang-Big Crunch (BBBC)	(Erol & Eksin, 2006)	The evolution of the universe
	Gravitational Search Algorithm (GSA)	(Rashedi et al., 2009)	The law of gravity and mass interactions
	Multi-verse Optimizer (MVO)	(Mirjalili et al., 2016)	multi-verses theory
	Central Force Optimization (CFO)	(Formato, 2007)	The metaphor of gravitational kinematics
	Henry Gas Solubility Optimization (HGSO)	(Hashim et al., 2019)	The behavior of Henry's law
	Thermal Exchange Optimization (TEO)	(Kaveh & Dadras, 2017)	Newton's law of cooling
	Electromagnetic Field Optimization (EFO)	(Abedinpourshotorban et al., 2016)	The behavior of electromagnets
Human based	Teaching based learning algorithm (TBLA)	(Rao et al., 2012)	The influence of a teacher on the output of learners
	Collective Decision Optimization (CSO)	(Zhang et al., 2017)	Human decision-making characteristics
	Socio Evolution & Learning Optimization Algorithm (SELOA)	(Kumar et al., 2018)	Social learning behavior of humans
Others	Volleyball Premier League Algorithm (VPLA)	(Moghdani & Salimifard, 2018)	Competitions of volleyball teams
	Sine Cosine Algorithm (SCA)	(Mirjalili, 2016a)	Sine and cosine functions

Chau, & Huang, 2019). Socio Evolution Learning Optimization Algorithm (SELOA) (Kumar et al., 2018) is proposed by simulating the social learning behavior of humans organized as families in a societal setup. Furthermore, other popular MH algorithms, such as sine cosine algorithm (Mirjalili, 2016a), and volleyball Premier League Algorithm (Moghdani & Salimifard, 2018). Table 1 summarizes several popular MH algorithms.

Generally, population-based optimization algorithms start the optimization processes (improvement processes) by selecting candidate solutions randomly. These created solutions are evolutionary improved by the optimization rules and evaluated by a specific objective function iteratively, the optimization techniques' nature. Since optimization algorithms endeavor to find the optimal or near-optimal solution for the given optimization problems stochastically, finding a solution in a single run is not guaranteed (Abualigah, 2020b; Abualigah, Shehab, Alshinwan, & Alabool, 2019). However, the probability of obtaining the optimal global solution for the given problem is grown by a sufficient number of random solutions and evolutionary iterations.

Despite the variations between optimization algorithms in population-based meta-heuristic methods, the optimization process is divided into two main phases: exploration/diversification versus exploitation/intensification (Abualigah, Shehab, Alshinwan, Mirjalili, & Abd Elaziz, 2020; Shehab et al., 2019). This refers to the wide coverage of the search space by utilizing different search solutions of the used algorithm to avoid the searching problems.

3. Aquila Optimizer (AO)

In this section, the proposed nature-inspired algorithm, called Aquila Optimizer (AO), is presented as follows.

3.1. Inspiration and behavior of Aquila during hunting

In the Northern Hemisphere, the Aquila is one of the most popular birds of prey. Aquila is the most common spread species of the Aquila. Similar to all birds, Aquila belongs to the group "Accipitridae". Typically, Aquila is dark brown, with lighter Golden-brown plumage on their back of a neck. Young Aquila of this group mainly has white color on the tail, and usually, their wings have minor white marks. Aquila uses its speed and agility united with sturdy feet and large, sharpened talons to grab various prey, mainly rabbits, hares, deeps, marmots, squirrels, and other ground animals (Steenhof, Kochert, & Mcdonald, 1997). Aquila and their distinctive behaviors can be observed in nature.

Aquila keeps territories that may be as high as 200 km^2. They create large nests in mountains and other high positions. The breeding actions occur in the spring; they are monogamous and may survive together for many years or probably throughout life. Females produce up to 4 eggs and later incubate them for 6 weeks. Typically, one or two newborns live to fledge in about 12 weeks. These young Aquila usually achieve complete confidence in the fall, following which they move widely to building territory for themselves.

Due to its hunting bravery, Aquila is one of the most studied birds globally. Male Aquila got significantly more prey when solo-hunting. Aquila utilizes their speed and sharp talons to hunt squirrels, rabbits, and many other animals. They have even been recognized as an attacker for full-grown deer (Hatch, 1968). The next most notable animal in the diet of Aquila is the ground squirrels.

Mainly, four hunting methods are recognized to be used by the Aquila, with many distinct differences and most Aquila's ability to cleverly and quickly vary back and forth between hunting methods relying on the situations. The following points express the hunting methods of Aquila.

- The first method, high soar with a vertical stoop, is utilized for hunting birds in flight, where the Aquila rises at a high level over the ground. Once it explores prey, the Aquila enters a long, low-angled glide with speed rising as the wings close further. The Aquila needs a height feature over its prey for the success of this method. Just before the engagement, the wings and tail are unfolded, and feet are pushed ahead to grab the prey to look like a clap of thunder (Carnie, 1954).
- The second method, contour flight with short glide attack, is recognized as the most usually used method by Aquila, where the Aquila rises at a low level over the ground. The prey is then hounded closely, whether the prey is running or flying. This method is beneficial for hunting ground squirrels, breeding grouse, or seabirds (Meinertzhagen, 1940).
- The third method is a low flight with a slow descent attack. In this, the Aquila lows to the ground, and next onslaught progressively on the prey. The Aquila selects its victim and lands on the prey's neck and back, trying to penetrate. This hunting method is utilized for slow prey, such as rattlesnakes, hedgehogs, foxes, and tortoises, or any prey with an absence of escape response (Dekker, 1985).
- The fourth method is walking and grab prey, in which the Aquila walks on the land and tries to pull its prey. It is utilized for pulling the young of large prey (i.e., deer or sheep) out of the coverage area (Watson, 2010).

In conclusion, Aquila is one of the most intelligent and skillful hunters and probably next after humans. The main inspiration for the proposed AO algorithm is derived from the methods mentioned above. The following subsections describe how these processes are modeled in the AO.

3.2. Solutions initialization

In AO, it is a population-based method, the optimization rule begins with the population of candidate solutions (X) as presented in Eq. (1), which is generated stochastically between the upper bound (UB) and lower bound (LB) of the given problem. The best-obtained solution, so far, is determined as the optimal solution approximately in each iteration.

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & x_{1,Dim-1} & x_{1,Dim} \\ x_{2,1} & \cdots & x_{2,j} & \cdots & x_{2,Dim} \\ \cdots & \cdots & x_{i,j} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{N-1,1} & \cdots & x_{N-1,j} & \cdots & x_{N-1,Dim} \\ x_{N,1} & \cdots & x_{N,j} & x_{N,Dim-1} & x_{N,Dim} \end{bmatrix} \tag{1}$$

where X denotes a set of current candidate solutions, which are generated randomly by using Eq. (2), X_i denotes to the decision values (positions) of the i^{th} solution, N is the total number of candidate solutions (population), and Dim denotes to the dimension size of the problem.

$$X_{ij} = rand \times \left(UB_j - LB_j\right) + LB_j, \; i = 1,2,\ldots..,N \; j = 1,2,\ldots,Dim \tag{2}$$

where $rand$ is a random number, LB_j denotes to the j^{th} lower bound, and UB_j denotes to the j^{th} upper bound of the given problem.

3.3. Mathematical model of AO

The proposed AO algorithm simulates Aquila's behavior during hunting in which showing the actions of each step of the hunt. Hence, the optimization procedures of the proposed AO algorithm are represented in four methods; selecting the search space by high soar with the vertical stoop, exploring within a diverge search space by contour flight with short glide attack, exploiting within a converge search space by low flight with slow descent attack, and swooping by walk and grab prey. The AO algorithm can transfer from exploration steps to exploitation steps using different behaviors based on this condition if $t \leqslant (\frac{2}{3}) * T$ the

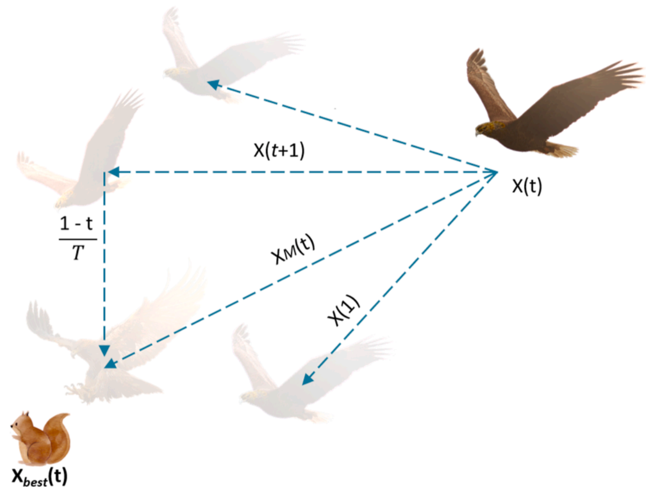

Fig. 1. The behavior of the Aquila high soar with the vertical stoop.

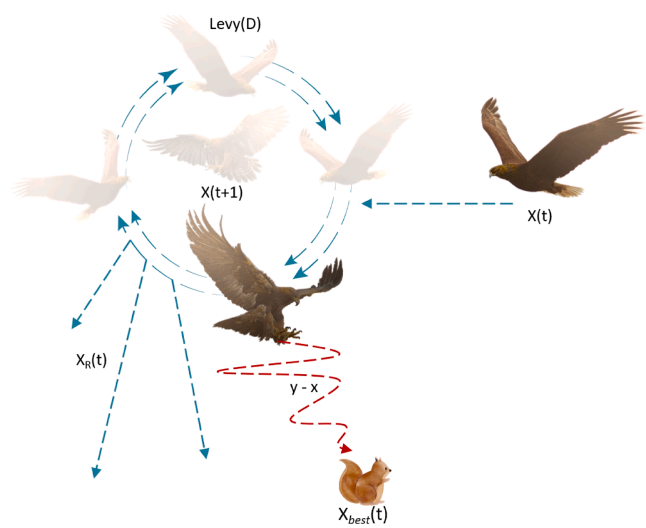

Fig. 2. The behavior of the Aquila contour flight with short glide attack.

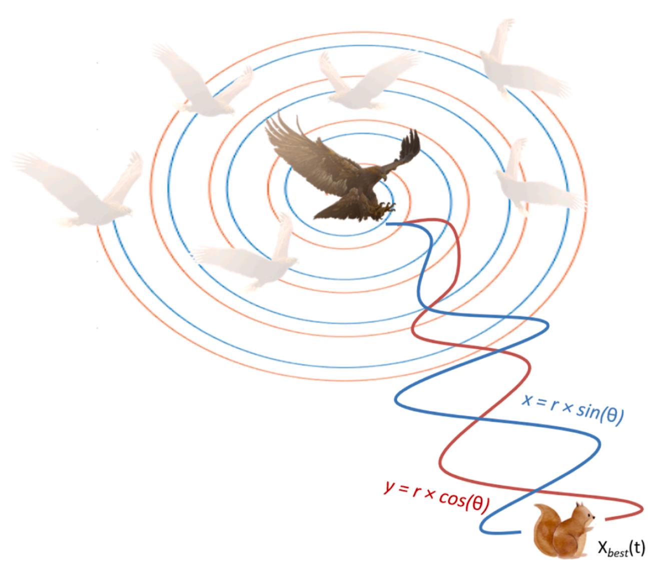

Fig. 3. The behavior of the AO in a spiral shape.

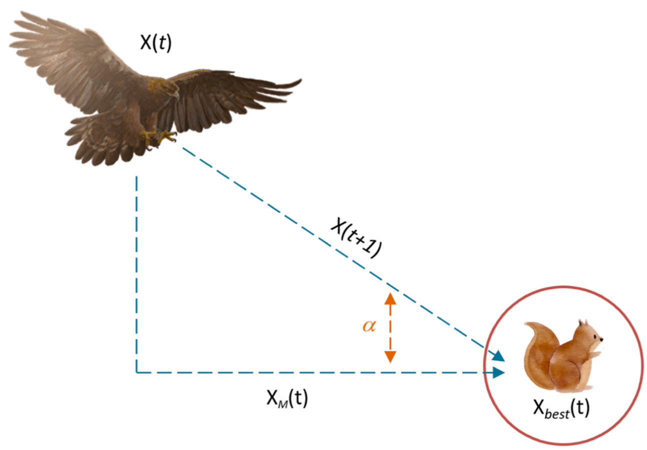

Fig. 4. The behavior of the Aquila low flight with slow descent attack.

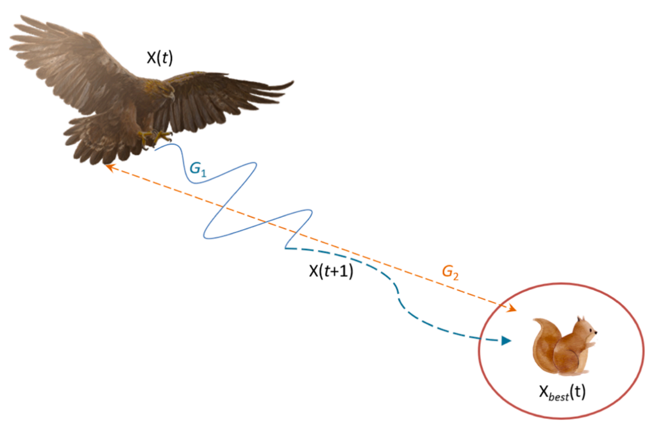

Fig. 5. The behavior of the Aquila walk and grab prey.

exploration steps will be excited; otherwise, the exploitation steps well be executed.

We modeled Aquila behaviors as a mathematical optimization paradigm, and it is determining the best solution subjected to specific constraints. The mathematical model of the AO is proposed as follows.

3.3.1. Step 1: Expanded exploration (X_1)

In the first method (X_1), the Aquila recognizes the prey area and selects the best hunting area by high soar with the vertical stoop. Here, the AO widely explorers from high soar to determine the area of the search space, where the prey is. Fig. 1 shows the behavior of the Aquila high soar with the vertical stoop. This behavior is mathematically presented as in Eq. (3).

$$X_1\left(t+1\right) = X_{best}\left(t\right) \times \left(1-\frac{t}{T}\right) + \left(X_M\left(t\right) - X_{best}\left(t\right) * rand\right), \tag{3}$$

where, $X_1(t+1)$ is the solution of the next iteration of t, which is generated by the first search method (X_1). $X_{best}(t)$ is the best-obtained solution until t^{th} iteration, this reflects the approximate place of the prey. This equation $\left(\frac{1-t}{T}\right)$ is used to control the expanded search (exploration) through the number of iterations. $X_M(t)$ denotes to the locations mean value of the current solutions connected at t^{th} iteration, which is calculated using Eq. (4). *rand* is a random value between 0 and 1. t and T present the current iteration and the maximum number of iteration, respectively.

$$X_M\left(t\right) = \frac{1}{N}\sum_{i=1}^{N} X_i\left(t\right), \forall j = 1,2,\ldots,Dim \tag{4}$$

where, *Dim* is the dimension size of the problem and N is the number of candidate solution (population size).

3.3.2. Step 2: Narrowed exploration (X_2)

In the second method (X_2), when the prey area is found from a high soar, the Aquila circles above the target prey, prepares the land, and then attacks. This method called contour flight with short glide attack. Here, AO narrowly explores the selected area of the target prey in preparation for the attack. Fig. 2 shows the behavior of the Aquila contour flight with short glide attack. This behavior is mathematically presented as in Eq. (5).

$$X_2(t+1) = X_{best}(t) \times Levy(D) + X_R(t) + (y-x)*rand, \tag{5}$$

where $X_2(t+1)$ is the solution of the next iteration of t, which is generated by the second search method (X_2). D is the dimension space, and $Levy(D)$ is the levy flight distribution function, which is calculated using Eq. (6). $X_R(t)$ is a random solution taken in the range of [1 N] at the i^{th} iteration.

$$Levy\left(D\right) = s \times \frac{u \times \sigma}{\left|\upsilon\right|^{\frac{1}{\beta}}} \tag{6}$$

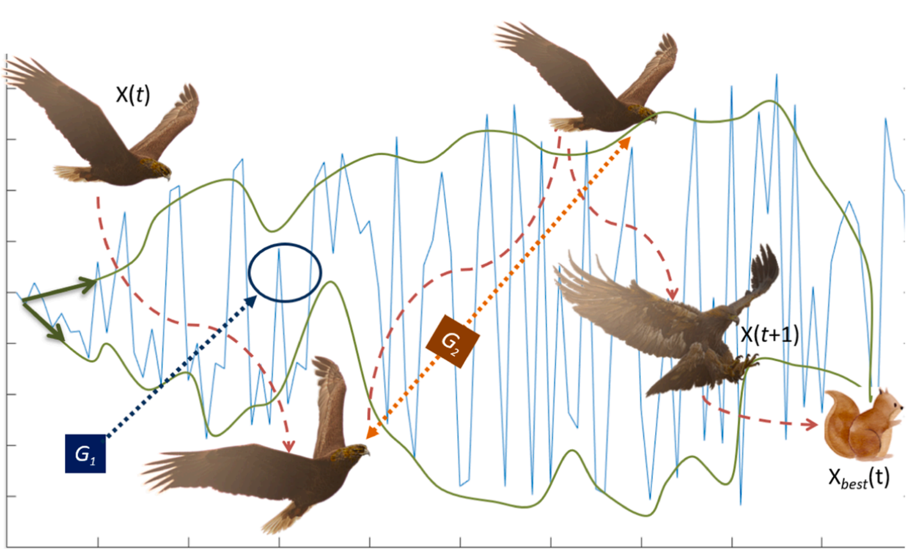

Fig. 6. The effects of the quality function (QF), G_1, and G_2 on the behavior of the AO.

Table 2
Unimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F1	$f(x) = \sum_{i=1}^{n} x_i^2$	10, 50, 100, 500	[−100,100]	0
F2	$f(x) = \sum_{i=0}^{n} \lvert x_i \rvert + \prod_{i=0}^{n} \lvert x_i \rvert$	10, 50, 100, 500	[−10,10]	0
F3	$f(x) = \sum_{i=1}^{d} (\sum_{j=1}^{i} x_j)^2$	10, 50, 100, 500	[−100,100]	0
F4	$f(x) = max_i\{\lvert x_i \rvert, 1 \leqslant i \leqslant n\}$	10, 50, 100, 500	[−100,100]	0
F5	$f(x) = \sum_{i=1}^{n-1} [100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2]$	10, 50, 100, 500	[−30,30]	0
F6	$f(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	10, 50, 100, 500	[−100,100]	0
F7	$f(x) = \sum_{i=0}^{n} ix_i^4 + \text{random}[0,1)$	10, 50, 100, 500	[−128,128]	0

where s is a constant values fixed to 0.01, u, and υ are random numbers between 0 and 1. σ is calculated using Eq. (7).

$$\sigma = \left(\frac{\Gamma\left(1+\beta\right) \times sine\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}} \right) \tag{7}$$

where β is a constant value fixed to 1.5. In Eq. (5), y and x are used to present the spiral shape in the search, which are calculated as follows.

$$y = r \times cos(\theta) \tag{8}$$

$$x = r \times sin(\theta) \tag{9}$$

where,

$$r = r_1 + U \times D_1 \tag{10}$$

$$\theta = -\omega \times D_1 + \theta_1 \tag{11}$$

$$\theta 1 = \frac{3 \times \pi}{2} \tag{12}$$

r_1 takes a value between 1 and 20 for fixed the number of search cycles, and U is a small value fixed to 0.00565. D_1 is integer numbers from 1 to the length of the search space (*Dim*), and ω is a small value fixed to 0.005. Fig. 3 shows the behavior of the AO in a spiral shape.

3.3.3. Step 3: Expanded exploitation (X_3)

In the third method (X_3), when the prey area is specified accurately, and the Aquila is ready for landing and attack, the Aquila descends vertically with a preliminary attack to discover the prey reaction. This method called low flight with slow descent attack. Here, AO exploits the selected area of the target to get close of prey and attack. Fig. 4 shows the behavior of the Aquila low flight with a slow descent attack. This behavior is mathematically presented as in Eq. (13).

Table 3
Multimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F8	$f(x) = \sum_{i=1}^{n} (-x_i sin(\sqrt{\lvert x_i \rvert}))$	10, 50, 100, 500	[−500,500]	−418.9829× n
F9	$f(x) = \sum_{i=1}^{n} [x_i^2 - 10cos(2\pi x_i) + 10]$	10, 50, 100, 500	[−5.12,5.12]	0
F10	$f\left(x\right) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)\right) + 20 + e$	10, 50, 100, 500	[−32,32]	0
F11	$f\left(x\right) = 1 + \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right)$	10, 50, 100, 500	[−600,600]	0
F12	$f\left(x\right) = \frac{\pi}{n}\{10sin(\pi y_1)\} + \sum_{i=1}^{n-1} (y_i - 1)^2 \left[1 + 10sin^2\left(\pi y_{i+1}\right) + \sum_{i=1}^{n} u(x_i, 10, 100, 4)\right]$, where $y_i = 1 + \frac{x_i + 1}{4}$ $, u\left(x_i, a, k, m\right) \begin{cases} K(x_i - a)^m & \text{if} x_i > a \\ 0 & -a \leqslant x_i \geqslant a \\ K(-x_i - a)^m & -a \leqslant x_i \end{cases}$	10, 50, 100, 500	[−50,50]	0
F13	$f(x) = 0.1(sin^2(3\pi x_1) + \sum_{i=1}^{n} (x_i - 1)^2 [1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2 1 + sin^2(2\pi x_n)) + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	10, 50, 100, 500	[−50,50]	0

Table 4
Fixed-dimension multimodal benchmark functions.

Function	Description	Dimensions	Range	f_{min}
F14	$f\left(x\right) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2}} (x_i - a_{ij})\right)^{-1}$	2	[−65,65]	1
F15	$f\left(x\right) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	[−5,5]	0.00030
F16	$f\left(x\right) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[−5,5]	−1.0316
F17	$\text{f(x)}=\left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cos\, x_1 + 10$	2	[−5,5]	0.398
F18	$f(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_i + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	[−2,2]	3
F19	$f(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{i=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	3	[−1,2]	−3.86
F20	$f(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{i=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	6	[0,1]	−.32
F21	$f(x) = -\sum_{i=1}^{5} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,1]	−10.1532
F22	$f(x) = -\sum_{i=1}^{7} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,1]	−10.4028
F23	$f(x) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,1]	−10.5363

$$X_3(t+1) = (X_{best}(t) - X_M(t)) \times \alpha - rand + ((UB - LB) \times rand + LB) \times \delta, \quad (13)$$

where $X_3(t+1)$ is the solution of the next iteration of t, which is generated by the third search method (X_3). $X_{best}(t)$ refers to the approximate location of the prey until i^{th} iteration (the best-obtained solution), and $X_M(t)$ denotes to the mean value of the current solution at t^{th} iteration, which is calculated using Eq. (4). *rand* is a random value between 0 and 1. α and δ are the exploitation adjustment parameters fixed in this paper to a small value (0.1). *LB* denotes to the lower bound and *UB* denotes to the upper bound of the given problem.

3.3.4. Step 4: Narrowed exploitation (X_4)

In the fourth method (X_4), when the Aquila got close to the prey, the Aquila attacks the prey over the land according to their stochastic movements. This method called walk and grab prey. Here, and finally, AO attacks the prey in the last location. Fig. 5 shows the behavior of the Aquila walk and grab prey. This behavior is mathematically presented as in Eq. (14).

$$X_4(t+1) = QF \times X_{best}(t) - (G_1 \times X(t) \times rand) - G_2 \times Levy(D) + rand \times G_1, \quad (14)$$

where $X_4(t+1)$ is the solution of the next iteration of t, which is generated by the fourth search method (X_4). *QF* denotes to a quality function used to equilibrium the search strategies, which is calculated using Eq. (15). G_1 denotes various motions of the AO that are used to track the prey during the elope, which is generated using Eq. (16). G_2 presents decreasing values from 2 to 0, which denote the flight slope of the AO that is used to follow the prey during the elope from the first location (1) to the last location (t), which is generated using Eq. (17). $X(t)$ is the current solution at the t^{th} iteration.

$$QF(t) = t^{\frac{2 \times rand - 1}{(1-T)^2}} \quad (15)$$

$$G_1 = 2 \times rand - 1 \quad (16)$$

$$G_2 = 2 \times \left(1 - \frac{t}{T}\right) \quad (17)$$

$QF(t)$ is the quality function value at the t^{th} iteration, and *rand* is a random value between 0 and 1. t and T present the current iteration and the maximum number of iteration, respectively. $Levy(D)$ is the levy flight distribution function calculated using Eq. (6). Fig. 6 shows the effects of the quality function (QF), G_1, and G_2 on the behavior of the AO.

Table 5
Parameter values for the comparative algorithms.

Algorithm	Parameter	Value
GOA	l	1.5
	f	0.5
EO	r	0.5
	a	4
	GP	0.5
PSO	Topology	Fully connected
	Cognitive and social constant	(C1, C2) 2, 2
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of dimension range
DA	w	0.2–0.9
	s, a, and c	0.1
	f and e	1
ALO	I ratio	10^w
	w	2–6
GWO	Convergence parameter (a)	Linear reduction from 2 to 0
MPA	γ	γ>1
	P	0.0
SSA	v_0	0
SCA	–	–
WOA	α	Decreased from 2 to 0
	b	2
SMA	vb and vc	Decreased from 2 to 0

3.4. Pseudo-code of the Aquila Optimizer (AO)

To recap, in AO, the optimization start the improvement procedures by generating a random predefined set of candidate solutions, called population. Through the trajectory of repetition, the search strategies of the AO explore the reasonable positions of the near-optimal solution or the best-obtained solution. Each solution updates its positions according to the best-obtained solution by the optimization processes of the AO. To emphasize the equilibrium between the search strategies of the AO (i.e., exploration and exploitation), four different search strategies for the exploration and exploitation are provided (i.e., expanded exploration, narrowed exploration, expanded exploitation, and narrowed exploitation). Finally, the search process of the AO is terminated when the end criterion is met. The Pseudo-code of the AO is detailed in Algorithm 1.

Algorithm 1. Aquila Optimizer

Initialization phase:
Initialize the population X of the AO.
Initialize the parameters of the AO (i.e., α, δ, etc).
WHILE (The end condition is not met) **do**
  Calculate the fitness function values.
  $X_{best}(t)$= Determine the best obtained solution according to the fitness values.
  for (i = 1,2…,N) **do**
    Update the mean value of the current solution X_M(t).
    Update the x, y, G_1, G_2, Levy(D), etc.
    if $t \leqslant (\frac{2}{3}) * T$ **then**
    if $rand \leqslant 0.5$ **then**
      ▷ **Step 1: Expanded exploration** (X_1)
      Update the current solution using Eq. (3).
      if Fitness($X_1(t+1)$) < Fitness($X(t)$) **then**
      $X(t) = (X_1(t+1))$
      if Fitness($X_1(t+1)$) < Fitness($X_{best}(t)$) **then**
        $X_{best}(t) = X_1(t+1)$
      end if
    end if
  else
      {▷ **Step 2: Narrowed exploration** (X_2)}
      Update the current solution using Eq. (5).
      if Fitness($X_2(t+1)$) < Fitness($X(t)$) **then**
        $X(t) = (X_2(t+1))$
        if Fitness($X_2(t+1)$) < Fitness($X_{best}(t)$) **then**
          $X_{best}(t) = X_2(t+1)$
        end if
      end if
    end if
  else
    if $rand \leqslant 0.5$ **then**
      {▷ **Step 3: Expanded exploitation** (X_3)}
      Update the current solution using Eq. (13).
      if Fitness($X_3(t+1)$) < Fitness($X(t)$) **then**
        $X(t) = (X_3(t+1))$
        if Fitness($X_3(t+1)$) < Fitness($X_{best}(t)$) **then**
      $X_{best}(t) = X_3(t+1)$
      end if
    end if
  else
    ▷ **Step 4: Narrowed exploitation** (X_4)
    Update the current solution using Eq. (14).
    if Fitness($X_4(t+1)$) < Fitness($X(t)$) **then**
      $X(t) = (X_4(t+1))$
        ifFitness($X_4(t+1)$) < Fitness($X_{best}(t)$) **then**
          $X_{best}(t) = X_4(t+1)$
        end if
      end if
    end if
      end if
  end for
end while
return The best solution (X_{best}).

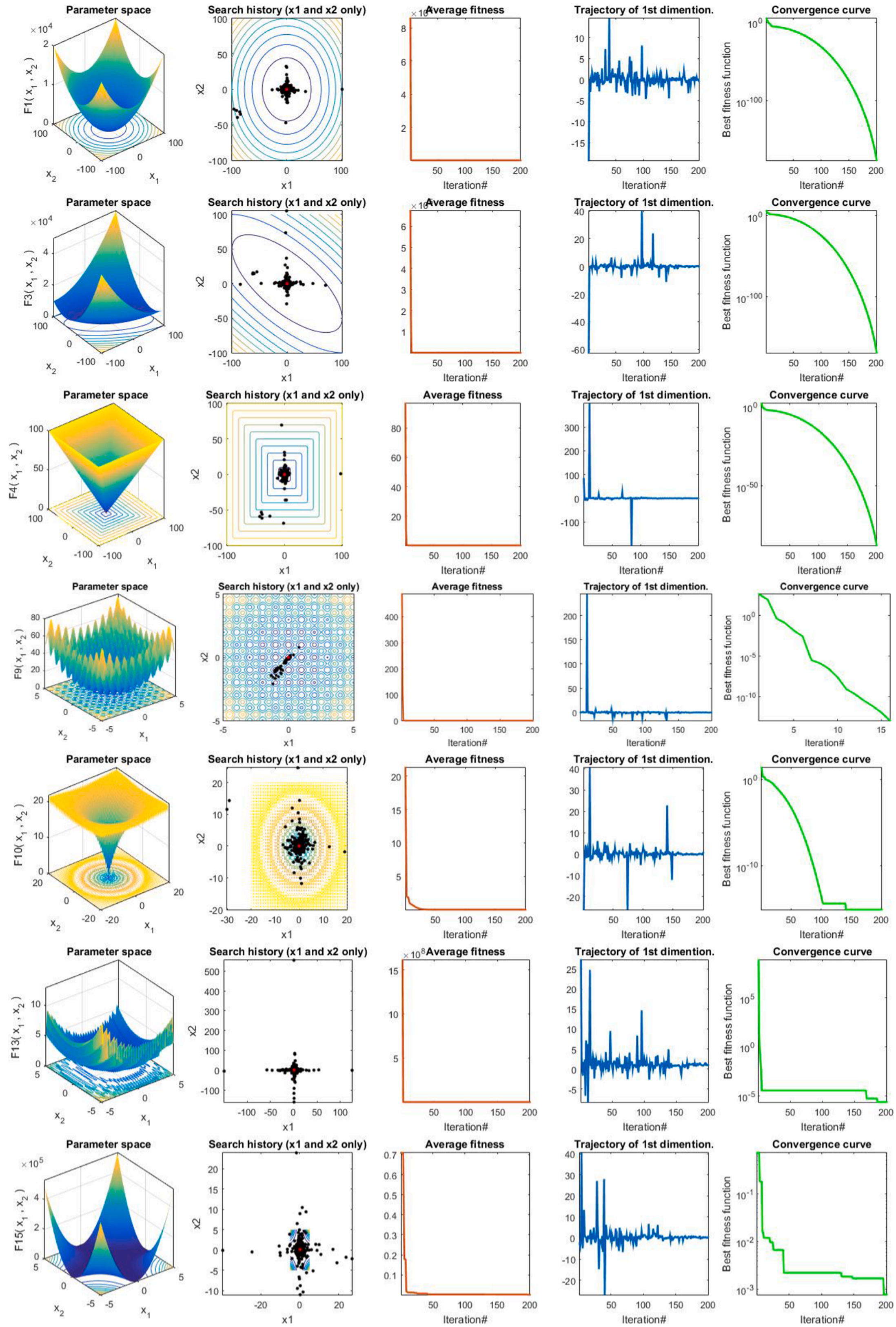

Fig. 7. Qualitative results for the studied problems.

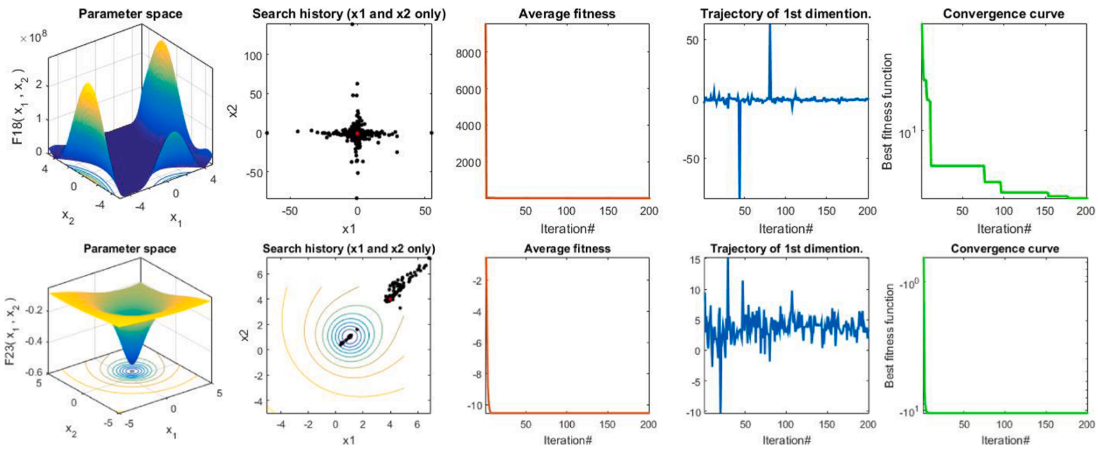

Fig. 7. (*continued*).

Table 6
Scenarios of the tuning parameters.

Scenario No.	α value	δ value
1	0.1	0.1
2	0.1	0.5
3	0.1	0.9
4	0.5	0.1
5	0.5	0.5
6	0.5	0.9
7	0.9	0.1
8	0.9	0.5
9	0.9	0.9

3.5. Computational complexity of the Aquila Optimizer (AO)

In this section, the general computational complexity of the AO is presented. The computational complexity of the AO typically relies on three rules: solutions initialization, calculate the fitness functions, and updating of solutions. Assume that, N is the number of solutions, $O(N)$ is the computational complexity of the solutions' initialization processes. The computational complexity of the solutions' updating processes is $O(T \times N) + O(T \times N \times Dim)$, which consists of exploring for the best positions and updating the solutions' positions of all solutions, where the total number of iterations is called T and the dimension size of the given problem is called Dim. Accordingly, the total computational complexity of the proposed AO is $O(N \times (T \times D + 1))$.

4. Experimental results and discussions

The current investigation appraises the performance of the AO using numerous test benchmark functions and real-world optimization problems. Three series of well-regarded functions suit including twenty-three classical functions, twenty-nine CEC2017 benchmarks, and ten CEC2019 benchmarks are utilized in the numerical validation stage; moreover, seven engineering optimization problems are employed as examples for the real-world applications. The AO has been implemented for 500 iterations with 30 search agents to solve the considered test benchmarks and engineering applications. In order to examine the consistency and reliability of the AO, it has been executed for 30 independent times at this the average of the results (Average), standard deviation (STD), worst-so-far solutions, and best-so-far ones have been reported. To confirm the AO quality, it has been compared with a wide range of meta-heuristic optimization algorithms by using Friedman's mean rank test to demonstrate the AO superiority described in the following sections. To achieve a fair comparison, the considered algorithms have implemented the same number of iterations and population size of AO 500, 30, respectively.

4.1. Definition of twenty-three classical test functions

To assess the AO's in exploring the search space, exploiting the global solutions, and escaping from the local minima, twenty-three benchmark functions include unimodal, multimodal, and fixed dimension multimodal functions. The Unimodal test benchmarks (F1-F7), in Table 2, are applied to examine the AO's exploitation ability. While, the set of multimodal benchmarks (F8-F13), in Table 3, are designed to test the AO's exploration tendency. These two sets of benchmarks are employed in 10, 50, 100, and 500 dimensions. The fixed dimension multimodal test benchmarks (F14-F23) of Table 4 show the ability of AO in exploration the search space in the low dimensions. Numerous of well-established optimization algorithms including Grasshopper Optimization Algorithm (GOA) (Abualigah & Diabat, 2020), Equilibrium Optimizer (EO) (Faramarzi, Heidarinejad, Stephens, & Mirjalili, 2020), Particle Swarm Optimization (PSO) (Abualigah, Khader, & Hanandeh, 2018), Dragonfly Algorithm (DA) (Mirjalili, 2016b), Ant Lion Optimizer (ALO) (Abualigah, Shehab, Alshinwan, Mirjalili, et al., 2020), Grey Wolf Optimizer (GWO) (Abualigah, Shehab, Alshinwan, Alabool, et al., 2020), Marine Predators Algorithm (MPA) (Al-Qaness, Ewees, Fan, Abualigah, & Abd Elaziz, 2020), Salp Swarm Algorithm (SSA) (Abualigah et al., 2019), Sine Cosine Algorithm (SCA) (Mirjalili, 2016a), Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), and Slime Mould Algorithm (SMA) (Zhao, Gao, & Sun, 2020) have been tested with the same benchmarks set to show the superiority of the proposed AO. Table 5 summaries the parameter settings of the counterparts algorithms. The iterations number and population size have been tuned as 500 and 50, respectively, for all the investigated methods with 30 independent runs. The analysis has been performed on MATLAB 2018 platform using PC an Intel Core i5, 2.2 GHz CPU and 16 GB of RAM.

4.1.1. Qualitative analysis for the convergence of AO

To validate the developed AO model's performance, the convergence and the trajectory are used as in Fig. 7. This figure depicts the qualitative measures, such as the 2D shape of the function (as in the first column), to discuss the search space's topology-the search history of the solutions (second column). The average fitness value, trajectory, and convergence curve in the third, fourth, and fifth columns.

In general, the search's history discusses the interaction between the glad and prey. It improves the behavior of the AO to determine the modality of the collective search of glads. This modality represents the movements of the glads around the optimal point in the case of unimodal functions. While in the case of multimodal functions, the modality

Table 7
The influence of the AO parameters (i.e., α and δ) tested on various classical test functions.

Fun No.	Scenario No.								
Measure	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
F1									
Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
F3									
Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
F5									
Worst	3.5452E−03	1.2003E−02	1.0688E+00	2.4860E−03	7.7249E−02	1.5643E−01	9.2393E−03	8.7626E−02	1.9360E−01
Average	7.5563E−04	5.3272E−03	3.7669E−01	9.3808E−04	2.3046E−02	6.1452E−02	2.9616E−03	3.0283E−02	3.5318E−02
Best	3.0716E−05	4.9156E−05	2.5972E−03	2.1377E−05	2.6727E−04	3.3591E−03	4.8392E−05	2.5782E−04	2.3909E−04
STD	1.2825E−03	4.6059E−03	4.3612E−01	8.1676E−04	2.4598E−02	6.8500E−02	2.9794E−03	3.6559E−02	6.6042E−02
F6									
Worst	9.3433E−04	1.4871E−03	1.6305E−03	4.5867E−04	1.6101E−03	1.8361E−03	2.5836E−04	5.0271E−04	1.2255E−02
Average	1.2866E−04	3.2509E−04	2.6693E−04	8.3540E−05	5.7930E−04	7.1889E−04	6.4938E−05	1.3677E−04	3.2565E−03
Best	8.9132E−08	1.2406E−05	1.8149E−07	4.0921E−09	9.8538E−07	2.0150E−05	9.8426E−08	2.4579E−06	1.9958E−05
STD	3.2581E−04	4.8124E−04	5.5469E−04	1.5817E−04	5.6937E−04	6.5062E−04	9.6101E−05	2.0381E−04	4.2484E−03
F7									
Worst	2.4857E−03	6.0771E−04	9.8470E−04	1.8054E−03	7.5880E−04	1.5938E−03	2.5689E−03	2.2417E−03	1.5480E−03
Average	6.7149E−04	3.4123E−04	3.8074E−04	6.1805E−04	3.0839E−04	9.4638E−04	8.1345E−04	5.7466E−04	5.3287E−04
Best	7.4363E−05	9.3075E−05	7.2355E−05	1.6019E−05	2.6660E−06	3.0243E−04	5.4588E−05	7.3517E−05	2.0936E−05
STD	8.1515E−04	1.8032E−04	3.1235E−04	5.8056E−04	2.8467E−04	4.2589E−04	7.7100E−04	7.0843E−04	4.7333E−04
F9									
Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
F11									
Worst	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Average	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Best	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
STD	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
F12									
Worst	2.5953E−06	2.4913E−04	6.0485E−04	1.4918E−05	6.8346E−03	3.9597E−04	8.0765E−05	2.5108E−04	1.8653E−03
Average	1.0423E−06	7.3327E−05	1.4308E−04	6.6588E−06	8.7494E−04	1.3414E−04	2.4816E−05	4.3581E−05	2.9459E−04
Best	2.2313E−08	7.3942E−07	6.3818E−07	3.1857E−07	4.2849E−08	6.2585E−06	2.2365E−07	3.7318E−08	3.2924E−07
STD	9.3764E−07	9.8852E−05	2.2879E−04	5.9844E−06	2.4083E−03	1.3325E−04	3.3871E−05	8.5033E−05	6.4205E−04
F13									
Worst	4.4032E−04	2.1265E−03	4.8500E−03	2.0505E−04	5.4268E−04	1.9890E−03	3.1063E−04	9.6081E−04	2.4658E−03
Average	7.2669E−05	3.8436E−04	6.7492E−04	8.4223E−05	1.2665E−04	5.3678E−04	4.8247E−05	4.0248E−04	8.1892E−04
Best	9.3142E−08	5.7119E−06	7.0224E−06	7.4176E−07	2.6086E−06	2.8230E−06	2.9500E−07	1.8673E−05	4.4407E−05
STD	1.4966E−04	7.2191E−04	1.6890E−03	8.3966E−05	1.8606E−04	7.5516E−04	1.0756E−04	4.5882E−04	8.4563E−04
(W|L|T)	(2|3|4)	(0|5|4)	(0|5|4)	(0|5|4)	(1|4|4)	(0|5|4)	(2|3|4)	(0|5|4)	(0|5|4)
Mean	2.0000E+00	2.8889E+00	4.0000E+00	2.1111E+00	3.3333E+00	4.6667E+00	2.2222E+00	3.2222E+00	4.5556E+00
Ranking	1	4	7	2	6	9	3	5	8

represents the scattering properties of glads. All of these modality characteristics enhance the exploration and exploitation abilities when AO is used to solve multimodal and unimodal functions, respectively.

Moreover, from the third column that represents the average fitness value overall the solutions among the number of iteration, it can be observed that the average fitness value at the beginning iterations is high. However, before the number of iterations reached 50, the average becomes small, which indicates that the AO requires a small number of iterations to convergence to the optimal solution. From the trajectory of the solution (as in the fourth column), it can be seen that the solution has a high magnitude and frequency at the early iterations. At the last iterations, they have nearly vanished. This illustrates AO's high exploration ability at the early iterations and good exploitation at the last iterations. Based on this behavior, AO has a high chance of reaching the optimal solution.

It can be noticed from the convergence curve (last column in Fig. 7) that there are variant types of patterns based on the tested functions. For example, for the unimodal functions, their conference curves are smooth. It needs a small number of iterations to be enhanced. The convergence curve is similar to step-wise for the multimodal functions because this type is more complex than unimodal functions.

4.1.2. Parameters analysis of the AO

In this section, we test the effect of changing the value of AO's parameters on its performance. In general, there are two main parameters in AO, so we put different scenarios according to the value of these parameters. These parameters are evaluated at one value from 0.1, 0.5, and 0.9; therefore, we have nine scenarios (as in Table 6).

Table 7 depicts the statistical results obtained at each scenario among nine functions (i.e., F1, F3, F5, F6, F7, F9, F11, F12, and F13). From these results, it can be noticed that the first scenario (i.e., $\alpha = 0.1$ and $\delta = 0.1$) among all the tested functions has better results overall in other scenarios. Followed by fourth and seventh scenarios that allocated the second and third rank, respectively. However, it can be observed that AO's performance at all of these scenarios is similar at F1, F3, F9, and F11. Also, by comparing the first scenario (at low value) with scenario 9 (at high value), it can be noticed that the AO at the first scenario wins two times while at another scenario not win. AO lost only three times in

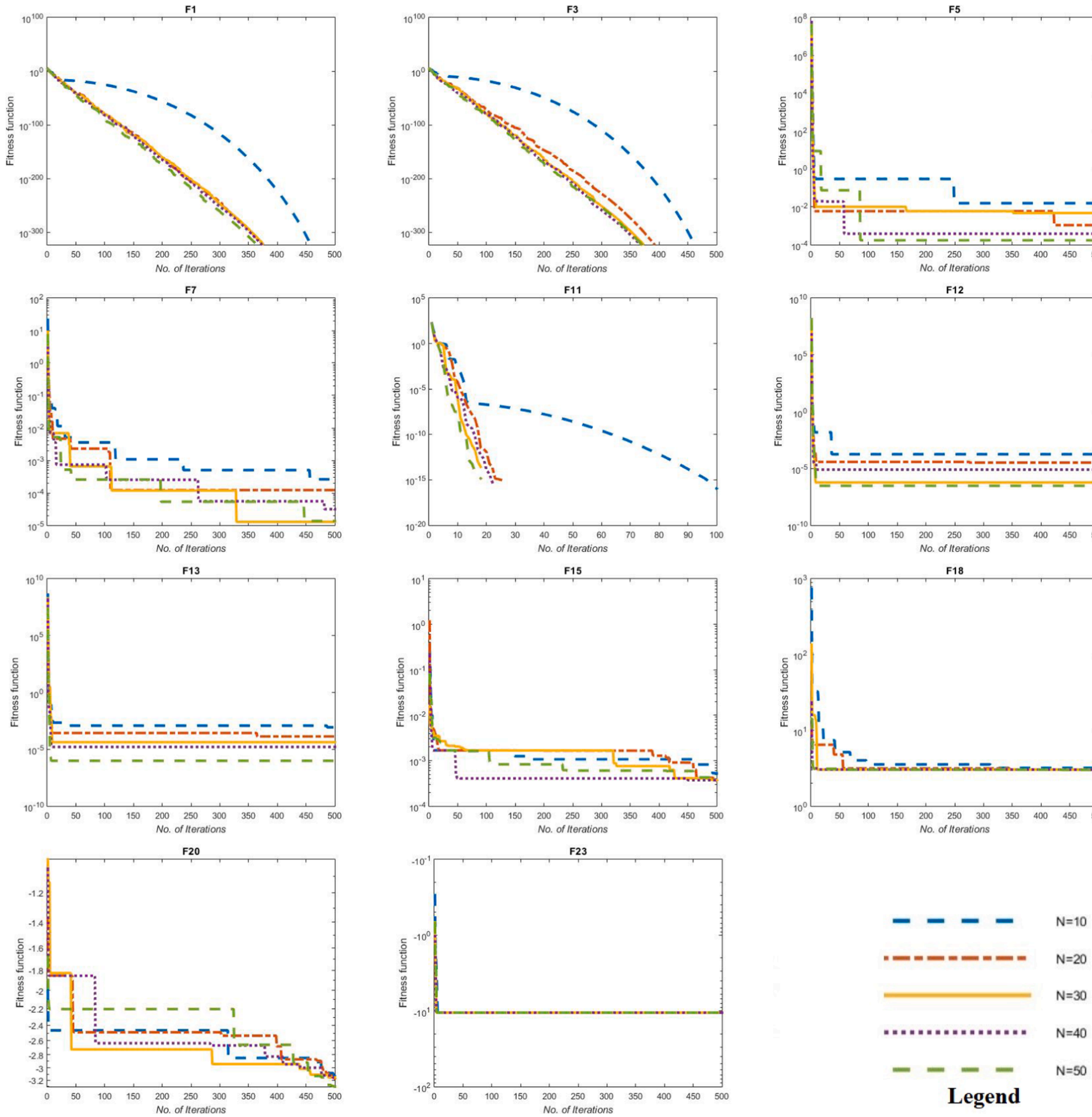

Fig. 8. The influence of the AO population size (i.e., number of solutions (*N*)) tested on various classical test functions.

the first scenario, but AO using the ninth scenario, lost five times. The seventh scenario has the same analysis as the first scenario. However, it allocates the third rank with a small difference from the first scenario.

The impact of the number of solutions (i.e., *N*) is examined on several benchmark functions. To appropriately analyze the AO's parameter sensitivity, we tested several numbers of solutions (i.e., 10, 20, 30, 40, and 50) by comparing the changes in the number of solution parameters throughout 100 iterations. It can be seen from the obtained results in Fig. 8 that when uses many population sizes, the proposed AO method keeps its advantages, which means that AO is more robust and less overwhelmed by the population size. As shown in this figure, it is also clear that the change in the number of solutions in the first example (F23) did not affect the algorithm's ability to solve this problem. For F12 and F13, it was clear that the number of smaller solutions was less efficient than the rest at solving problems. Thus, we can say that the best number of solutions is 30.

4.1.3. Intensification capability of the AO

The set of unimodal functions of (F1-F7) have been employed to examine AO's ability in exploiting the global solutions as this suite of functions has only one global optimum. Table 8 displays AO and other recent competitors' results on the studied unimodal benchmarks (F1-F7). The results include the worst-so-far solutions, the Average and STD values across the number of independent runs, and the best-so-far solutions that have been involved. The attained values confirm the superiority of the AO as it prevails over the other peers in achieving the minimal values of the best-so-far solutions and the worst-so-far solutions and the Average values with high reliability and consistency (minimum STD) for six functions of this set. The SMA achieves the least mean value for F7 while the GWO and AO occupy the second and third positions for this function's mean value.

Nonetheless, the AO offers the least best value for F7. Through this observation, one can see the AO's high ability in the intensification

Table 8
Results of the comparative methods on classical test functions (F1-F13), the dimension is fixed to 10.

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F1												
Worst	0.0000E+00	3.6969E−04	7.7535E−55	6.2268E−19	2.3600E+01	1.4323E−07	1.4594E−48	1.0861E−29	1.4896E−09	8.0970E−10	2.9973E−69	1.6487E−209
Average	0.0000E+00	9.2301E−05	1.9446E−55	1.6757E−19	1.3050E+01	4.3784E−08	7.0295E−49	5.1571E−30	8.8917E−10	2.0351E−10	1.0157E−69	2.3553E−210
Best	0.0000E+00	4.5311E−06	4.3340E−58	2.8877E−21	9.4062E−01	1.3931E−08	3.3951E−51	4.8287E−31	4.5257E−10	2.2968E−15	1.2964E−74	0.0000E+00
STD	0.0000E+00	1.5720E−04	3.3159E−55	2.5923E−19	8.5620E+00	5.6212E−08	5.5804E−49	3.9049E−30	4.0621E−10	3.4922E−10	1.4278E−69	0.0000E+00
F2												
Worst	3.2140E−217	8.0714E+00	1.7385E−32	5.6805E−09	5.0004E+00	1.8234E+00	5.0478E−27	2.6606E−16	1.6008E−01	1.9284E−08	7.0476E−49	3.3909E−102
Average	9.4973E−218	1.9913E+00	4.7530E−33	1.8197E−09	2.6815E+00	6.5730E−01	1.8314E−27	1.1119E−16	3.2973E−02	7.1625E−09	1.4120E−49	4.8441E−103
Best	6.5477E−225	4.2831E−02	4.4565E−34	1.8509E−10	1.1170E+00	2.0566E−05	5.2330E−29	1.5592E−17	7.8584E−06	2.2667E−11	2.3267E−54	4.0840E−185
STD	0.0000E+00	3.4564E+00	7.1132E−33	2.3197E−09	1.4933E+00	9.0885E−01	2.2740E−27	1.2968E−16	7.1078E−02	9.7874E−09	3.1504E−49	1.2816E−102
F3												
Worst	0.0000E+00	7.9216E+00	1.3672E−26	8.9441E−05	2.2990E+02	3.5658E+01	5.9319E−19	8.8997E−13	1.6144E−02	1.5833E+00	9.1052E+02	5.9474E−170
Average	0.0000E+00	2.3721E+00	2.7965E−27	2.4370E−05	1.5135E+02	9.7732E+00	1.4430E−19	2.9489E−13	3.2460E−03	3.1907E−01	4.1977E+02	8.4963E−171
Best	0.0000E+00	7.6675E−01	3.2828E−30	3.0206E−07	4.9375E+01	7.7055E−01	5.8410E−22	4.3286E−15	5.5017E−06	8.7897E−06	1.8044E+01	0.0000E+00
STD	0.0000E+00	3.1152E+00	6.0803E−27	3.7242E−05	6.6986E+01	1.4868E+01	2.5214E−19	4.1003E−13	7.2100E−03	7.0675E−01	3.7377E+02	0.0000E+00
F4												
Worst	3.4546E−217	3.6825E−01	4.5050E−18	9.0690E−04	5.4895E+00	1.4412E−01	5.9612E−15	8.9567E−13	6.6264E−04	4.2456E−03	1.0180E+01	4.1196E−106
Average	9.9112E−218	1.6042E−01	9.0972E−19	2.4924E−04	3.1205E+00	3.4159E−02	2.1428E−15	4.8253E−13	1.7262E−04	1.3333E−03	3.1560E+00	6.2423E−107
Best	2.3220E−221	8.7472E−02	7.2504E−22	2.0935E−05	1.5899E+00	1.0461E−03	2.3617E−16	2.3066E−13	1.6171E−05	4.3787E−06	2.1035E−03	4.1535E−184
STD	0.0000E+00	1.1778E−01	2.0099E−18	3.7336E−04	1.6164E+00	6.1690E−02	2.5515E−15	2.7060E−13	2.7860E−04	1.8792E−03	4.0482E+00	1.5441E−106
F5												
Worst	7.2509E−03	4.7288E+02	5.8178E+00	7.6530E+00	1.2150E+04	2.1338E+02	8.0599E+00	8.8155E+00	6.4316E+02	8.2314E+00	8.9311E+00	8.2833E+00
Average	1.8851E−03	1.3186E+02	5.5290E+00	4.9164E+00	2.7086E+03	1.1610E+02	6.9905E+00	8.7760E+00	1.8700E+02	7.6830E+00	7.6503E+00	6.6668E+00
Best	5.8588E−05	3.5924E+00	5.3001E+00	1.0189E−01	1.3482E+02	5.9959E+00	6.2379E+00	8.7560E+00	7.8930E+00	7.2782E+00	6.8245E+00	1.3987E−01
STD	2.5151E−03	1.9815E+02	2.4395E−01	2.8429E+00	5.2795E+03	1.0054E+02	7.6379E−01	2.3773E−02	2.5958E+02	4.4474E−01	8.2650E−01	2.9165E+00
F6												
Worst	1.9689E−04	7.2259E−04	8.5541E−14	2.0342E−16	4.5832E+01	1.1796E−07	5.3267E−06	8.6068E−11	9.3625E−01	5.1611E−01	2.3389E−01	1.1110E−03
Average	6.3114E−05	2.2305E−04	4.1139E−14	4.3574E−17	1.6905E+01	3.2933E−08	3.5900E−06	2.8785E−11	6.1533E−01	4.1137E−01	5.0174E−02	4.7827E−04
Best	7.2800E−08	3.7020E−07	4.8416E−16	2.8863E−21	1.0348E+00	7.9036E−09	2.5332E−06	3.7619E−12	3.1269E−01	2.9111E−01	1.1980E−03	2.0744E−04
STD	8.9617E−05	2.9549E−04	3.2871E−14	8.9572E−17	1.8622E+01	4.7742E−08	1.1089E−06	3.2764E−11	2.3594E−01	9.6704E−02	1.0272E−01	3.0449E−04
F7												
Worst	1.5929E−03	1.5150E−01	1.8674E−03	1.6135E−02	4.2162E−02	6.1820E−02	1.3368E−03	2.1653E−03	3.6384E−02	3.8409E−03	8.9085E−03	9.9489E−04
Average	8.2851E−04	1.0781E−01	1.3129E−03	9.4087E−03	2.4046E−02	3.4906E−02	6.4406E−04	1.0836E−03	2.1775E−02	2.4214E−03	5.2117E−03	4.9240E−04
Best	3.6048E−05	5.4523E−02	7.1835E−04	3.9634E−03	7.3884E−03	1.6443E−02	3.9526E−04	3.9490E−04	1.3986E−02	1.4779E−03	1.9935E−03	6.3985E−05
STD	4.8479E−04	4.3620E−02	4.8920E−04	4.3669E−03	1.6416E−02	1.9059E−02	3.8983E−04	7.1962E−04	8.7354E−03	9.6575E−04	3.3316E−03	3.9987E−04
F8												
Worst	−1.4073E+03	−2.4131E+03	−2.6410E+03	−1.5161E+03	−2.5770E+03	−2.0333E+03	−2.2885E+03	−3.2378E+03	−2.4891E+03	−1.9022E+03	−2.5990E+03	−4.1892E+03
Average	−2.2694E+03	−2.7474E+03	−3.1952E+03	−2.1103E+03	−2.7496E+03	−2.4552E+03	−2.7077E+03	−3.5326E+03	−2.9844E+03	−2.0030E+03	−3.0851E+03	−4.1895E+03
Best	−2.6958E+03	−3.3212E+03	−3.5946E+03	−2.5933E+03	−2.9985E+03	−3.2226E+03	−3.1415E+03	−3.9514E+03	−3.2590E+03	−2.1001E+03	−3.8213E+03	−4.1898E+03
STD	4.6187E+02	3.4712E+02	3.9090E+02	4.4304E+02	1.6458E+02	4.9743E+02	3.4845E+02	2.7177E+02	3.0091E+02	7.7952E+01	4.4698E+02	1.8346E−01
F9												
Worst	0.0000E+00	6.2682E+01	0.0000E+00	1.2934E+01	4.7290E+01	4.2783E+01	1.5144E+01	5.0275E−07	2.5869E+01	6.3709E+00	1.4211E−14	0.0000E+00
Average	0.0000E+00	4.3380E+01	0.0000E+00	5.6148E+00	3.1340E+01	3.1441E+01	4.3101E+00	1.0055E−07	1.4327E+01	1.2742E+00	2.8422E−15	0.0000E+00
Best	0.0000E+00	2.8854E+01	0.0000E+00	1.9899E+00	1.0558E+01	1.5919E+01	0.0000E+00	0.0000E+00	5.9697E+00	2.7143E−12	0.0000E+00	0.0000E+00
STD	0.0000E+00	1.4099E+01	0.0000E+00	4.3077E+00	1.5146E+01	1.1401E+01	6.6614E+00	2.2484E−07	8.7477E+00	2.8492E+00	6.3553E−15	0.0000E+00
F10												
Worst	8.8818E−16	3.5742E+00	4.4409E−15	1.1957E−09	1.2660E+01	2.0133E+00	1.5099E−14	7.9936E−15	2.8136E+00	6.9120E−06	7.9936E−15	8.8818E−16
Average	8.8818E−16	1.6781E+00	4.4409E−15	7.0605E−10	4.9842E+00	4.0282E−01	1.1546E−14	5.1514E−15	1.4882E+00	1.7246E−06	4.4409E−15	8.8818E−16
Best	8.8818E−16	1.0370E−03	4.4409E−15	3.2066E−10	1.4270E+00	8.1753E−05	7.9936E−15	4.4409E−15	9.2279E−06	5.6642E−08	8.8818E−16	8.8818E−16
STD	0.0000E+00	1.3033E+00	0.0000E+00	3.7348E−10	4.4356E+00	9.0030E−01	3.5527E−15	1.5888E−15	1.1045E+00	2.9457E−06	3.5527E−15	0.0000E+00
F11												
Worst	0.0000E+00	2.9456E−01	2.9518E−02	5.3434E−01	1.0454E+00	3.7635E−01	5.9217E−02	0.0000E+00	2.6543E−01	4.7349E−02	2.4588E−01	0.0000E+00

(*continued on next page*)

Table 8 (*continued*)

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
Average	0.0000E+00	2.5765E−01	5.9036E−03	2.4519E−01	5.8851E−01	2.3516E−01	1.3919E−02	0.0000E+00	1.9428E−01	1.2173E−02	9.7462E−02	0.0000E+00
Best	0.0000E+00	1.9581E−01	0.0000E+00	1.0083E−01	6.5156E−02	1.2060E−01	0.0000E+00	0.0000E+00	1.3262E−01	6.6321E−12	0.0000E+00	0.0000E+00
STD	0.0000E+00	4.1907E−02	1.3201E−02	1.7935E−01	3.7515E−01	1.1319E−01	2.5718E−02	0.0000E+00	5.8327E−02	2.0376E−02	1.3346E−01	0.0000E+00
F12												
Worst	1.0421E−04	3.5311E+00	6.8465E−14	2.8069E−19	4.6366E+00	5.6267E+00	2.0267E−02	4.3457E−11	5.1834E−01	2.1882E−01	2.3163E−02	2.9263E−02
Average	2.9262E−05	1.2733E+00	1.7762E−14	5.8169E−20	1.6329E+00	2.9035E+00	1.0206E−02	2.5447E−11	1.9305E−01	1.3370E−01	1.1534E−02	1.1291E−02
Best	1.1309E−07	8.3499E−04	5.9790E−17	2.0418E−21	7.5827E−02	2.6276E−01	2.7411E−06	1.1688E−11	5.1467E−11	6.6012E−02	1.4821E−03	8.9676E−05
STD	3.5700E−05	1.4307E+00	2.8777E−14	1.2439E−19	1.7959E+00	1.9525E+00	9.9777E−03	1.3247E−11	2.1431E−01	6.0210E−02	9.0233E−03	1.2002E−02
F13												
Worst	2.7104E−04	5.5560E−02	4.3949E−02	1.0987E−02	1.5863E+00	1.0991E−02	1.9995E−01	9.4975E−11	1.0987E−02	4.7545E−01	1.5739E−01	5.1401E−01
Average	9.5026E−05	2.2263E−02	8.7898E−03	2.1975E−03	9.3911E−01	4.3965E−03	4.0000E−02	5.6741E−11	2.1975E−03	3.9263E−01	7.2692E−02	1.1294E−01
Best	2.7176E−06	8.2498E−04	4.0278E−16	8.6127E−22	2.8877E−01	2.2534E−07	6.5501E−06	3.5877E−11	5.7806E−10	3.3532E−01	2.4993E−03	6.6998E−03
STD	1.0312E−04	2.2905E−02	1.9655E−02	4.9137E−03	4.6888E−01	6.0192E−03	8.9412E−02	2.3790E−11	4.9137E−03	5.0910E−02	6.4705E−02	1.8368E−01
(W|L|T)	(6|5|2)	(0|13|0)	(0|12|1)	(2|10|0)	(0|13|0)	(0|13|0)	(0|13|0)	(1|12|0)	(0|13|0)	(1|12|0)	(0|13|0)	(1|10|2)
Mean	2.0000E+00	8.8667E+00	4.1333E+00	5.9333E+00	9.8667E+00	8.4667E+00	5.0000E+00	4.8667E+00	8.8000E+00	7.3333E+00	6.2000E+00	4.7333E+00
Ranking	1	11	2	6	12	9	5	4	10	8	7	3

(exploitation) of the optimal solutions. This feature proceeds from its main procedures; the proposed AO has two exploitation strategies to focus on the local search area mainly (i.e., expanded exploitation and narrowed exploitation). This encourages the optimization process during the search to focus on the local area widely and narrowly.

4.1.4. Diversification capability of AO

The multiple local optima of the multimodal benchmarks are the recommended gate to examine the MHs' exploration (diversification) capacity. Two multimodal test functions are applied while testing the AO, multidimensional (F8-F13) and fixed-dimensional test functions (F14-F23). The worst, average, best, and STD values of the studied functions set are illustrated in Tables 8,9 for multidimensional (F8-F13), and fixed-dimensional (F14-F23) functions, respectively. The results of the tables reveal that the AO outperforms the other peers in the majority of the high dimensional multimodal benchmarks (see Table 8) as it provides the minimum statistical metrics values (worst, average, best, and STD) in five functions out of six from this set. The SMA is the best competitor for F8, as shown in Table 8. For low (fixed) dimensional multimodal benchmarks (F14-F23) (see Table 9), the AO attains the best performance for half of this set. In order to have meaningful statistical results, three lines of Tables 8,9 have been reported to display statistical analysis for the implemented algorithms across the course of the studied functions to demonstrate the superiority of AO. The first line shows three symbols (W|L|T) that denote the number of the functions in which the performance of the algorithm is the best (win) | indistinguishable (tie) | inferior to the others (loss). The second line refers to the Friedman mean rank. The third line illustrates the final rank values of the implemented algorithms. As per those lines, one can see that the AO exhibits the best performance in the majority of the multimodal benchmarks; that is why it occupied the first position as a final rank in comparison with the other peers. The reason behind this performance is that the proposed AO has two exploration strategies to focus on the global search area mainly (i.e., expanded exploration and narrowed exploitation). This helps the algorithm discover the search-land space efficiently and with high quality compared to the other recently proved algorithms. These strategies lead the optimization process in two ways during the search on a wide search area (i.e., widely and narrowly).

4.1.5. Acceleration convergence analysis of AO

The convergence of the AO for the optimal solutions across the iteration numbers is an essential factor that should be investigated, therefore Fig. 9 depicts the best solutions attained so far across the iterations number. The acceleration convergence curves of the proposed AO show a detectable decaying rate in the cases of the unimodal functions (F1-F7). Meanwhile, other algorithms suffer from an intense stagnation to the local solutions that confirm the AO's exploitation stage is high capacity with a reliable exploration. The AO's convergence curves of the multimodal functions show the smooth transition between its exploration and exploitation phases. It convergences for optimal solutions with an observable speed compared with the other peers in F10, F11, F12, F13, F14, F16, F17. F21, F22, and F23. For these functions, it can be seen that the AO has a high balance between exploration and exploitation stages as it catches the nearest values for the optimal solutions with the fastest response in comparison with the other peers then these solutions have been exploited efficiently across the number of iterations to provide the optimal solutions. For F18, F19, and F20, the AO gradually recognizes the optimal solutions and updates the solutions across the iteration numbers that confirm the preceding observation.

4.1.6. Stability analysis of AO

To evaluate the stability and quality of the proposed AO's performance while dealing with high dimensional optimization problems, thirteen functions of Tables 2 and 3 have been employed with three dimensions levels of 50, 100, and 500. The AO and the other counterparts (GOA, EO, PSO, DA, ALO, GWO, MPA, SSA, SCA, WOA, and SMA)

Table 9
Results of the comparative methods on classical test functions (F14-F23).

Fun No.	Comparative methods										
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F14												
Worst	2.9821E+00	1.4600E+01	9.9800E−01	3.9700E+00	1.9900E+00	5.9300E+00	1.0800E+01	9.9800E−01	2.9800E+00	2.9800E+00	1.0800E+01	4.6705E+00
Average	2.3207E+00	1.2300E+01	9.9800E−01	2.7800E+00	1.4000E+00	3.5700E+00	6.2900E+00	9.9800E−01	1.5900E+00	1.4900E+00	4.1400E+00	2.4741E+00
Best	9.9800E−01	9.8000E+00	9.9800E−01	1.9900E+00	9.9800E−01	9.9800E−01	2.9800E+00	9.9800E−01	9.9800E−01	9.9800E−01	9.9800E−01	9.9800E−01
STD	1.0246E+00	1.8600E+00	1.5700E−16	1.0800E+00	5.4400E−01	2.2700E+00	4.1000E+00	0.0000E+00	8.8800E−01	8.5900E−01	4.2300E+00	4.6712E+00
F15												
Worst	8.4575E−04	2.1800E−02	3.5300E−04	9.9300E−04	2.0400E−02	2.0400E−02	2.0400E−02	3.0700E−04	2.0800E−02	1.9100E−03	1.5200E−03	1.4362E−03
Average	5.5089E−04	9.7900E−03	3.2100E−04	9.2000E−04	5.7200E−03	4.9500E−03	8.4500E−03	3.0700E−04	4.8000E−03	1.2800E−03	7.7400E−04	8.3936E−04
Best	4.0581E−04	7.8500E−04	3.0800E−04	8.6600E−04	1.5000E−03	7.7800E−04	3.0800E−04	3.0700E−04	7.4300E−04	6.9200E−04	3.1300E−04	5.1901E−04
STD	1.9027E−04	1.0500E−02	1.9800E−05	6.6400E−05	8.1900E−03	8.6500E−03	1.0900E−02	2.9000E−15	8.9500E−03	4.9000E−04	4.5600E−04	3.6588E−04
F16												
Worst	−1.0316E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0290E+00
Average	−1.0316E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0306E+00
Best	−1.0316E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0300E+00	−1.0313E+00
STD	6.5206E−08	2.1400E−13	1.9200E−16	1.9200E−16	4.2300E−07	2.9300E−13	2.8000E−08	0.0000E+00	1.5200E−14	2.1700E−05	1.1800E−09	8.5115E−04
F17												
Worst	3.9789E−01	0.0000E+00	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	0.0000E+00	4.0200E−01	3.9800E−01	3.9933E−01
Average	3.9789E−01	0.0000E+00	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	0.0000E+00	4.0000E−01	3.9800E−01	3.9829E−01
Best	3.9789E−01	0.0000E+00	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	3.9800E−01	0.0000E+00	3.9900E−01	3.9800E−01	3.9798E−01
STD	4.7081E−07	0.0000E+00	0.0000E+00	0.0000E+00	1.1900E−06	1.8000E−13	1.0500E−05	4.3700E−14	0.0000E+00	1.3700E−03	1.9800E−04	5.2390E−04
F18												
Worst	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	8.4000E+01	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00
Average	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	1.9200E+01	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00
Best	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00	3.0000E+00
STD	3.0000E+00	1.7500E−12	1.1300E−15	1.2600E−15	4.6300E−13	1.5400E−12	3.6200E+01	1.1300E−15	5.9500E−13	1.1300E−03	1.5500E−04	4.6300E−13
F19												
Worst	−3.8607E+00	−3.0800E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8500E+00	−3.8500E+00	−3.8336E+00
Average	−3.8624E+00	−3.5400E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8500E+00	−3.8500E+00	−3.8499E+00
Best	−3.8628E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8600E+00	−3.8500E+00	−3.8600E+00	−3.8601E+00
STD	8.4657E−04	3.2900E−01	6.2800E−16	3.1400E−16	1.4800E−03	4.6500E−11	1.8200E−03	3.1400E−16	2.8100E−10	1.3200E−03	4.4600E−03	1.0229E−02
F20												
Worst	−3.1994E+00	−3.2000E+00	−3.2000E+00	−3.2000E+00	−3.1800E+00	−3.1900E+00	−3.0800E+00	−3.2000E+00	−3.1600E+00	−2.6000E+00	−3.0900E+00	−2.8432E+00
Average	−3.3014E+00	−3.3000E+00	−3.3000E+00	−3.2500E+00	−3.2100E+00	−3.2700E+00	−3.2000E+00	−3.3000E+00	−3.2400E+00	−2.9300E+00	−3.2600E+00	−3.3011E+00
Best	−3.3220E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.3200E+00	−3.0700E+00	−3.3200E+00	−3.2223E+00
STD	4.9969E−02	5.5600E−02	5.3200E−02	6.5100E−02	6.0500E−02	6.9000E−02	8.5900E−02	5.3200E−02	7.7000E−02	1.8900E−01	9.9200E−02	1.3453E−01
F21												
Worst	−1.0140E+01	−2.6800E+00	−2.9400E−01	−2.6300E+00	−2.6300E+00	−2.6800E+00	−1.0100E+01	−1.0002E+01	−5.0600E+00	−4.9600E−01	−5.0500E+00	−5.0240E+00
Average	−1.0149E+01	−6.1500E+00	−4.6600E−01	−7.1500E+00	−6.6300E+00	−4.6100E+00	−1.0100E+01	−1.0013E+01	−6.0800E+00	−8.9400E−01	−8.0700E+00	−7.5258E+00
Best	−1.0153E+01	−1.0200E+01	−7.1100E−01	−1.0200E+01	−1.0200E+01	−5.1000E+00	−1.0200E+01	−1.0200E+01	−1.0200E+01	−2.1000E+00	−1.0100E+01	−1.0152E+01
STD	4.9281E−03	3.7800E+00	1.5900E−01	4.1100E+00	3.3700E+00	1.0800E+00	2.9900E−03	1.7800E−11	2.2700E+00	6.9300E−01	2.7500E+00	2.7307E+00
F22												
Worst	−1.0395E+01	−2.7500E+00	−5.0900E+00	−2.7500E+00	−2.7500E+00	−3.7200E+00	−1.0400E+01	−1.0400E+01	−3.7200E+00	−9.0600E−01	−2.9400E−01	−5.0840E+00
Average	−1.0401E+01	−7.3400E+00	−9.3400E+00	−6.4700E+00	−6.3800E+00	−7.7300E+00	−1.0400E+01	−1.0400E+01	−8.0100E+00	−1.9000E+00	−5.5600E−01	−1.0304E+01
Best	−1.0403E+01	−1.0400E+01	−1.0400E+01	−1.0400E+01	−1.0300E+01	−1.0400E+01	−1.0400E+01	−1.0400E+01	−1.0400E+01	−4.0200E+00	−8.5800E−01	−1.0401E+01
STD	2.9768E−03	4.1900E+00	2.3800E+00	3.6800E+00	3.5400E+00	3.6600E+00	1.3400E−03	1.0400E−11	3.3100E+00	1.4300E+00	2.7200E−01	2.7263E+00
F23												
Worst	−1.0535E+01	−1.6800E+00	−3.8400E+00	−1.0500E+01	−1.6800E+00	−3.8400E+00	−1.0500E+01	−7.8900E−01	−1.0500E+01	−2.4600E+00	−5.1000E+00	−5.1209E+00
Average	−1.0536E+01	−4.2600E+00	−9.2000E+00	−1.0500E+01	−6.5200E+00	−9.2000E+00	−1.0500E+01	−2.0900E+00	−1.0500E+01	−4.1400E+00	−9.4200E+00	−7.6143E+00
Best	−1.0536E+01	−1.0500E+01	−1.0500E+01	−1.0500E+01	−1.0500E+01	−1.0500E+01	−1.0500E+01	−5.1300E+00	−1.0500E+01	−7.2700E+00	−1.0500E+01	−1.0485E+01
STD	2.7562E−04	3.6000E+00	3.0000E+00	1.5400E−15	3.7400E+00	3.0000E+00	1.9800E−03	1.9000E+00	2.1000E−11	1.9000E+00	2.4200E+00	2.7558E+00
(W|L|T)	(6|6|1)	(1|11|1)	(0|11|2)	(0|12|1)	(0|12|1)	(0|12|1)	(0|13|0)	(2|9|2)	(1|11|1)	(0|12|1)	(0|12|1)	(0|12|1)
Mean	2.0000E+00	7.0000E+00	3.9000E+00	4.9000E+00	5.9000E+00	5.7000E+00	6.1000E+00	3.0000E+00	4.6000E+00	8.1000E+00	5.9000E+00	5.6000E+00
Ranking	1	11	3	5	8	7	10	2	4	12	8	6

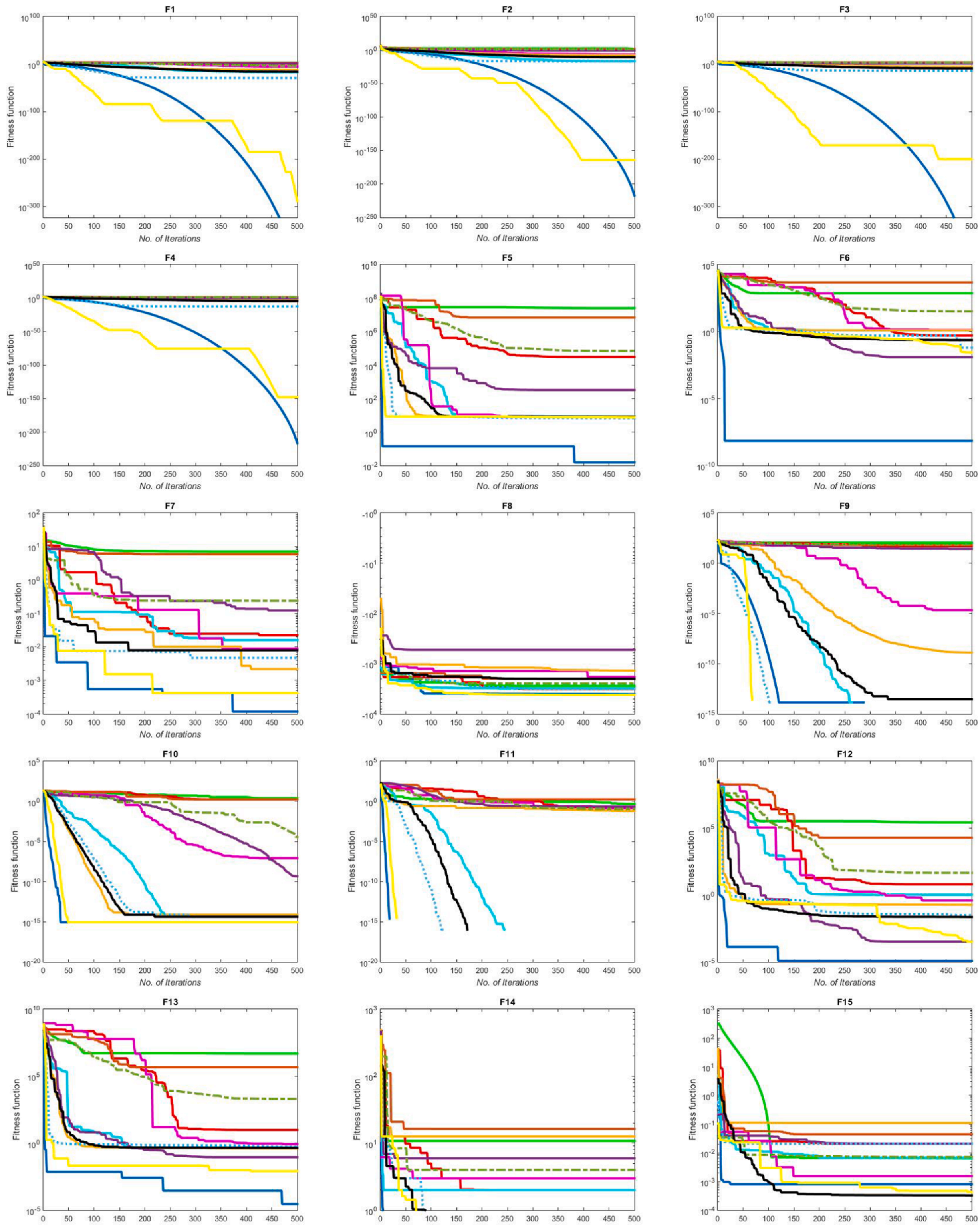

Fig. 9. Convergence behavior of the comparative algorithms on classical test functions (F1-F23).

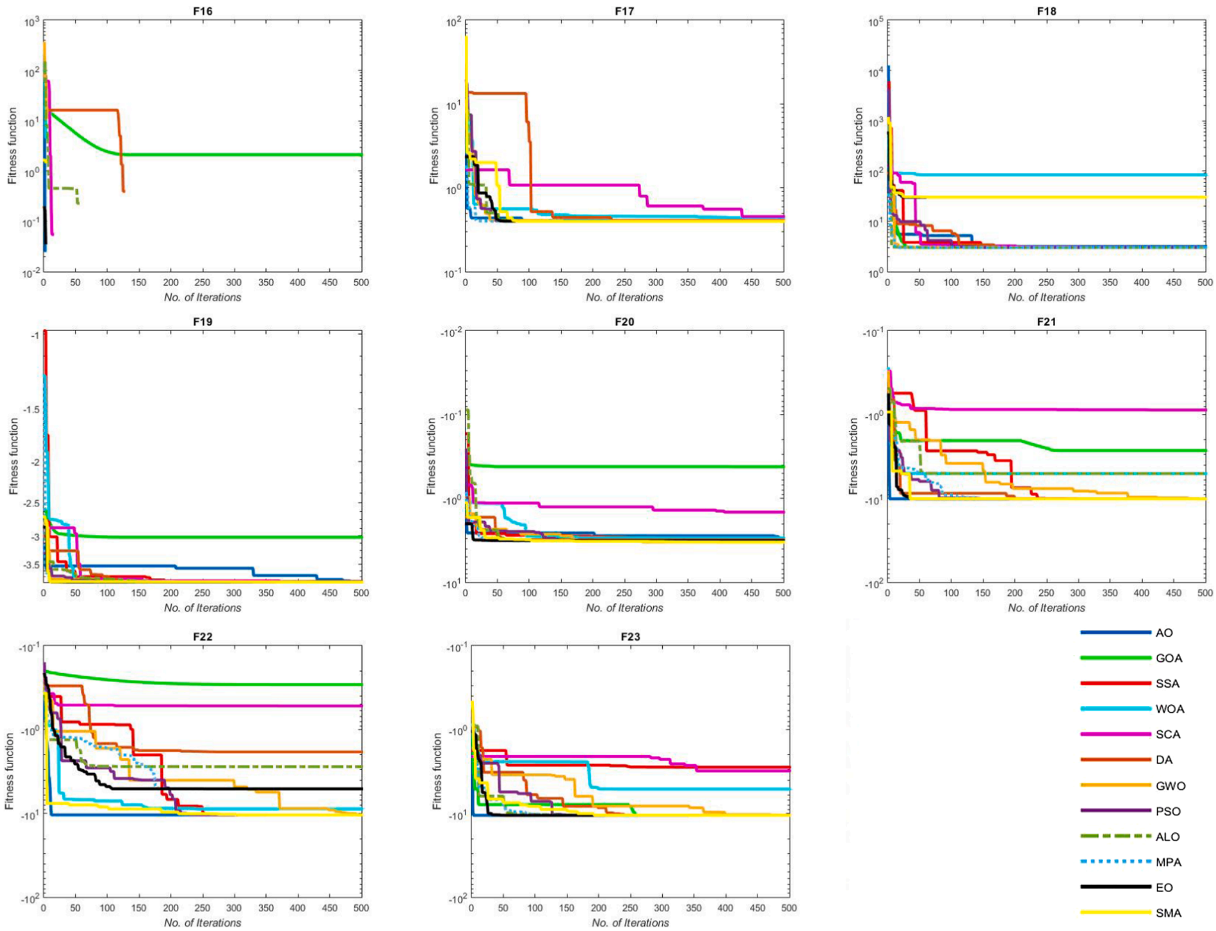

Fig. 9. (*continued*).

have been implemented for 30 independent runs with 500 iteration and 50 search agents. The worst, average, best, and STD values of the considered benchmarks' test-bed by the executed algorithms have been computed as in Tables 10,11 for the three studied dimensions of 50, 100, and 500, respectively. Furthermore, the number of the functions in which the algorithm can be classified as winning | tie | loss (W | L | T), as well as the total rank values of the algorithms, have been reported in the last lines of the Tables 10,11 to have a detailed statistical analysis. The obtained results confirm that the AO shows its reliability, stability, and robustness by increasing the optimized functions' dimension where it has the best performance for seven 50 dimensional functions ***(see Table 12).

Moreover, it is the best for nine functions for the cases of 100 and 500-dimensional test functions. This observation approves the AO's efficient exploration and exploitation capabilities in discovering the search domain while dealing with the many local optima of high dimensional multimodal functions and exploit the optimal solutions accurately. For the other counterparts, the SMA occupied the second rank after AO with success performance for only one function in 100 and 500-dimensional benchmarks. In contrast, the GOA, EO, PSO, DA, ALO, GWO, MPA, SSA, and SCA failed to show the best performance for these high dimensional functions. Sequentially, the AO outperforms the other counterparts and occupies the first rank for the three cases of high dimensional functions.

4.2. Results comparisons using CEC2017 test functions

To further investigate the quality of the proposed AO and examine its capabilities of the exploration, exploitation, and local optimum avoidance, one of the most challenging benchmarks has been employed is the CEC2017 suite. Four classes have been covered in the twenty-nigh CEC2017 functions that are unimodal(F1-F3), multimodal (F4-F11), hybrid (F12-F20), and finally, composition functions (F21-F30). The specifications of the CEC2017 has been reported in Table 13. We examined the AO with these benchmarks and compared it with well-regarded algorithms such as GA, PSO, CS, GWO, SSA, EO, GSA, and Covariance Matrix Adaptation Evolution Strategy (CMA-ES). All the algorithms have been executed for 30 independent runs with 500 iterations and 50 search agents. The average and STD values of the studied benchmarks set, moreover, the number of functions in which the algorithms can be accounted as the best | indistinguishable | inferior to the others, and the Friedman mean rank value, as well as the final ranks of the algorithms, are listed in Table 14.

The results divulge that the AO has a comparable performance concerning the SI techniques (PSO, CS, GWO, SSA, and EO), the EA algorithms (GA, CMA-ES), as well as the PhA one (GSA). The AO reaches for the optimal solutions of F9. It shows the nearest average values for the optimal solutions for other eight functions (F5, F14, F16, F20, F21, F24, F25, F30) meanwhile the GA, PSO, CS, GWO, EO, and GSA failed to

Table 10
Results of the comparative methods on classical test functions (F1-F13), the dimension is fixed to 50.

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F1												
Worst	0.0000E+00	7.8000E+03	4.0700E−21	6.9700E+00	3.6400E+03	1.1100E+04	3.7000E−11	1.5500E−21	1.0200E+03	7.0200E+03	5.5800E−42	1.9744E−220
Average	0.0000E+00	6.7200E+03	1.0800E−21	3.9900E+00	3.0200E+03	6.7300E+03	1.2100E−11	4.2500E−22	6.1400E+02	3.2800E+03	1.4000E−42	2.8206E−221
Best	0.0000E+00	4.7800E+03	3.2000E−23	2.3600E+00	2.4800E+03	3.7400E+03	3.0100E−12	3.6600E−24	2.8000E+02	5.7400E+02	4.5600E−58	0.0000E+00
STD	0.0000E+00	1.3600E+03	1.9900E−21	2.0300E+00	5.5200E+02	3.1100E+03	1.6600E−11	7.4800E−22	3.0500E+02	2.7000E+03	2.7900E−42	0.0000E+00
F2												
Worst	3.3730E−216	4.1325E+24	1.2911E−11	1.0885E+02	1.2331E+02	3.8276E+02	7.1711E−05	7.9169E−12	1.7569E+06	1.5811E+01	9.3673E−36	3.4565E−107
Average	1.2823E−216	1.0331E+24	9.3615E−12	9.3889E+01	7.9961E+01	2.1559E+02	6.7175E−05	3.4440E−12	4.4487E+05	7.9742E+00	3.6638E−36	4.9380E−108
Best	4.2658E−220	5.7983E+02	5.3561E−12	7.5642E+01	5.4524E+01	1.4414E+02	6.3534E−05	5.3007E−13	2.3050E+02	1.4311E+00	2.9744E−43	1.0290E−239
STD	0.0000E+00	2.0662E+24	3.3725E−12	1.3906E+01	3.0258E+01	1.1203E+02	3.5184E−06	3.5477E−12	8.7472E+05	7.0079E+00	4.0658E−36	1.3064E−107
F3												
Worst	0.0000E+00	8.1600E+04	2.1600E+02	1.6500E+04	1.6400E+05	1.6700E+05	3.3700E+03	1.2200E+01	6.1400E+04	1.5200E+05	5.5300E+05	6.7725E−202
Average	0.0000E+00	7.1900E+04	6.0200E+01	1.3900E+04	1.2800E+05	1.2000E+05	1.6200E+03	3.2800E+00	5.6400E+04	8.9000E+04	3.8900E+05	9.6750E−203
Best	0.0000E+00	5.7400E+04	1.8300E+00	9.0400E+03	8.9100E+04	8.7600E+04	2.7700E+02	8.4100E−02	5.2100E+04	3.3500E+04	3.1100E+05	0.0000E+00
STD	0.0000E+00	1.0300E+04	1.0400E+02	3.4400E+03	3.8900E+04	3.7200E+04	1.5800E+03	5.9400E+00	4.6600E+03	5.6500E+04	1.1100E+05	0.0000E+00
F4												
Worst	3.8171E−217	7.0400E+08	6.8700E+08	7.5600E+08	6.8000E+08	8.8900E+08	8.5300E+08	5.9300E+08	8.0000E+08	7.3000E+08	7.9800E+08	2.1946E−103
Average	1.1726E−217	6.5100E+08	6.1800E+08	6.5100E+08	5.5900E+08	7.6100E+08	6.4500E+08	5.3700E+08	6.7700E+08	6.3100E+08	6.9800E+08	3.1352E−104
Best	1.7827E−220	5.5800E+08	4.4000E+08	5.5300E+08	4.4700E+08	6.7300E+08	4.8300E+08	4.9100E+08	4.5800E+08	4.5000E+08	6.4900E+08	2.6621E−187
STD	0.0000E+00	6.4000E+07	1.1900E+08	8.3200E+07	9.5400E+07	9.2300E+07	1.5600E+08	4.2300E+07	1.5100E+08	1.2400E+08	6.7800E+07	8.2950E−104
F5												
Worst	1.6754E−02	7.2100E+07	4.8600E+01	1.4500E+05	9.1000E+07	2.5400E+07	4.8800E+01	4.8700E+01	3.2100E+07	5.6800E+07	4.8800E+01	4.8890E+01
Average	7.2723E−03	5.0400E+07	4.7900E+01	6.0800E+04	4.2900E+07	1.6500E+07	4.8700E+01	4.8700E+01	2.4000E+07	2.5200E+07	4.8800E+01	3.3403E+01
Best	3.5200E−05	3.8000E+07	4.7200E+01	1.6800E+04	7.7700E+06	5.5200E+06	4.8600E+01	4.8500E+01	1.7800E+07	1.9700E+06	4.8800E+01	6.3248E+00
STD	7.1591E−03	1.5000E+07	7.6000E−01	6.0300E+04	3.7000E+07	8.2200E+06	1.0600E−01	1.0900E−01	6.5700E+06	2.7600E+07	2.8700E−02	2.1293E+01
F6												
Worst	3.9756E−03	2.8600E+04	6.7700E+00	8.9100E+01	3.5200E+04	2.9000E+04	7.9100E+00	5.2600E+00	1.6600E+04	8.9800E+03	8.3900E+00	4.4631E+00
Average	1.0981E−03	2.0300E+04	6.3700E+00	6.9000E+01	2.3700E+04	2.4100E+04	7.1100E+00	4.3300E+00	1.0700E+04	4.0200E+03	6.5500E+00	3.0079E+00
Best	9.1128E−07	1.4200E+04	5.6700E+00	4.9100E+01	1.3600E+04	2.1700E+04	6.2300E+00	3.6900E+00	5.7800E+03	4.0400E+02	5.4400E+00	2.3585E−01
STD	1.6565E−03	6.0600E+03	4.7900E−01	1.7900E+01	9.3400E+03	3.3400E+03	6.9100E−01	7.1400E−01	4.4800E+03	3.7600E+03	1.3500E+00	1.6904E+00
F7												
Worst	1.2267E−03	4.0400E+02	1.7000E−02	3.2000E+02	3.0500E+01	3.1500E+01	5.0400E−02	8.1400E−03	1.2200E+01	2.0200E+01	3.0800E−02	3.0226E−03
Average	5.3134E−04	3.0000E+02	6.7800E−03	2.0100E+02	1.5200E+01	1.8800E+01	2.6900E−02	4.8000E−03	1.0500E+01	1.5200E+01	1.6200E−02	1.0741E−03
Best	1.5038E−05	2.4800E+02	1.0800E−03	1.2200E+02	1.9000E+00	9.0300E+00	1.2100E−02	2.4700E−03	8.5100E+00	4.5600E+00	7.2900E−03	1.4281E−04
STD	5.1254E−04	7.0200E+01	7.1200E−03	9.0000E+01	1.1800E+01	1.0300E+01	1.7400E−02	2.4900E−03	1.6900E+00	7.2200E+00	1.0900E−02	1.1286E−03
F8												
Worst	−6.9100E+03	−8.2500E+03	−7.9100E+03	−2.8200E+03	−4.2000E+03	−9.0300E+03	−6.8400E+03	−1.0400E+04	−8.6400E+03	−3.6700E+03	−8.6000E+03	−2.0822E+04
Average	−1.0100E+04	−9.3000E+03	−1.0700E+04	−3.9000E+03	−5.6700E+03	−9.0300E+03	−7.7000E+03	−1.1400E+04	−9.0800E+03	−4.0200E+03	−1.2400E+04	−2.0887E+04
Best	−1.2800E+04	−9.8500E+03	−1.2800E+04	−5.2700E+03	−6.6100E+03	−9.0300E+03	−8.8800E+03	−1.2600E+04	−9.6100E+03	−4.2500E+03	−1.5000E+04	−2.0947E+04
STD	2.0800E+03	7.2300E+02	2.0800E+03	1.0200E+03	1.1200E+03	0.0000E+00	8.9100E+02	9.6400E+02	4.3700E+02	2.4800E+02	2.9900E+03	5.2103E+01
F9												
Worst	0.0000E+00	6.9200E+02	9.9800E−01	4.7400E+02	5.3200E+02	4.3700E+02	3.0600E+01	0.0000E+00	4.3600E+02	3.5600E+02	0.0000E+00	0.0000E+00
Average	0.0000E+00	6.0200E+02	4.9900E−01	3.5900E+02	4.5100E+02	3.8000E+02	2.0800E+01	0.0000E+00	3.8300E+02	2.0300E+02	0.0000E+00	0.0000E+00
Best	0.0000E+00	5.2200E+02	1.1400E−13	3.0200E+02	3.0900E+02	2.9500E+02	8.9400E+00	0.0000E+00	3.1100E+02	1.0700E+02	0.0000E+00	0.0000E+00
STD	0.0000E+00	7.0000E+01	5.7600E−01	8.0700E+01	9.8900E+01	6.7400E+01	1.0800E+01	0.0000E+00	5.8400E+01	1.1000E+02	0.0000E+00	0.0000E+00
F10												
Worst	8.8818E−16	2.0600E+01	3.5200E−08	6.3800E+00	1.8900E+01	1.7900E+01	2.1800E−04	2.3100E−12	1.9000E+01	2.0600E+01	1.4700E−13	8.8818E−16
Average	8.8818E−16	2.0000E+01	1.7600E−08	4.9700E+00	1.5500E+01	1.2700E+01	1.4400E−04	1.6000E−12	1.8300E+01	1.9500E+01	5.0600E−14	8.8818E−16
Best	8.8818E−16	1.9300E+01	8.2300E−09	4.3700E+00	1.2700E+01	1.7300E+00	9.6100E−05	2.9200E−13	1.7200E+01	1.6200E+01	4.4400E−15	8.8818E−16
STD	0.0000E+00	5.4300E−01	1.2000E−08	9.4200E−01	2.5600E+00	7.4200E+00	5.2200E−05	8.9500E−13	7.7600E−01	2.1900E+00	6.5800E−14	0.0000E+00
F11												
Worst	0.0000E+00	2.1700E+02	7.9900E−15	1.3300E+02	1.2800E+02	2.5700E+02	7.6800E−02	1.0500E−02	1.6300E+02	2.2100E+01	0.0000E+00	0.0000E+00

(*continued on next page*)

Table 10 (*continued*)

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
Average	0.0000E+00	1.5300E+02	2.4700E−15	1.1700E+02	9.0500E+01	2.2600E+02	4.7100E−02	2.6300E−03	1.0300E+02	1.4600E+01	0.0000E+00	0.0000E+00
Best	0.0000E+00	1.1300E+02	0.0000E+00	8.9700E+01	4.3000E+01	2.0500E+02	5.2200E−06	0.0000E+00	7.0300E+01	2.5800E+00	0.0000E+00	0.0000E+00
STD	0.0000E+00	4.4500E+01	3.7400E−15	1.9400E+01	3.7900E+01	2.4700E+01	3.3000E−02	5.2600E−03	4.1600E+01	8.8100E+00	0.0000E+00	0.0000E+00
F12												
Worst	3.4745E−06	7.2887E+07	3.5673E−01	2.2947E+01	7.4787E+05	4.7065E+07	7.4009E−01	2.0077E−01	1.9431E+07	9.2098E+07	7.0376E−01	5.7213E−01
Average	1.0767E−06	3.6137E+07	3.0089E−01	1.4439E+01	4.7637E+05	1.5716E+07	6.1270E−01	1.3741E−01	1.0097E+07	6.4479E+07	4.7881E−01	1.5052E−01
Best	6.2530E−08	1.6941E+07	2.5770E−01	9.8132E+00	1.6807E+01	2.1230E+06	4.2030E−01	7.6591E−02	2.1004E+06	1.3318E+07	2.2526E−01	1.9706E−03
STD	1.4585E−06	2.5708E+07	4.6545E−02	5.8116E+00	3.3844E+05	2.1337E+07	1.5289E−01	5.5152E−02	7.4632E+06	3.5021E+07	2.3845E−01	2.4430E−01
F13												
Worst	1.4249E−04	1.7700E+08	4.3400E+00	1.2400E+03	5.5200E+07	1.7800E+08	4.6200E+00	4.7000E+00	4.0000E+07	2.1200E+08	4.4500E+00	4.9432E+00
Average	6.4589E−05	1.2900E+08	4.1100E+00	3.3800E+02	1.9100E+07	9.6700E+07	4.0500E+00	4.2200E+00	2.5500E+07	9.3600E+07	3.7500E+00	2.0861E+00
Best	1.2283E−05	7.4000E+07	3.8200E+00	3.2200E+01	4.1300E+06	2.6500E+07	3.5900E+00	3.7100E+00	7.5900E+06	1.9100E+07	3.2100E+00	4.2197E−04
STD	6.6508E−05	4.4100E+07	2.2000E−01	5.9900E+02	2.4300E+07	7.7100E+07	4.9100E−01	4.0800E−01	1.3400E+07	8.2600E+07	5.4500E−01	2.5937E+00
(W|L|T)	(7|3|3)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|12|1)	(0|12|1)	(0|13|0)	(0|11|2)	(2|9|2)
Mean	1.3077E+00	1.0538E+01	4.3846E+00	8.1538E+00	8.9231E+00	1.0154E+01	5.9231E+00	3.4615E+00	8.8462E+00	8.9231E+00	4.6154E+00	1.7692E+00
Ranking	1	12	4	7	9	11	6	3	8	9	5	2

Table 11
Results of the comparative methods on classical test functions (F1-F13), the dimension is fixed to 100.

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F1												
Worst	0.0000E+00	3.9700E+04	2.3000E−17	1.3900E+02	1.7200E+04	5.7700E+04	1.0400E−06	2.5900E−21	2.0400E+04	1.5700E+04	9.7200E−41	2.6189E−160
Average	0.0000E+00	3.5100E+04	6.2100E−18	1.0300E+02	1.1800E+04	3.4800E+04	5.1900E−07	1.5600E−21	1.8900E+04	9.4400E+03	2.4300E−41	5.2392E−161
Best	0.0000E+00	3.1200E+04	2.0900E−19	7.6100E+01	7.0700E+03	1.7900E+04	2.1300E−07	3.7200E−22	1.7500E+04	4.8200E+03	2.7400E−47	0.0000E+00
STD	0.0000E+00	3.6500E+03	1.1200E−17	2.9100E+01	5.2900E+03	2.0100E+04	3.6400E−07	1.0300E−21	1.1900E+03	4.9700E+03	4.8600E−41	1.1711E−160
F2												
Worst	9.2889E−217	1.1452E+21	5.2919E−09	7.7670E+01	2.0112E+02	2.1207E+02	1.6321E−04	4.2119E−13	2.9650E+06	3.7443E+00	1.0113E−20	2.2415E−108
Average	5.0995E−217	2.8631E+20	3.3539E−09	6.0455E+01	1.4725E+02	2.0087E+02	9.6421E−05	1.5599E−13	7.4137E+05	2.1442E+00	2.5286E−21	4.4830E−109
Best	2.1750E−218	2.2804E+07	1.0910E−09	4.1852E+01	7.4817E+01	1.8230E+02	4.5429E−05	5.0701E−14	1.4328E+02	6.7898E−01	6.1115E−30	1.1235E−256
STD	0.0000E+00	5.7262E+20	1.8478E−09	1.8648E+01	5.8095E+01	1.3305E+01	5.3157E−05	1.7731E−13	1.4824E+06	1.6112E+00	5.0564E−21	1.0024E−108
F3												
Worst	0.0000E+00	8.7200E+04	5.9900E+01	1.5700E+04	1.9800E+05	1.7800E+05	6.8400E+02	1.8400E+00	6.5900E+04	9.5600E+04	4.7900E+05	4.9870E−188
Average	0.0000E+00	6.7100E+04	1.7900E+01	1.2400E+04	1.2800E+05	1.3000E+05	4.6100E+02	4.9400E−01	5.6000E+04	6.7800E+04	3.8200E+05	9.9741E−189
Best	0.0000E+00	5.4200E+04	4.4000E−02	1.0700E+04	5.5900E+04	9.1100E+04	1.4000E+02	5.1200E−07	4.6300E+04	4.0300E+04	3.0400E+05	0.0000E+00
STD	0.0000E+00	1.4500E+04	2.8500E+01	2.3400E+03	5.7900E+04	3.9000E+04	2.2900E+02	8.9900E−01	8.6300E+03	2.8100E+04	7.6700E+04	0.0000E+00
F4												
Worst	8.1471E−219	6.6000E+01	7.8600E−03	2.8400E+01	5.5300E+01	7.0900E+01	1.4800E+00	1.0900E−08	6.9600E+01	8.9700E+01	9.8200E+01	1.3547E−98
Average	2.5154E−219	6.2700E+01	4.2000E−03	2.4800E+01	4.3900E+01	5.5400E+01	1.1700E+00	5.9700E−09	6.5100E+01	7.9500E+01	8.5400E+01	2.7108E−99
Best	1.2027E−221	5.5400E+01	4.2300E−04	2.1900E+01	3.2900E+01	3.5100E+01	8.8100E−01	2.8900E−09	5.2700E+01	6.9000E+01	5.4600E+01	2.9552E−153
STD	0.0000E+00	4.9500E+00	3.1900E−03	3.2900E+00	9.3700E+00	1.4900E+01	3.1200E−01	3.7100E−09	8.2800E+00	8.6500E+00	2.0600E+01	6.0578E−99
F5												
Worst	1.0092E−01	1.3800E+08	4.8000E+01	3.6600E+04	1.6900E+07	3.1200E+07	4.8800E+01	4.8700E+01	3.3300E+07	3.1200E+07	4.8900E+01	9.8885E+01

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Table 11 (*continued*)

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
Average	2.4782E−02	9.1100E+07	4.7900E+01	2.8000E+04	9.5300E+06	1.9400E+07	4.8700E+01	4.8400E+01	1.2300E+07	1.6600E+07	4.8800E+01	6.0627E+01
Best	6.5611E−04	4.4100E+07	4.7500E+01	1.9200E+04	2.2100E+05	8.8800E+06	4.8600E+01	4.7900E+01	3.8100E+06	7.6500E+05	4.8700E+01	4.9378E−01
STD	4.2755E−02	4.4500E+07	2.6000E−01	9.6200E+03	8.6800E+06	9.2700E+06	1.1200E−01	3.7400E−01	1.4100E+07	1.6700E+07	6.8400E−02	5.2351E+01
F6												
Worst	2.1929E−03	2.5744E+04	6.2188E+00	1.3322E+02	3.3508E+04	4.2545E+04	7.3563E+00	5.1809E+00	1.4103E+04	5.2709E+03	7.3056E+00	1.4490E+01
Average	5.4323E−04	1.9595E+04	5.9007E+00	6.3916E+01	1.6152E+04	2.7833E+04	6.9873E+00	4.0499E+00	1.0800E+04	3.0192E+03	6.9144E+00	1.0255E+01
Best	1.2605E−05	1.2858E+04	5.4759E+00	3.0782E+01	5.2722E+03	1.9855E+04	5.9623E+00	2.7927E+00	7.1065E+03	4.9268E+02	6.3303E+00	8.7579E−01
STD	9.2961E−04	5.2781E+03	3.5111E−01	4.7639E+01	1.3542E+04	1.0379E+04	6.8377E−01	1.1065E+00	3.3932E+03	1.9831E+03	4.1399E−01	5.5614E+00
F7												
Worst	1.5608E−03	4.8350E+02	9.1935E−03	2.6149E+02	4.3638E+01	3.0933E+01	2.7556E−02	4.5624E−03	1.6360E+01	2.2357E+01	1.7910E−02	2.6144E−03
Average	8.9547E−04	3.6677E+02	6.4348E−03	9.5848E+01	1.9206E+01	1.7896E+01	1.9682E−02	2.1244E−03	1.1079E+01	1.3982E+01	9.3010E−03	1.4817E−03
Best	1.1121E−04	2.4063E+02	5.1110E−03	3.1747E+01	8.1894E+00	7.1799E+00	1.2770E−02	3.0150E−04	8.7359E+00	5.7753E+00	4.0999E−03	2.8307E−04
STD	7.0387E−04	1.1768E+02	1.8724E−03	1.1103E+02	1.6431E+01	1.0755E+01	7.0929E−03	1.7809E−03	3.5519E+00	7.4866E+00	6.2593E−03	1.0280E−03
F8												
Worst	−6.4064E+03	−8.9826E+03	−8.4348E+03	−2.7692E+03	−3.6225E+03	−9.0295E+03	−6.6831E+03	−1.0349E+04	−8.1524E+03	−3.4601E+03	−9.5209E+03	−4.0264E+04
Average	−9.9172E+03	−9.8724E+03	−1.0147E+04	−3.9191E+03	−5.0974E+03	−9.0536E+03	−7.8497E+03	−1.1362E+04	−8.5828E+03	−3.8878E+03	−1.0332E+04	−4.1488E+04
Best	−1.2910E+04	−1.0458E+04	−1.0847E+04	−5.8528E+03	−6.3641E+03	−9.1259E+03	−8.8646E+03	−1.3138E+04	−9.0583E+03	−4.0823E+03	−1.2162E+04	−4.1895E+04
STD	2.4531E+03	6.5467E+02	1.1508E+03	1.3438E+03	1.2780E+03	4.8210E+01	1.1649E+03	1.2581E+03	4.0865E+02	2.8768E+02	1.2436E+03	6.9282E+02
F9												
Worst	0.0000E+00	7.0663E+02	2.0900E+00	3.3931E+02	5.6783E+02	3.7412E+02	1.7930E+01	0.0000E+00	4.0301E+02	1.7052E+02	0.0000E+00	0.0000E+00
Average	0.0000E+00	6.6341E+02	5.2251E−01	2.9352E+02	4.5123E+02	3.3656E+02	1.4038E+01	0.0000E+00	3.5050E+02	7.2797E+01	0.0000E+00	0.0000E+00
Best	0.0000E+00	6.2961E+02	5.6843E−14	2.4692E+02	3.8412E+02	2.9418E+02	8.7782E+00	0.0000E+00	3.2658E+02	2.6505E+01	0.0000E+00	0.0000E+00
STD	0.0000E+00	3.1910E+01	1.0450E+00	3.7772E+01	8.2045E+01	4.0240E+01	4.1669E+00	0.0000E+00	3.6131E+01	6.6103E+01	0.0000E+00	0.0000E+00
F10												
Worst	8.8818E−16	1.9924E+01	1.6431E−13	1.9307E−02	8.4589E+00	1.6272E+01	7.8494E−10	2.5757E−14	3.4042E+00	4.3402E−01	2.9310E−14	8.8818E−16
Average	8.8818E−16	1.9423E+01	1.0303E−13	9.4472E−03	7.7682E+00	9.8105E+00	4.1539E−10	9.7700E−15	1.8245E+00	1.0968E−01	1.1546E−14	8.8818E−16
Best	8.8818E−16	1.8952E+01	4.3521E−14	3.3824E−04	7.0928E+00	2.3170E+00	6.4845E−11	4.4409E−15	1.1551E+00	6.6117E−04	8.8818E−16	8.8818E−16
STD	0.0000E+00	4.0023E−01	5.3909E−14	8.0172E−03	6.7957E−01	5.7527E+00	3.2156E−10	1.0658E−14	1.0672E+00	2.1623E−01	1.2644E−14	0.0000E+00
F11												
Worst	0.0000E+00	5.8841E−01	2.0230E−02	3.1030E+01	1.5024E+01	4.5986E−01	5.9418E−02	3.9454E−02	3.2876E−01	6.7533E−01	3.0136E−01	0.0000E+00
Average	0.0000E+00	5.4089E−01	5.0574E−03	9.0836E+00	5.9464E+00	2.2969E−01	2.7528E−02	1.6716E−02	1.9871E−01	3.7989E−01	1.3220E−01	0.0000E+00
Best	0.0000E+00	4.5020E−01	0.0000E+00	5.4626E−01	2.6364E+00	1.1258E−01	1.1102E−16	0.0000E+00	7.1478E−02	4.4130E−04	0.0000E+00	0.0000E+00
STD	0.0000E+00	6.1730E−02	1.0115E−02	1.4657E+01	6.0586E+00	1.5614E−01	3.1985E−02	1.9918E−02	1.0506E−01	3.3104E−01	1.5560E−01	0.0000E+00
F12												
Worst	2.2090E−05	5.3071E+04	1.9912E−02	4.2285E−04	4.4872E+06	4.3171E+01	6.4279E−02	2.5172E−03	9.9366E+00	3.2393E−01	3.5385E−01	1.0142E+00
Average	7.3398E−06	1.3376E+04	7.2748E−03	1.3397E−04	1.1218E+06	2.4414E+01	5.6330E−02	7.8564E−04	4.5891E+00	2.2991E−01	1.9968E−01	4.3739E−01
Best	6.8016E−08	2.2231E+01	4.8091E−05	3.6038E−06	1.0234E+01	1.3639E+01	4.1262E−02	3.8885E−10	1.2334E+00	1.5004E−01	1.3770E−01	2.5910E−03
STD	8.7088E−06	2.6463E+04	9.4262E−03	1.9609E−04	2.2436E+06	1.3499E+01	1.0417E−02	1.1914E−03	3.7974E+00	7.6710E−02	1.0328E−01	4.0437E−01
F13												
Worst	4.2673E−05	3.7036E+04	7.7126E−01	2.1318E−02	8.4145E+05	3.1735E+01	6.0130E−01	1.1813E−01	3.0647E−01	6.8286E−01	7.3070E−01	9.8429E+00
Average	1.8432E−05	1.4949E+04	2.8896E−01	5.4671E−03	2.3951E+05	1.3111E+01	3.8014E−01	3.7320E−02	1.6538E−01	5.4638E−01	4.4940E−01	5.2416E+00
Best	2.5705E−07	5.8035E+01	9.9259E−02	1.7087E−05	2.4995E+03	1.9905E+00	2.1468E−01	2.7350E−04	1.7032E−02	4.6047E−01	1.5378E−01	4.0715E−01
STD	1.6556E−05	1.8163E+04	3.2255E−01	1.0568E−02	4.0346E+05	1.2893E+01	1.6531E−01	5.4378E−02	1.1949E−01	9.6117E−02	3.2217E−01	4.3678E+00
(W|L|T)	(9|1|3)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|12|1)	(1|9|3)
Mean	1.3077E+00	1.0615E+01	4.0769E+00	7.2308E+00	9.9231E+00	9.7692E+00	5.6923E+00	2.9231E+00	8.4615E+00	8.6154E+00	5.4615E+00	3.3077E+00
Ranking	1	12	4	7	11	10	6	2	8	9	5	3

Table 12
Results of the comparative methods on classical test functions (F1-F13), the dimension is fixed to 500.

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F1												
Worst	0.0000E+00	9.1900E+04	4.2300E−12	2.2800E+03	4.8100E+04	1.2500E+05	3.0300E−03	2.6200E−22	9.4500E+04	2.1100E+04	9.4900E−24	1.8453E−149
Average	0.0000E+00	8.8900E+04	1.5300E−12	1.6700E+03	3.7400E+04	9.0600E+04	1.6800E−03	1.3400E−22	7.6100E+04	1.5500E+04	2.6700E−24	3.6907E−150
Best	0.0000E+00	8.1900E+04	1.3700E−13	1.2300E+03	2.7400E+04	5.9200E+04	7.6900E−04	4.7200E−24	5.8900E+04	1.0900E+04	1.4800E−37	0.0000E+00
STD	0.0000E+00	4.7000E+03	1.9100E−12	4.5400E+02	8.7000E+03	3.0700E+04	9.8500E−04	1.0600E−22	1.4600E+04	5.1900E+03	4.5800E−24	8.2526E−150
F2												
Worst	2.6672E−215	1.7200E+137	5.0800E−06	1.6200E+161	1.2400E+03	2.6900E+247	2.9500E+00	1.9400E−10	1.3300E+171	2.3100E+02	3.9300E−21	5.8913E−92
Average	1.4218E−215	4.3100E+136	3.8000E−06	4.0500E+160	1.0400E+03	6.7200E+246	1.8200E+00	1.0600E−10	3.3200E+170	1.7800E+02	1.1800E−21	1.1783E−92
Best	1.4735E−217	2.2500E+103	2.7300E−06	1.0400E+67	7.1600E+02	2.0600E+03	3.9600E−01	1.3800E−11	1.6900E+132	1.3500E+02	6.4100E−25	3.7965E−197
STD	0.0000E+00	8.6100E+136	1.1200E−06	6.5500E+04	2.2500E+02	6.5500E+04	1.1400E+00	9.5200E−11	6.5500E+04	4.3900E+01	1.8700E−21	2.6347E−92
F3												
Worst	0.0000E+00	7.6400E+06	2.0100E+05	2.7100E+06	1.7000E+07	1.0500E+07	1.0500E+06	8.7700E+04	8.5100E+06	1.2300E+07	1.4400E+08	5.9228E−193
Average	0.0000E+00	6.0500E+06	1.1800E+05	1.5500E+06	1.1500E+07	8.1300E+06	8.8900E+05	4.8900E+04	5.6900E+06	8.5600E+06	8.5600E+07	1.2030E−193
Best	0.0000E+00	3.3900E+06	3.6100E+04	7.8500E+05	5.9700E+06	4.8100E+06	7.3800E+05	1.5100E+04	2.7900E+06	6.2900E+06	3.3800E+07	0.0000E+00
STD	0.0000E+00	1.8700E+06	9.3200E+04	8.7800E+05	4.6900E+06	2.3900E+06	1.4700E+05	3.9200E+04	2.3500E+06	2.6000E+06	5.7100E+07	0.0000E+00
F4												
Worst	6.3989E−217	9.5500E+01	9.8200E+01	6.3300E+01	9.9500E+01	8.8300E+01	9.0900E+01	1.2100E−05	9.7600E+01	9.9700E+01	9.9600E+01	2.3802E−82
Average	2.5832E−217	8.9700E+01	7.5500E+01	5.8300E+01	9.8600E+01	8.4100E+01	8.0600E+01	3.5700E−06	9.7100E+01	9.9600E+01	9.2400E+01	4.7605E−83
Best	6.2842E−219	8.7000E+01	5.2500E+01	5.5300E+01	9.6800E+01	7.9300E+01	7.1200E+01	2.9600E−07	9.6400E+01	9.9500E+01	7.9900E+01	1.1106E−204
STD	0.0000E+00	3.9100E+00	1.9400E+01	3.5100E+00	1.2300E+00	4.4100E+00	1.0100E+01	5.7000E−06	5.5500E−01	1.1500E−01	8.7600E+00	1.0645E−82
F5												
Worst	6.4568E−01	2.5900E+09	4.9900E+02	5.8700E+08	1.8100E+09	3.3700E+09	1.0700E+06	4.9900E+02	4.9200E+09	3.2100E+09	4.9800E+02	2.9883E+02
Average	1.6470E−01	2.4100E+09	4.9900E+02	5.5100E+08	1.1600E+09	2.2000E+09	6.5400E+05	4.9800E+02	4.3800E+09	2.5400E+09	4.9800E+02	2.0173E+02
Best	8.1831E−03	2.2900E+09	4.9900E+02	4.5100E+08	6.9000E+08	1.7700E+09	1.3000E+05	4.9800E+02	3.9700E+09	1.7700E+09	4.9700E+02	1.5641E+01
STD	2.6993E−01	1.2700E+08	2.5100E−02	6.6700E+07	5.3300E+08	7.8100E+08	3.9700E+05	8.9800E−02	4.0300E+08	6.1300E+08	4.2100E−01	1.3600E+02
F6												
Worst	6.1400E−03	8.0834E+05	1.1851E+02	2.3018E+05	3.5784E+05	1.0929E+06	1.6278E+02	1.0815E+02	1.0545E+06	2.6911E+05	1.0385E+02	6.2300E+01
Average	1.8268E−03	7.6086E+05	1.1766E+02	2.2730E+05	3.1464E+05	1.0002E+06	1.4463E+02	1.0732E+02	9.5250E+05	2.5202E+05	1.0107E+02	5.0294E+01
Best	6.7168E−05	7.3346E+05	1.1669E+02	2.2237E+05	2.3105E+05	8.7263E+05	1.1964E+02	1.0565E+02	8.5874E+05	2.3662E+05	9.8640E+01	1.0348E+01
STD	2.5734E−03	4.1287E+04	9.1754E−01	4.2884E+03	7.2403E+04	1.1421E+05	2.2367E+01	1.4438E+00	9.8146E+04	1.6312E+04	2.6227E+00	2.2375E+01
F7												
Worst	3.4682E−03	5.1100E+04	3.0500E−02	6.1800E+04	7.9200E+03	1.3900E+04	9.8500E−01	8.0900E−03	1.8700E+04	2.0700E+04	4.9200E−02	6.2744E−04
Average	9.8927E−04	4.7300E+04	1.8200E−02	5.8400E+04	4.0600E+03	1.0200E+04	7.1100E−01	4.4800E−03	1.1700E+04	1.7600E+04	3.3100E−02	5.0630E−04
Best	2.2066E−04	4.2500E+04	9.0200E−03	5.2300E+04	1.2100E+03	5.9500E+03	5.2800E−01	2.4600E−03	8.6500E+03	1.5200E+04	1.3000E−02	4.0625E−04
STD	1.3900E−03	3.5700E+03	1.0100E−02	4.3000E+03	2.8200E+03	4.1300E+03	2.1800E−01	2.4900E−03	4.7000E+03	2.7400E+03	1.5500E−02	7.9220E−05
F8												
Worst	−1.3853E+04	−3.3200E+04	−3.8500E+04	−9.5600E+03	−1.5000E+04	−9.0300E+04	−1.0200E+04	−5.1000E+04	−2.6700E+04	−1.2100E+04	−1.2600E+05	−1.1666E+05
Average	−1.8631E+04	−3.7000E+04	−4.2600E+04	−1.1900E+04	−1.7000E+04	−9.0300E+04	−2.9100E+04	−5.5300E+04	−3.2500E+04	−1.3300E+04	−1.3900E+05	−1.2362E+05
Best	−2.2142E+04	−3.9500E+04	−4.5800E+04	−1.5600E+04	−1.9000E+04	−9.0300E+04	−3.6300E+04	−5.9600E+04	−3.7300E+04	−1.4500E+04	−1.5000E+05	−1.2564E+05
STD	2.9872E+03	2.6900E+03	3.2600E+03	2.8500E+03	1.6400E+03	0.0000E+00	1.2600E+04	4.7200E+03	4.7100E+03	1.0400E+03	9.9700E+03	3.9005E+03
F9												
Worst	0.0000E+00	8.4800E+03	1.0200E+00	8.3500E+03	6.6500E+03	6.1800E+03	3.5800E+02	0.0000E+00	6.0000E+03	1.4800E+03	0.0000E+00	0.0000E+00
Average	0.0000E+00	8.1600E+03	2.5500E−01	7.8200E+03	5.9700E+03	5.7900E+03	2.7700E+02	0.0000E+00	5.9500E+03	1.2700E+03	0.0000E+00	0.0000E+00
Best	0.0000E+00	8.0400E+03	5.0600E−10	7.2600E+03	5.2700E+03	5.2900E+03	1.2100E+02	0.0000E+00	5.8700E+03	1.1400E+03	0.0000E+00	0.0000E+00
STD	0.0000E+00	2.1100E+02	5.0900E−01	5.3400E+02	5.8100E+02	3.7900E+02	1.0600E+02	0.0000E+00	5.4600E+01	1.4400E+02	0.0000E+00	0.0000E+00
F10												
Worst	8.8800E−16	2.0400E+01	1.5800E−03	1.8900E+01	1.9100E+01	1.9800E+01	2.5200E+00	3.7700E−09	2.0800E+01	2.0900E+01	3.9200E−09	8.8818E−16
Average	8.8800E−16	2.0100E+01	7.0900E−04	1.8600E+01	1.8500E+01	1.3500E+01	2.1400E+00	2.4200E−09	2.0500E+01	2.0900E+01	1.2100E−09	8.8818E−16
Best	8.8800E−16	1.9800E+01	1.9100E−04	1.8300E+01	1.7300E+01	8.8800E−16	1.4800E+00	1.4100E−09	2.0200E+01	2.0900E+01	1.1100E−10	8.8818E−16
STD	0.0000E+00	2.6100E−01	6.2700E−04	2.5200E−01	8.6500E−01	9.0800E+00	4.5700E−01	1.0100E−09	2.6000E−01	2.1700E−02	1.8200E−09	0.0000E+00
F11												
Worst	0.0000E+00	6.3700E+03	1.4600E−09	1.9700E+03	2.8000E+03	8.2800E+03	1.3400E+00	0.0000E+00	6.6500E+03	2.2400E+03	0.0000E+00	0.0000E+00

(*continued on next page*)

Table 12 (*continued*)

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
Average	0.0000E+00	5.7200E+03	7.4500E−10	1.8900E+03	2.0500E+03	6.4600E+03	1.1400E+00	0.0000E+00	6.5100E+03	1.6500E+03	0.0000E+00	0.0000E+00
Best	0.0000E+00	5.2900E+03	2.6200E−10	1.8100E+03	1.4400E+03	5.5700E+03	9.1900E−01	0.0000E+00	6.1500E+03	7.9500E+02	0.0000E+00	0.0000E+00
STD	0.0000E+00	4.8400E+02	5.1900E−10	6.6700E+01	5.6400E+02	1.2300E+03	1.7100E−01	0.0000E+00	2.4100E+02	6.2600E+02	0.0000E+00	0.0000E+00
F12												
Worst	2.2090E−05	6.3500E+03	7.0800E−09	2.0600E+03	3.2400E+03	6.2400E+03	1.2800E+00	0.0000E+00	6.1600E+03	3.3600E+03	1.1100E−16	1.0142E+00
Average	7.3398E−06	5.8700E+03	3.6600E−09	1.9300E+03	2.1300E+03	5.4900E+03	1.0100E+00	0.0000E+00	5.6500E+03	2.2300E+03	2.7800E−17	4.3739E−01
Best	6.8016E−08	5.4400E+03	5.5000E−10	1.8300E+03	1.2300E+03	4.2900E+03	3.6700E−01	0.0000E+00	5.2500E+03	1.3000E+03	0.0000E+00	2.5910E−03
STD	8.7088E−06	3.7500E+02	2.8000E−09	9.7300E+01	8.6500E+02	8.6100E+02	4.3100E−01	0.0000E+00	3.8300E+02	8.8600E+02	5.5500E−17	4.0437E−01
F13												
Worst	4.2673E−05	7.7972E+09	4.9839E+01	2.8311E+08	2.7634E+09	7.8310E+09	3.0577E+02	4.9837E+01	8.6217E+09	1.0576E+10	4.7792E+01	9.8429E+00
Average	1.8432E−05	7.1971E+09	4.9581E+01	2.6358E+08	1.5912E+09	7.2341E+09	2.0353E+02	4.9799E+01	7.8299E+09	8.8182E+09	3.9005E+01	5.2416E+00
Best	2.5705E−07	6.5728E+09	4.9172E+01	2.4510E+08	1.1179E+09	6.6313E+09	1.0472E+02	4.9764E+01	7.2658E+09	6.5647E+09	3.0093E+01	4.0715E−01
STD	1.6556E−05	6.0494E+08	2.9263E−01	1.5540E+07	7.8420E+08	5.7852E+08	8.3825E+01	3.7117E−02	6.3361E+08	1.7158E+09	8.0546E+00	4.3678E+00
(W|L|T)	(9|3|1)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(0|13|0)	(1|11|1)	(0|13|0)	(0|13|0)	(1|11|1)	(1|11|1)
Mean	1.9231E+00	9.6923E+00	4.6154E+00	8.2308E+00	8.9231E+00	9.0769E+00	6.0769E+00	3.0769E+00	1.0077E+01	9.5385E+00	3.7692E+00	2.0000E+00
Ranking	1	11	5	7	8	9	6	3	12	10	4	2

classified as the best algorithms for any function. The CMA-ES is considered the stronger competitor for AO with achieving the best response for eight functions. Most of the functions in which the AO has the best response from the hybrid and composition functions affirm the high quality of the exploration and exploitation phases of the AO and its ability to deal with the high local optimal of the composition functions. The AO has no significant difference from the other algorithms in 13 functions and has a minimum Friedman mean rank across the studied test-bed suit course. Therefore it has the first rank as a final score over these studied functions. The best convergence curves of the implemented algorithms have been depicted in Fig. 10. By inspecting these curves, one can see that the AO exhibits the fastest convergence for (F5, F9, F16, F20, F21, F24, F28, and F30) and a comparable convergence for (F1, F11, F14, F15, F18, F22, and F 27). This observation states that AO can be considered as one of the reliable algorithms.

4.3. Results comparisons using CEC2019 test functions

This section details the AO analysis while testing with ten functions of recent CEC benchmarks (CEC2019); the specification of the functions has been listed in Table 15. The AO has been implemented with 500 iterations and 50 population sizes for 30 independent runs. Its results compared with GOA EO, PSO, DA, ALO, and GWO, MPA, SSA, SCA, WOA, and SMA. The comparison has performed through the worst, average, best, and STD values by the considered algorithms across the course of the functions, as reported in Table 16. Moreover, the Friedman mean rank values and final ranks have involved in the last lines of the table (see Table 16). The results confirm the proposed AO's superiority in dealing with these challenging test-bed functions, as it is classified as

Table 13
Review of CEC2017 benchmark function problems.

Type	No.	Description	Fi*
Unimodal functions	1	Shifted and Rotated Bent Cigar Function	100
	3	Shifted and Rotated Zakharov Function	300
Simple Multimodal Functions	4	Shifted and Rotated Rosenbrock's Function	400
	5	Shifted and Rotated Rastrigin's Function	500
	6	Shifted and Rotated Expanded Scaffer's F6 Function	600
	7	Shifted and Rotated Lunacek Bi-Rastrigin Function	700
	8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	9	Shifted and Rotated Levy Function	900
	10	Shifted and Rotated Schwefel's Function	1000
Hybrid functions	11	Hybrid Function 1 (N = 3)	1100
	12	Hybrid Function 2 (N = 3)	1200
	13	Hybrid Function 3 (N = 3)	1300
	14	Hybrid Function 4 (N = 4)	1400
	15	Hybrid Function 5 (N = 4)	1500
	16	Hybrid Function 6 (N = 4)	1600
	17	Hybrid Function 6 (N = 5)	1700
	18	Hybrid Function 6 (N = 5)	1800
	19	Hybrid Function 6 (N = 5)	1900
	20	Hybrid Function 6 (N = 6)	2000
Composition Functions	21	Composition Function 1 (N = 3)	2100
	22	Composition Function 2 (N = 3)	2200
	23	Composition Function 3 (N = 4)	2300
	24	Composition Function 4 (N = 4)	2400
	25	Composition Function 5 (N = 5)	2500
	26	Composition Function 6 (N = 5)	2600
	27	Composition Function 7 (N = 6)	2700
	28	Composition Function 8 (N = 6)	2800
	29	Composition Function 9 (N = 3)	2900
	30	Composition Function 10 (N = 3)	3000

Table 14
Results of the comparative methods on CEC2017 test functions.

Fun No.	Comparative methods								
Measure	AO	GA	PSO	CS	GWO	SSA	EO	GSA	CMA-ES
F1									
Average	2.9600E+02	9.8000E+03	3.9600E+03	2.9600E+02	3.2500E+05	3.4000E+03	2.4700E+03	2.9600E+02	1.0000E+02
STD	2.7500E+02	5.9400E+03	4.4600E+03	2.7500E+02	1.0700E+05	3.6700E+03	2.2100E+03	2.7500E+02	0.0000E+00
F3									
Average	1.1420E+03	8.7200E+03	3.0000E+02	1.0800E+04	1.5400E+03	3.0000E+02	3.0000E+02	1.0800E+04	3.0000E+02
STD	2.3500E+02	5.9000E+03	1.9000E−10	1.6000E+03	1.8900E+03	0.0000E+00	2.4000E−08	1.6200E+03	0.0000E+00
F4									
Average	4.0600E+02	4.1100E+02	4.0600E+02	4.0700E+02	4.1000E+02	4.0600E+02	4.0400E+02	4.0700E+02	4.0000E+02
STD	8.5600E+00	1.8500E+01	3.2800E+00	2.9200E+00	7.5500E+00	1.0100E+01	7.9100E−01	2.9200E+00	0.0000E+00
F5									
Average	5.1100E+02	5.1600E+02	5.1300E+02	5.5700E+02	5.1400E+02	5.2200E+02	5.1100E+02	5.5700E+02	5.3000E+02
STD	7.1540E+01	6.9300E+00	6.5400E+00	8.4100E+00	6.1000E+00	1.0500E+01	3.6700E+00	8.4000E+00	5.8300E+01
F6									
Average	6.2376E+02	6.0000E+02	6.0000E+02	6.2200E+02	6.0100E+02	6.1000E+02	6.0000E+02	6.2200E+02	6.8200E+02
STD	1.3928E+01	6.6800E−02	9.8000E−01	9.0200E+00	8.8000E−01	8.2600E+00	1.5000E−04	9.0200E+00	3.5400E+01
F7									
Average	7.1500E+02	7.2800E+02	7.1900E+02	7.1500E+02	7.3000E+02	7.4100E+02	7.2100E+02	7.1500E+02	7.1300E+02
STD	1.8520E+00	7.2900E+00	5.1000E+00	1.5600E+00	8.6000E+00	1.6600E+01	5.7400E+00	1.5500E+00	1.6300E+00
F8									
Average	8.2000E+02	8.2100E+02	8.1100E+02	8.2100E+02	8.1400E+02	8.2300E+02	8.1000E+02	8.2100E+02	8.2900E+02
STD	7.2560E+00	8.9600E+00	5.4700E+00	4.6900E+00	8.2600E+00	9.9500E+00	2.9200E+00	4.6900E+00	5.3000E+01
F9									
Average	9.0000E+02	9.1000E+02	9.0000E+02	9.0000E+02	9.1100E+02	9.4400E+02	9.0000E+02	9.0000E+02	4.6700E+03
STD	0.0000E+00	1.5200E+01	5.9000E−14	0.0000E+00	1.9500E+01	1.0500E+02	2.2700E−02	5.9000E−15	2.0600E+03
F10									
Average	1.7060E+03	1.7200E+03	1.4700E+03	2.6900E+03	1.5300E+03	1.8600E+03	1.4200E+03	2.6900E+03	2.5900E+03
STD	3.6200E+02	2.5200E+02	2.1500E+02	2.9800E+02	2.8700E+02	2.9500E+02	2.6200E+02	2.9800E+02	4.1400E+02
F11									
Average	1.1250E+03	1.1300E+03	1.1100E+03	1.1300E+03	1.1400E+03	1.1800E+03	1.1100E+03	1.1300E+03	1.1100E+03
STD	2.3150E+01	2.3800E+01	6.2800E+00	1.0500E+01	5.4100E+01	5.9800E+01	5.0200E+00	1.0500E+01	2.5400E+01
F12									
Average	1.0031E+04	3.7300E+04	1.4500E+04	7.1000E+05	6.2500E+05	1.9800E+06	1.0300E+04	7.0300E+05	1.6300E+03
STD	2.3200E+02	3.4800E+04	1.1300E+04	4.2000E+05	1.1300E+06	1.9100E+06	9.7900E+03	4.2100E+04	1.9800E+02
F13									
Average	8.0190E+03	1.0800E+04	8.6000E+03	1.1100E+04	9.8400E+03	1.6100E+04	8.0200E+03	1.1100E+04	1.3200E+03
STD	5.6234E+03	8.9300E+03	5.1200E+03	2.1100E+03	5.6300E+03	1.0500E+04	6.7200E+03	2.1100E+03	7.8300E+01
F14									
Average	1.4490E+03	7.0500E+03	1.4800E+03	7.1500E+03	3.4000E+03	1.5100E+03	1.4600E+03	7.1500E+03	1.4500E+03
STD	5.4000E+01	8.1600E+03	4.2500E+01	1.4900E+03	1.9500E+03	5.1100E+01	3.2500E+01	1.4900E+03	5.6000E+01
F15									
Average	1.7100E+03	9.3000E+03	1.7100E+03	1.8000E+04	3.8100E+03	2.2400E+03	1.5900E+03	1.8000E+04	1.5100E+03
STD	2.7600E+02	8.9800E+03	2.8300E+02	5.5000E+03	3.8600E+03	5.7100E+02	4.8000E+01	5.5000E+03	1.6400E+01
F16									
Average	1.6240E+03	1.7900E+03	1.8600E+03	2.1500E+03	1.7300E+03	1.7300E+03	1.6500E+03	2.1500E+03	1.8200E+03
STD	4.0000E+01	1.2900E+02	1.2800E+02	1.0600E+02	1.2400E+02	1.2700E+02	5.0900E+01	1.0600E+02	2.3000E+02
F17									
Average	1.7420E+03	1.7500E+03	1.7600E+03	1.8600E+03	1.7600E+03	1.7700E+03	1.7300E+03	1.8600E+03	1.8300E+03
STD	2.9000E+01	3.9800E+01	4.7500E+01	1.0800E+02	3.1300E+01	3.4200E+01	1.8100E+01	1.0800E+02	1.7600E+02
F18									
Average	8.7120E+03	1.5700E+04	1.4600E+04	8.7200E+03	2.5800E+04	2.3400E+04	2.2500E+04	8.7200E+03	1.8300E+03
STD	3.2510E+03	1.2800E+04	1.1900E+04	5.0600E+03	1.5800E+04	1.4000E+04	1.1400E+04	5.0600E+03	1.3500E+01
F19									
Average	1.9440E+03	9.6900E+03	2.6000E+03	4.5000E+04	9.8700E+03	2.9200E+03	1.9500E+03	1.3700E+04	1.9200E+03
STD	3.0000E+01	6.7700E+03	2.1900E+03	1.9000E+04	6.3700E+03	1.8700E+03	4.7100E+01	1.9200E+04	2.8700E+01
F20									
Average	2.0180E+03	2.0600E+03	2.0900E+03	2.2700E+03	2.0800E+03	2.0900E+03	2.0200E+03	2.2700E+03	2.4900E+03
STD	2.1000E+01	6.0000E+01	6.2300E+01	8.1700E+01	5.2000E+01	4.9300E+01	2.2300E+01	8.1700E+01	2.4300E+02
F21									
Average	2.2050E+03	2.3000E+03	2.2800E+03	2.3600E+03	2.3200E+03	2.2500E+03	2.3100E+03	2.3600E+03	2.3200E+03
STD	4.0000E+01	4.3800E+01	5.4000E+01	2.8200E+01	7.0000E+00	6.0400E+01	2.1000E+01	2.8200E+01	6.7800E+01
F22									
Average	2.3050E+03	2.3000E+03	2.3100E+03	2.3000E+03	2.3100E+03	2.3000E+03	2.3000E+03	2.3000E+03	3.5300E+03
STD	2.2000E+01	2.3800E+00	6.6100E+01	7.0000E−02	1.6800E+01	1.1800E+01	1.8400E+01	7.2000E−02	8.4800E+02
F23									
Average	2.6200E+03	2.6300E+03	2.6200E+03	2.7400E+03	2.6200E+03	2.6200E+03	2.6200E+03	2.7400E+03	2.7300E+03
STD	1.2000E+01	1.3400E+01	9.2300E+00	3.9100E+01	8.4700E+00	8.6900E+00	5.5300E+00	3.9100E+01	2.4300E+02
F24									
Average	2.6860E+03	2.7600E+03	2.6900E+03	2.7400E+03	2.7400E+03	2.7300E+03	2.7400E+03	2.7400E+03	2.7000E+03
STD	1.6000E+01	1.4900E+01	1.0800E+02	5.5500E+00	8.7300E+00	6.4400E+01	6.9000E+00	5.5200E+00	7.3400E+01
F25									
Average	2.9190E+03	2.9500E+03	2.9200E+03	2.9400E+03	2.9400E+03	2.9200E+03	2.9300E+03	2.9400E+03	2.9300E+03
STD	1.9100E+01	1.9300E+01	2.5000E+01	1.5300E+01	2.3600E+01	2.3900E+01	1.9800E+01	1.5400E+01	2.0900E+01
F26									

(continued on next page)

Table 14 (*continued*)

Fun No.	Comparative methods								
Measure	AO	GA	PSO	CS	GWO	SSA	EO	GSA	CMA-ES
Average	3.0060E+03	3.1100E+03	2.9500E+03	3.4400E+03	3.2200E+03	2.9000E+03	2.9700E+03	3.4400E+04	3.4600E+03
STD	1.4500E+02	3.3500E+02	2.5000E+02	6.2900E+02	4.2700E+02	3.6600E+01	1.6500E+02	6.2900E+02	5.9900E+02
F27									
Average	3.0900E+03	3.1200E+03	3.1200E+03	3.2600E+03	3.1000E+03	3.0900E+03	3.0910E+03	3.2600E+03	3.1400E+03
STD	8.3700E+00	1.9200E+01	2.5000E+01	4.1700E+01	2.1800E+01	2.7800E+00	2.2400E+00	4.1700E+01	2.1400E+01
F28									
Average	3.2110E+03	3.3200E+03	3.3200E+03	3.4600E+03	3.3900E+03	3.2100E+03	3.3000E+03	3.4600E+03	3.4000E+03
STD	4.6840E+01	1.2600E+02	1.2200E+02	3.3800E+01	1.0200E+02	1.1300E+02	1.3400E+02	3.3800E+01	1.3100E+02
F29									
Average	3.1900E+03	3.2500E+03	3.2000E+03	3.4500E+03	3.1900E+03	3.2100E+03	3.1700E+03	3.4500E+03	3.2100E+03
STD	2.9000E+01	8.2000E+01	5.2300E+01	1.7100E+02	4.2900E+01	5.1700E+01	2.4700E+01	1.7100E+02	1.1000E+02
F30									
Average	2.9014E+05	5.3700E+05	3.5100E+05	9.4000E+05	2.9800E+05	4.2100E+05	2.9700E+05	1.3000E+06	3.0500E+05
STD	5.2314E+04	6.3700E+05	5.0500E+05	3.6000E+05	5.2800E+05	5.6800E+05	4.5900E+05	3.6400E+05	4.4500E+05
(W|L|T)	(9|13|7)	(0|27|2)	(0|23|6)	(0|24|5)	(0|28|1)	(3|21|5)	(0|24|5)	(0|24|5)	(8|17|3)
Mean	2.4138E+00	5.6207E+00	3.4828E+00	6.3448E+00	5.3793E+00	4.8276E+00	2.5517E+00	6.3793E+00	4.6552E+00
Ranking	1	7	3	8	6	5	2	9	4

the best algorithm for half of these functions.

Meanwhile, the MPA success for three functions, the EO, and SMA for only one function out of this set. Sequentially, AO is located in the first position regarding the chain counterparts. The convergence curves of Fig. 11 shows the efficiency of the AO in converging for high qualified solutions with significant convergence speed as exhibited in F1, F7, F8, and F10.

4.4. Real-world applications

In this section, the proposed algorithm is evaluated in solving seven constrained real engineering problems, namely tension/compression spring design problem, pressure vessel design problem, welded beam design problem, 3-bar truss design problem, speed reducer problem, cantilever beam design problem, and multiple disc clutch brake problem. These problems contain several inequality constraints. The death penalty function is used in which the algorithm obtains significant values if it violates any of the constraints. The parameter settings are set as the previous experiments.

4.4.1. Tension/compression spring design problem

The goal of this problem is to minimize the weight of the tension/compression spring by selecting the best values of the following parameters wire diameter (d), the number of active coils (N), and mean coil diameter (D). Fig. 12 shows the problem style and the mathematical form is given in Eq. (18).

The proposed AO is compared with the following optimization algorithms, GSA (Saremi et al., 2017), OBSCA (Elaziz, Oliva, & Xiong, 2017), CPSO (He & Wang, 2007a), CC (Arora, 2004), RO (Kaveh & Khayatazad, 2012), HS (Mahdavi, Fesanghary, & Damangir, 2007), CSCA (Huang, Wang, & He, 2007), GA (Coello, 2000), WOA (Mirjalili & Lewis, 2016), MVO (Mirjalili et al., 2016), PSO (He & Wang, 2007a), ES (Mezura-Montes & Coello, 2008). The results are listed in Table 17 and showed that the AO obtained the best results than all other algorithms.

Furthermore, Fig. 13 illustrates the curves of the objective values, the trajectory of 1st solution, and the convergence of the AO during solving the problem.

$$\begin{aligned}
&\textit{Consider} && \vec{x} = [x_1\ x_2\ x_3] = [d\ D\ N],\\
&\textit{Minimize} && f(\vec{x}) = (x_3+2)x_2x_1^2,\\
&\textit{Subject to} && g_1\left(\vec{x}\right) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leqslant 0,\\
& && g_2\left(\vec{x}\right) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leqslant 0,\\
& && g_3\left(\vec{x}\right) = 1 - \frac{140.45x_1}{x_2^2x_3} \leqslant 0,\\
& && g_4\left(\vec{x}\right) = \frac{x_1+x_2}{1.5} - 1 \leqslant 0,\\
&\textit{Variables range} && 0.05 \leqslant x_1 \leqslant 2\\
& && 0.25 \leqslant x_2 \leqslant 1.30\\
& && 2.00 \leqslant x_3 \leqslant 15
\end{aligned} \tag{18}$$

4.4.2. Pressure vessel design problem

This problem tries to minimize the total cost of the cylindrical pressure vessel to match the pressure requirements. Fig. 14 exhibits the style of this problem. Four variables in this problem need to be minimized (i.e., the thickness of the shell (T_s), the inner radius (R), the thickness of the head (T_h), and the length of the cylindrical section(L)). There are also four constraints, as follows:

$$\begin{aligned}
&\textit{Consider} && \vec{x} = [x_1\ x_2\ x_3\ x_4] = [T_s\ T_h\ R\ L],\\
&\textit{Minimize} && 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3,\\
&\textit{Subject to} && g_1(\vec{x}) = -x_1 + 0.0193x_3 \leqslant 0,\\
& && g_2(\vec{x}) = -x_2 + 0.00954x_3 \leqslant 0,\\
& && g_3\left(\vec{x}\right) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leqslant 0,\\
& && g_4(\vec{x}) = x_4 - 240 \leqslant 0,\\
&\textit{Variables range} && 0 \leqslant x_1 \leqslant 99,\\
& && 0 \leqslant x_2 \leqslant 99,\\
& && 10 \leqslant x_3 \leqslant 200,\\
& && 10 \leqslant x_4 \leqslant 200
\end{aligned} \tag{19}$$

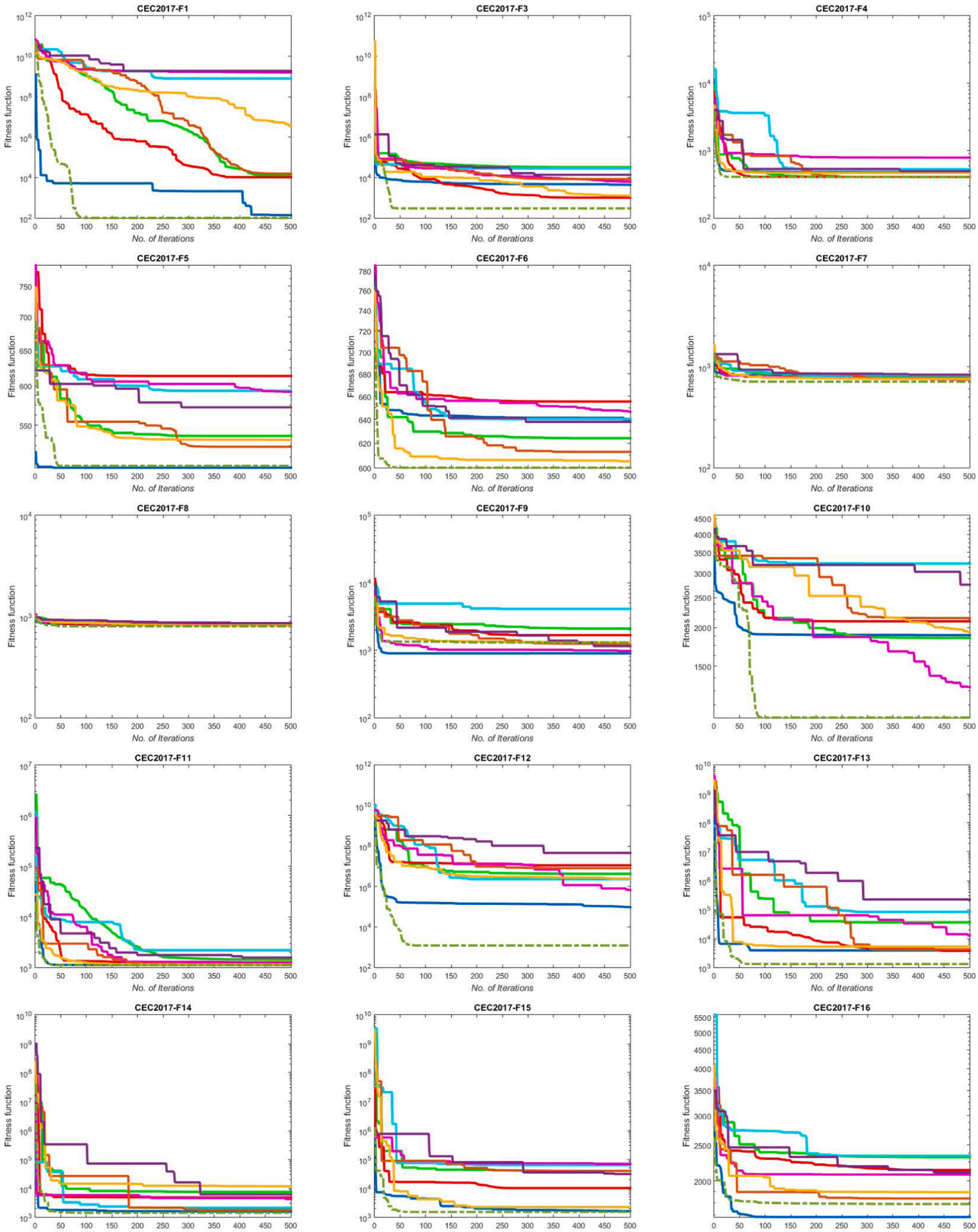

Fig. 10. Convergence behavior of the comparative algorithms on CEC2017 test functions.

The AO is compared with branch-bound (Sandgren, 1990), HS (Mahdavi et al., 2007), MVO (Mirjalili et al., 2016), WOA (Mirjalili & Lewis, 2016), ACO (Kaveh & Talatahari, 2010a), ES (Mezura-Montes & Coello, 2008), HPSO (He & Wang, 2007b), CSS (Kaveh & Talatahari, 2010b), CSCA (Huang et al., 2007), PSO-SCA (Liu, Cai, & Wang, 2010), GWO (Mirjalili, Mirjalili, & Lewis, 2014), GA (Coello, 2000), CPSO (He & Wang, 2007a), GSA (Rashedi et al., 2009), SMA (Zhao et al., 2020). From the results in Table 18 we can see that, the AO was ranked first and reached the minimum total cost in solving pressure vessel design problem. In addition, Fig. 15 shows the curves of the objective values, the trajectory of 1st solution, and the convergence of the AO during solving the problem.

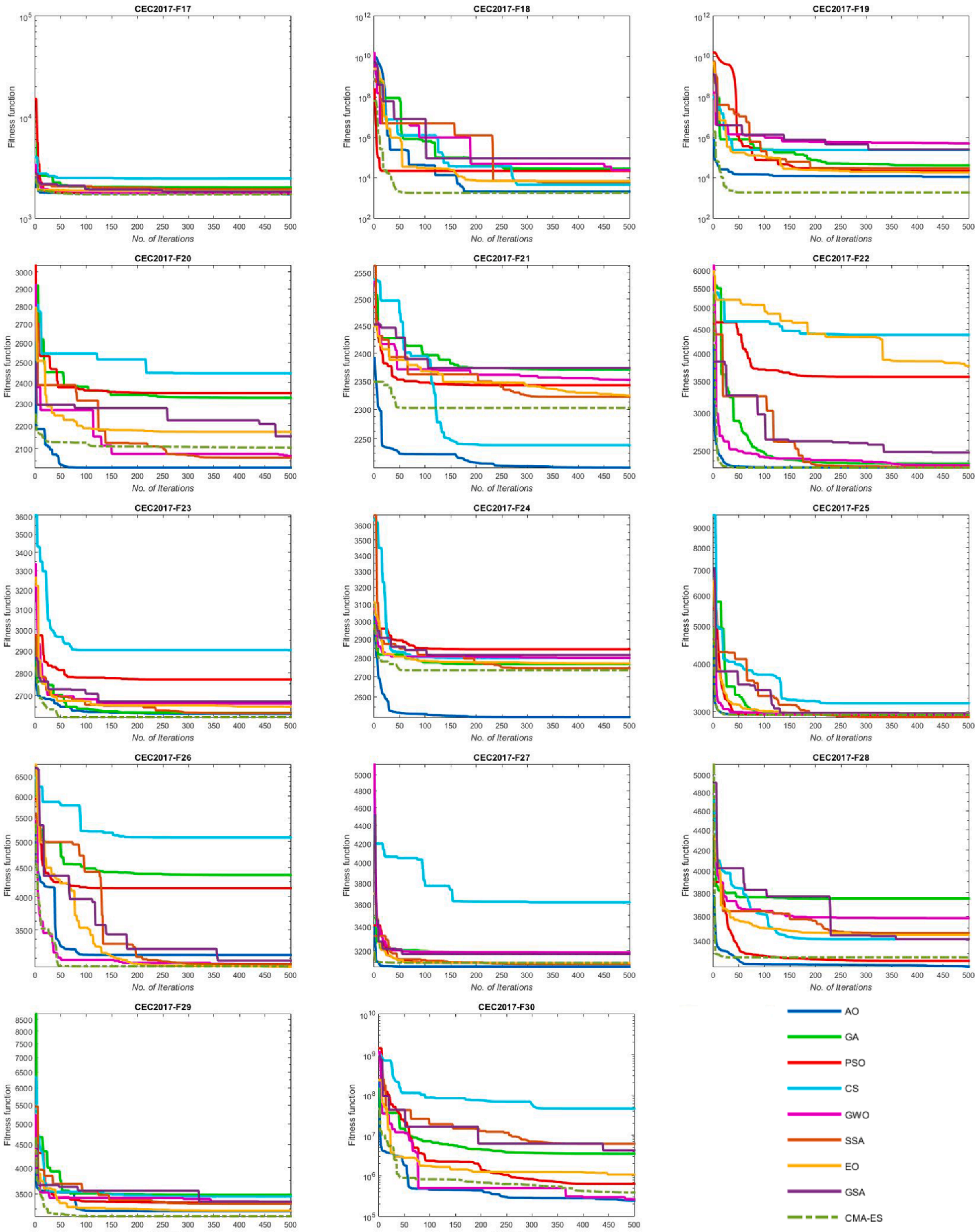

Fig. 10. (*continued*).

4.4.3. Welded beam design problem

This problem aims to minimize the economic cost of the welded beam design. The design of this problem is shown in Fig. 16. Four variables and seven constraints need to be optimized. These variables are the thickness of the weld (*h*), the thickness of the bar (*bb*), length of the attached part of the bar (*l*), and the height of the bar (*tt*). The mathematical form is listed in Eq. (20).

Table 19 listed the results of the AO and the compared algorithms namely OBSCA (Elaziz et al., 2017), GSA (Saremi et al., 2017), RO (Kaveh & Khayatazad, 2012), CSCA (Huang et al., 2007), GA (Deb, 1991), DAVID (Ragsdell & Phillips, 1976), SIMPLEX (Ragsdell & Phillips, 1976), APPROX (Ragsdell & Phillips, 1976), HS (Lee & Geem, 2005), SSA (Mirjalili et al., 2017), CPSO (He & Wang, 2007a), WOA (Mirjalili & Lewis, 2016), MVO (Mirjalili et al., 2016), MPA (Faramarzi, Heidarinejad, Mirjalili, et al., 2020), SMA (Zhao et al., 2020). From this table we can noticed that, the AO outperformed all other algorithms and obtained the minimum value of the total cost. Fig. 17 shows the curves of the AO during solving the problem.

$$
\begin{aligned}
&\textit{Consider} && \vec{x} = [x_1\ x_2\ x_3\ x_4] = [h\ l\ tt\ bb],\\
&\textit{Minimize} && f(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2),\\
&\textit{Subject to} && g_1(\vec{x}) = \tau(\vec{x}) - \tau_{max} \leqslant 0,\\
& && g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{max} \leqslant 0,\\
& && g_3(\vec{x}) = \delta(\vec{x}) - \delta_{max} \leqslant 0,\\
& && g_4(\vec{x}) = x_1 - x_4 \leqslant 0,\\
& && g_5(\vec{x}) = P - P_c(\vec{x}) \leqslant 0,\\
& && g_6(\vec{x}) = 0.125 - x_1 \leqslant 0,\\
& && g_7(\vec{x}) = 1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leqslant 0\\
&\textit{Variables range} && 0.1 \leqslant x_1 \leqslant 2,\\
& && 0.1 \leqslant x_2 \leqslant 10,\\
& && 0.1 \leqslant x_3 \leqslant 10,\\
& && 0.1 \leqslant x_4 \leqslant 2\\
&\textit{where} && \tau(\vec{x}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2},\\
& && \tau' = \frac{p}{\sqrt{2x_1x_2}},\quad \tau'' = \frac{MR}{J},\\
& && M = P\left(L + \frac{x_2}{2}\right),\\
& && R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},\\
& && J = 2\left\{\sqrt{2x_1x_2}\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\},\\
& && \sigma(\vec{x}) = \frac{6PL}{x_4x_3^2},\quad \delta(\vec{x}) = \frac{6PL^3}{Ex_3^2x_4}\\
& && P_c(\vec{x}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),\\
& && P = 6000\ lb, L = 14\ in.,\quad \delta_{max} = 0.25\ in.,\\
& && E = 30 \times 1^6\ psi,\quad G = 12 \times 10^6\ psi,\\
& && \tau_{max} = 13600\ psi,\quad \sigma_{max} = 30000\ psi
\end{aligned}
\tag{20}
$$

4.4.4. 3-bar truss design problem

The 3-bar truss design is a problem in the field of civil engineering. It tries to manipulate two parameters in order to achieve the minimum weight in designing a truss. Fig. 18 shows the design of this problem and its mathematical form is given in Eq. (21).

The AO is compared with Ray and Sain (Ray & Saini, 2001), AAA (Yildirim & Karci, 2018), SSA (Mirjalili et al., 2017), MBA (Sadollah, Bahreininejad, Eskandar, & Hamdi, 2013), DEDS (Zhang, Luo, & Wang, 2008), GOA (Saremi et al., 2017), PSO-DE (Liu et al., 2010), CS (Gandomi et al., 2013). The comparison results are listed in Table 20 and showed the superiority of the AO in solving this problem. Fig. 19 shows the curves of the AO during solving the problem.

Table 15
Review of CEC2019 benchmark function problems.

No.	Functions	$F_i^* = F_i(x^*)$	Dim	Search range
1	Storn's Chebyshev Polynomial Fitting Problem	1	9	[−8192, 8192]
2	Inverse Hilbert Matrix Problem	1	16	[−16384, 16384]
3	Lennard-Jones Minimum Energy Cluster	1	18	[−4,4]
4	Rastrigin's Function	1	10	[−100,100]
5	Griewangk's Function	1	10	[−100,100]
6	Weierstrass Function	1	10	[−100,100]
7	Modified Schwefel's Function	1	10	[−100,100]
8	Expanded Schaffer's F6 Function	1	10	[−100,100]
9	Happy Cat Function	1	10	[−100,100]
10	Ackley Function	1	10	[−100,100]

$$
\begin{aligned}
&\textit{Consider} && \vec{x} = [x_1\ x_2] = [A_1\ A_2],\\
&\textit{Minimize} && f(\vec{x}) = (2\sqrt{2x_1} + x_2) * l,\\
&\textit{Subject to} && g_1(\vec{x}) = \frac{\sqrt{2x_1} + x_2}{\sqrt{2x_1^2 + 2x_1x_2}}P - \sigma \leqslant 0,\\
& && g_2(\vec{x}) = \frac{x_2}{\sqrt{2x_1^2 + 2x_1x_2}}P - \sigma \leqslant 0,\\
& && g_3(\vec{x}) = \frac{1}{\sqrt{2x_2} + x_1}P - \sigma \leqslant 0,\\
&\textit{Variables range} && 0 \leqslant x_1, x_2 \leqslant 1,\\
&\textit{where} && l = 100\ cm, P = 2\ KN/cm^2, \sigma = 2\ KN/cm^2
\end{aligned}
\tag{21}
$$

4.4.5. Speed reducer problem

This problem tries to minimize the speed reducer's total weights by optimizing seven variables regarding the limitations of the gear teeth' curvature stress, transverse deflections of the shafts, and stresses in the shafts, and surface stress. Fig. 20 shows the design of this problem, and its mathematical form is given in Eq. (22).

The AO is compared with GA (Saruhan & Uygur, 2003), SES (Mezura-Montes, Coello, & Landa-Becerra, 2003), PSO (Stephen, Christu, & Dalvi, 2018), GSA (Rashedi et al., 2009), hHHO-SCA (Kamboj, Nandi, Bhadoria, & Sehgal, 2020), MDA (Lu & Kim, 2010), SCA (Mirjalili, 2016a), HS (Geem, Kim, & Loganathan, 2001), FA (Baykasoğlu & Ozsoydan, 2015), and SBSM (Akhtar, Tai, & Ray, 2002). The results of the AO and the compared methods are listed in Table 21. From this table, the AO is ranked first and outperformed all methods in solving this problem whereas, the SBSM is ranked second followed by FA, MDA, and SES, respectively. Fig. 21 shows the curves of the AO during solving the problem.

$$\begin{aligned}
&\textit{Minimize} \quad f(\vec{x}) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3)\\
&\textit{subject to}\\
&s.t.\, g_1(\vec{x}) = \frac{27}{x_1x_2^2x_3} - 1 \leqslant 0\\
&g_2(\vec{x}) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leqslant 0\\
&g_3(\vec{x}) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leqslant 0\\
&g_4(\vec{x}) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leqslant 0\\
&g_5(\vec{x}) = \frac{\sqrt{\left(\frac{745_{x_4}}{x_2x_3}\right)^2 + 16.9\times 10^6}}{110.0x_6^3} - 1 \leqslant 0\\
&g_6(\vec{x}) = \frac{\sqrt{\left(\frac{745_{x_4}}{x_2x_3}\right)^2 + 157.5\times 10^6}}{85.0x_6^3} - 1 \leqslant 0 \qquad (22)\\
&g_7(\vec{x}) = \frac{x_2x_3}{40} - 1 \leqslant 0\\
&g_8(\vec{x}) = \frac{5x_2}{x_1} - 1 \leqslant 0\\
&g_9(\vec{x}) = \frac{x_1}{12x_2} - 1 \leqslant 0\\
&g_{10}(\vec{x}) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leqslant 0\\
&g_{11}(\vec{x}) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leqslant 0\\
&\textit{where}\\
&2.6 \leqslant x1 \leqslant 3.6,\ 0.7 \leqslant x2 \leqslant 0.8,\ 17 \leqslant x3 \leqslant 28,\ 7.3 \leqslant x4 \leqslant 8.3,\ 7.8 \leqslant x5 \leqslant 8.3,\ 2.9 \leqslant x6 \leqslant 3.9,\ 5.0 \leqslant x7 \leqslant 5.5
\end{aligned}$$

4.4.6. Cantilever beam design problem

This problem is a type of concrete engineering problems. It works to minimize the total weight of a cantilever beam by optimizing the hollow square cross-section parameters. Fig. 22 shows the design of this problem and its mathematical form is given in Eq. (23). (see Fig. 23)

$$\begin{aligned}
&\textit{Consider} \quad x = [x_1\ x_2\ x_3\ x_4\ x_5]\\
&\textit{Minimise} \quad f(\vec{x}) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5),\\
&\textit{subject to}\\
&g(\vec{x}) = \frac{60}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leqslant 0 \qquad (23)\\
&\textit{Variable range}\\
&0.01 \leqslant x_1, x_2, x_3, x_4, x_5 \leqslant 100
\end{aligned}$$

To evaluate the performance of the proposed AO in solving this problem, it was compared with GCA_I (Chickermane & Gea, 1996), ALO (Mirjalili, 2015b), GCA_II (Chickermane & Gea, 1996), CS (Gandomi et al., 2013), MMA (Chickermane & Gea, 1996), SOS (Cheng & Prayogo, 2014), SMA (Zhao et al., 2020), and MFO (Mirjalili, 2015a). The results of the AO and the compared methods are listed in Table 22. From this table, the AO showed a good performance in solving the cantilever beam design problem, and it came in the first rank with little difference from CS followed by ALO and SMA, respectively. Fig. 21 shows the curves of the AO during solving the problem.

4.4.7. Multiple disc clutch brake problem

This problem works to minimize the total weight of a multiple disc clutch brake by optimizing five variables: the thickness of discs, outer radius, number of friction surfaces, inner radius, and actuating force. There are eight constraints for this problem based on the conditions of geometry and operating requirements. The feasible domain contains about 70% of the solution space. Fig. 24 illustrates the design of this problem and Eq. (24) represents its mathematical model.

Table 16
Results of the comparative methods on CEC2019 test functions.

Fun No.	Comparative methods											
Measure	AO	GOA	EO	PSO	DA	ALO	GWO	MPA	SSA	SCA	WOA	SMA
F1												
Worst	5.7185E+04	4.0133E+10	2.8491E+09	2.1499E+13	3.6220E+10	5.3112E+10	3.2839E+08	4.4140E+04	1.1307E+10	1.2002E+10	1.2404E+11	59119.21888
Average	4.8726E+04	1.3203E+10	9.3131E+08	8.0403E+12	2.1128E+10	2.4366E+10	1.0739E+08	3.7464E+04	7.3245E+09	4.6112E+09	5.7448E+10	49496.82449
Best	3.9908E+04	2.9754E+09	3.9735E+04	8.1972E+11	1.2918E+10	4.5166E+09	5.1689E+05	3.2396E+04	2.4621E+09	1.3097E+09	1.5327E+09	43022.95047
STD	6.4128E+03	1.5418E+10	1.3273E+09	8.6372E+12	9.2900E+09	2.2029E+10	1.5077E+08	4.7886E+03	3.4825E+09	4.2196E+09	4.7046E+10	7249.678206
F2												
Worst	1.7343E+01	1.7468E+01	1.7343E+01	1.5848E+04	6.0358E+02	1.7349E+01	1.7344E+01	1.7343E+01	1.7391E+01	1.7538E+01	1.7363E+01	17.37579915
Average	1.7343E+01	1.7397E+01	1.7343E+01	1.2323E+04	1.3515E+02	1.7345E+01	1.7344E+01	1.7343E+01	1.7355E+01	1.7468E+01	1.7354E+01	17.36710062
Best	1.7343E+01	1.7367E+01	1.7343E+01	8.2815E+03	1.7343E+01	1.7343E+01	1.7344E+01	1.7343E+01	1.7343E+01	1.7414E+01	1.7346E+01	17.35597052
STD	1.3292E−04	4.0848E−02	5.0243E−15	3.0180E+03	2.6186E+02	2.2146E−03	1.3962E−04	1.2596E−07	2.0765E−02	4.6095E−02	6.1916E−03	0.007884415
F3												
Worst	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2705E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2703E+01	1.2702E+01	12.70432773
Average	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2703E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2703E+01	1.2702E+01	12.70285328
Best	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2702E+01	1.2703E+01	1.2702E+01	12.70247222
STD	1.8286E−06	8.1884E−12	1.5384E−15	0.0000E+00	1.1169E−03	8.8818E−16	1.7866E−06	0.0000E+00	1.7764E−15	8.1441E−05	7.4263E−07	0.000824434
F4												
Worst	7.7626E+01	5.4726E+01	2.4077E+01	2.3904E+01	3.2190E+03	1.1342E+02	1.6793E+02	1.2958E+01	4.8753E+01	3.6227E+03	8.6610E+02	40.86124872
Average	6.1750E+01	3.2867E+01	1.7864E+01	1.7327E+01	1.2762E+03	5.6115E+01	7.5948E+01	8.9948E+00	3.4226E+01	1.9761E+03	5.8922E+02	26.12215256
Best	4.2306E+01	8.9569E+00	6.9995E+00	4.9752E+00	3.4288E+01	1.0945E+01	4.9114E+01	5.0030E+00	2.1889E+01	1.0981E+03	2.2660E+02	10.42762116
STD	1.3914E+01	1.9711E+01	6.8265E+00	7.3781E+00	1.4697E+03	3.7118E+01	5.1553E+01	3.3786E+00	1.0776E+01	9.6264E+02	2.8278E+02	11.4476303
F5												
Worst	1.4154E+00	9.7766E+00	1.1442E+01	1.1279E+01	1.0841E+01	8.5764E+00	1.2052E+01	4.0291E+00	6.2782E+00	1.1812E+01	1.1329E+01	1.754317948
Average	1.3545E+00	8.4847E+00	1.0754E+01	9.9834E+00	9.6346E+00	5.2938E+00	1.1649E+01	2.3349E+00	5.4861E+00	1.1146E+01	9.6370E+00	1.374072236
Best	1.1530E+00	7.6595E+00	9.8895E+00	8.8972E+00	8.5186E+00	2.9803E+00	1.1464E+01	1.2088E+00	4.5119E+00	9.8660E+00	8.2812E+00	1.136743818
STD	9.3665E−02	8.7632E−01	6.6035E−01	9.3240E−01	8.8471E−01	2.3569E+00	2.3951E−01	1.2762E+00	8.5332E−01	7.5113E−01	1.1535E+00	0.234609845
F6												
Worst	1.0058E+01	9.7766E+00	1.1442E+01	1.1279E+01	1.0841E+01	1.2191E+01	1.2052E+01	1.4029E+01	6.2782E+00	1.1812E+01	1.1329E+01	5.044147869
Average	8.3144E+00	8.4847E+00	1.0754E+01	9.9834E+00	9.6346E+00	1.1480E+01	1.1649E+01	1.2335E+01	5.4861E+00	1.1146E+01	9.6370E+00	4.43199477
Best	4.6493E+00	7.6595E+00	9.8895E+00	8.8972E+00	8.5186E+00	1.0921E+01	1.1464E+01	1.1209E+01	4.5119E+00	9.8660E+00	8.2812E+00	3.547038584
STD	2.1161E+00	8.7632E−01	6.6035E−01	9.3240E−01	8.8471E−01	4.7370E−01	2.3951E−01	1.1276E+01	8.5332E−01	7.5113E−01	1.1535E+00	0.666906413
F7												
Worst	3.8973E+02	7.6818E+02	5.4218E+02	4.1544E+02	1.2605E+03	1.3979E+03	6.5949E+02	1.4602E+02	5.6388E+02	1.0631E+03	1.1302E+03	556.5354916
Average	2.7780E+02	5.0079E+02	2.0373E+02	2.0108E+02	9.4774E+02	6.5560E+02	3.5348E+02	4.7289E+01	4.6133E+02	8.5032E+02	8.0560E+02	228.1812071
Best	2.0976E+02	2.4958E+02	−2.2288E+02	−3.4109E+00	4.9273E+02	1.8713E+02	1.2363E+02	−8.8834E+01	2.8857E+02	4.7767E+02	4.6215E+02	−95.13810593
STD	7.0498E+01	2.1911E+02	2.8572E+02	1.7996E+02	3.0667E+02	4.4719E+02	2.2858E+02	9.7761E+01	1.1757E+02	2.2416E+02	2.5333E+02	283.8794351
F8												
Worst	6.0065E+00	6.1986E+00	5.6001E+00	5.6265E+00	6.4732E+00	6.1027E+00	6.8118E+00	4.8994E+00	6.7080E+00	6.6863E+00	6.6408E+00	5.531143673
Average	5.0660E+00	5.5874E+00	3.9066E+00	4.9495E+00	5.9517E+00	5.7015E+00	5.4830E+00	4.0886E+00	5.4825E+00	5.9026E+00	6.1017E+00	5.312594537
Best	3.9924E+00	5.1592E+00	2.4154E+00	4.3550E+00	5.2004E+00	5.1209E+00	2.8996E+00	2.5565E+00	4.8619E+00	5.3787E+00	5.6658E+00	5.133001884
STD	7.8631E−01	4.3007E−01	1.1569E+00	6.1698E−01	4.9292E−01	3.9215E−01	1.6897E+00	9.1352E−01	7.1055E−01	5.7395E−01	4.2379E−01	0.171064613
F9												
Worst	2.4328E+00	2.9754E+00	2.4823E+00	2.4437E+00	1.9550E+02	2.8278E+00	7.1355E+00	2.4978E+00	2.9267E+00	4.5935E+02	6.4013E+00	2.566742354
Average	2.3908E+00	2.6643E+00	2.4025E+00	2.4115E+00	5.6366E+01	2.6040E+00	4.9324E+00	2.4163E+00	2.7329E+00	2.0256E+02	5.3971E+00	2.496126373
Best	2.3533E+00	2.5188E+00	2.3641E+00	2.3873E+00	2.7440E+00	2.4148E+00	3.2719E+00	2.3484E+00	2.3814E+00	4.5225E+01	3.9548E+00	2.455247496
STD	3.1720E−02	1.8276E−01	4.8202E−02	3.3764E−02	8.0061E+01	1.7406E−01	1.5294E+00	5.9561E−02	2.1946E−01	1.5775E+02	9.3385E−01	0.042546944
F10												
Worst	2.0279E+01	2.0001E+01	2.0475E+01	2.0671E+01	2.0639E+01	2.0088E+01	2.0603E+01	2.1035E+01	2.0003E+01	2.0552E+01	2.0459E+01	20.27673685
Average	1.6292E+01	2.0000E+01	2.0410E+01	2.0422E+01	2.0436E+01	2.0017E+01	2.0546E+01	2.1007E+01	2.0000E+01	2.0478E+01	2.0299E+01	20.09974459
Best	7.6041E−01	2.0000E+01	2.0323E+01	2.0122E+01	2.0288E+01	1.9999E+01	2.0482E+01	2.1000E+01	1.9998E+01	2.0417E+01	2.0208E+01	20.01121164
STD	8.6827E+00	3.7182E−04	5.6102E−02	2.0165E−01	1.3960E−01	3.9305E−02	4.5221E−02	3.5626E−02	1.8765E−03	5.8884E−02	1.0182E−01	0.103332342
(W|L|T)	(5|5|0)	(0|10|0)	(1|8|1)	(0|9|1)	(0|10|0)	(0|9|1)	(0|10|0)	(3|6|1)	(0|10|0)	(0|10|0)	(0|10|0)	(1|9|0)
Mean	2.7000E+00	6.4000E+00	4.6000E+00	6.0000E+00	9.8000E+00	6.6000E+00	8.2000E+00	4.1000E+00	5.6000E+00	1.0100E+01	8.7000E+00	4.8000E+00
Ranking	1	7	3	6	11	8	9	2	5	12	10	4

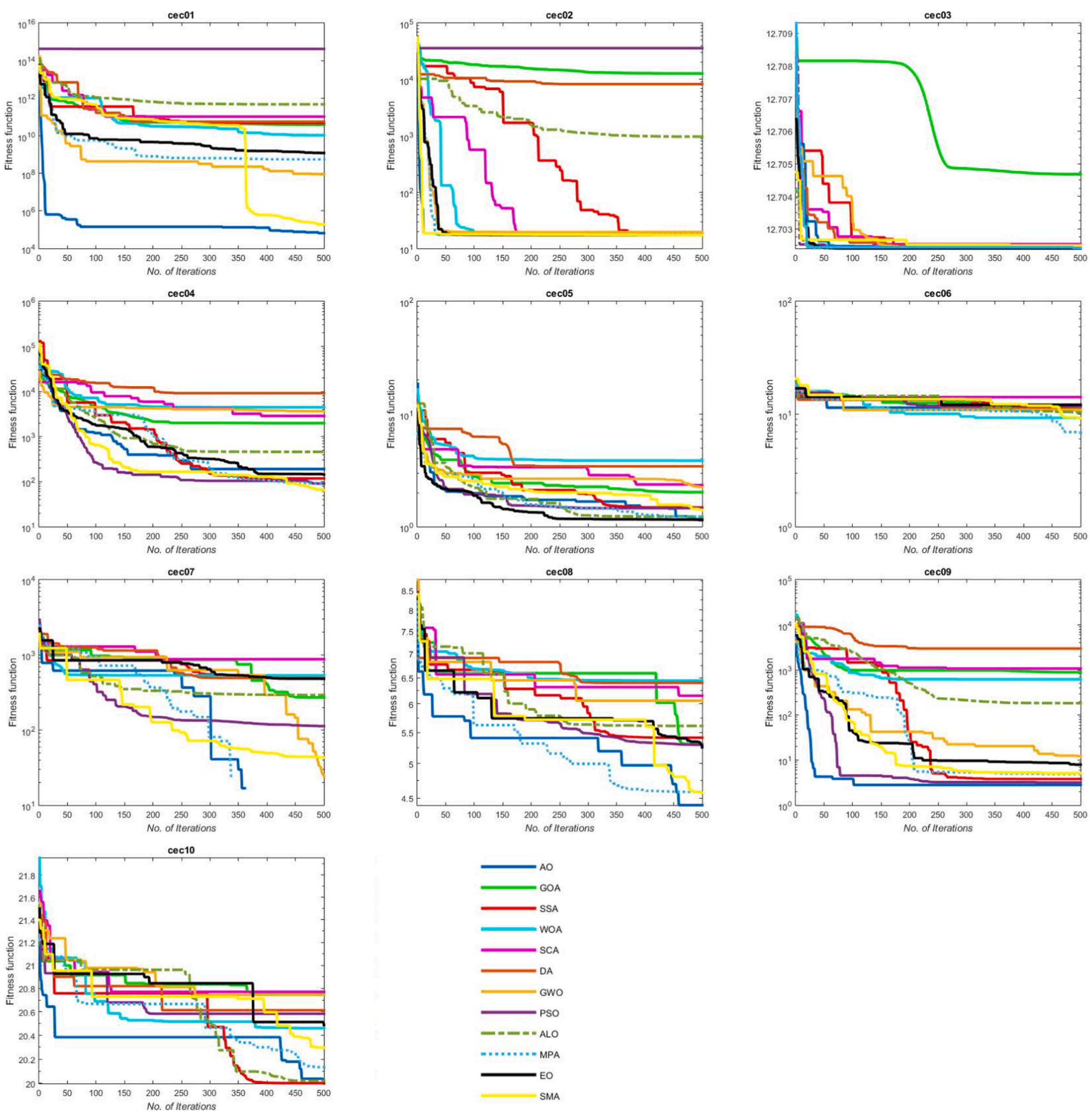

Fig. 11. Convergence behavior of the comparative algorithms on CEC2019 test functions.

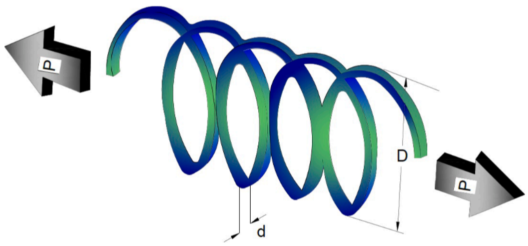

Fig. 12. Tension/compression spring design problem.

$$
\begin{aligned}
& f(x) = \Pi(r_o^2 - r_i^2)t(Z+1)\rho \\
\textit{subject to}\quad & g_1(x) = r_o - r_i - \Delta r \geqslant 0 \\
& g_2(x) = l_{max} - (Z+1)(t+\delta) \geqslant 0 \\
& g_3(x) = Pmax - Prz \geqslant 0 \\
& g_4(x) = P_{max}\ \nu_{sr\ max} - P_{rz}\ \nu_{sr} \geqslant 0 \\
& g_5(x) = \nu_{sr\ max} - \nu_{sr} \geqslant 0 \\
& g_6 = T_{max} - T \geqslant 0 \\
& g_7(x) = M_h - sM_s \geqslant 0 \\
& g_8(x) = T \geqslant 0 \\
\textit{where}\quad & M_h = \frac{2}{3}\mu FZ \frac{r_o^3 - r_i^2}{r_o^2 - r_i^3}, P_{rz} = \frac{F}{\Pi(r_o^2 - r_i^2)}, \\
& \nu_{rz} = \frac{2\Pi(r_o^3 - r_i^3)}{90(r_o^2 - r_i^2)}, T = \frac{I_z\ \Pi\ n}{30(M_h + M_f)} \\
& \Delta r = 20\ mm,\ I_z = 55\ kgmm^2,\ P_{max} = 1\ MPa,\ F_{max} = 1000\ N, \\
& T_{max} = 15\ s,\ \mu = 0.5,\ s = 1.5,\ M_s = 40Nm, \\
& M_f = 3Nm,\ n = 250\ rpm, \\
& \nu_{sr\ max} = 10\ m/s,\ l_{max} = 30\ mm,\ r_{i\ min} = 60, \\
& r_{i\ max} = 80, r_{o\ min} = 90, \\
& r_{o\ max} = 110,\ t_{min} = 1.5, t_{max} = 3,\ F_{min} = 600, \\
& F_{max} = 1000,\ Z_{min} = 2,\ Z_{max} = 9
\end{aligned} \tag{24}
$$

The proposed AO is compared with TLBO (Rao, Savsani, & Vakharia, 2011), MFO (Bhesdadiya, Trivedi, Jangir, & Jangir, 2018), NSGA-II (Deb & Srinivasan, 2008), MVO (Sayed, Darwish, & Hassanien, 2018), WCA (Eskandar, Sadollah, Bahreininejad, & Hamdi, 2012), and CMVO (Sayed et al., 2018). The comparison results are listed in Table 23 and showed the superiority of the AO in achieving the minimum total weights for this problem. Fig. 25 illustrates the curves of the AO during solving the problem.

5. Conclusion and potential future works

In this paper, a new alternative meta-heuristic technique, named Aquila Optimizer (AO), has been developed. This algorithm simulates the behaviors of Aquila in nature. In the AO, the optimization procedures are represented in four methods; selecting the search space by high soar with the vertical stoop, exploring within a diverge search space by contour flight with short glide attack, exploiting within a converge search space by low flight with slow descent attack, and swooping by walk and grab prey.

To validate the developed AO's ability to find the optimal solution, a set of different optimization problems were used. These problems include 23 classical benchmark functions, 29 functions from the CEC2017 benchmark, ten functions from the CEC2019 benchmark, and seven engineering problems. From the statistical results of benchmarks, it has been observed that the AO provided results either better than other well-known MH techniques or, at least, nearly equivalently. Moreover, from the empirical investigation of engineering problems, it can evaluate the developed AO's applicability to tackling real-world applications.

According to the previous discussion that illustrated the developed AO's superiority, it can open a wide range of future works. This including apply AO to various applications such as PV parameter estimation, neural network, image processing applications, text and data mining applications, big data applications, signal denoising, recourse management application, network applications, industry and engineering applications, other benchmark test functions, smart home applications, feature selection, image segmentation, task scheduling, and other. It can also be extended to real-world applications dependent on binary, discrete, and multiple objectives optimization. Moreover, AO's performance can be improved by combining it with levy flight, disruption, mutation, other stochastic components, whether the local search or global search methods and other evolutionary operators. Moreover, a discrete version of the AO can be proposed.

Table 17
Results of the comparative algorithms for solving the tension/compression spring design problem.

Algorithm	**Optimal values for variables**			Optimal	Ranking
	d	*D*	*N*	weight	
GSA (Saremi et al., 2017)	0.050276	0.323680	13.525410	0.0127022	10
OBSCA (Elaziz et al., 2017)	0.05230	0.31728	12.54854	0.012625	2
CPSO (He & Wang, 2007a)	0.051728	0.357644	11.244543	0.0126747	5
CC (Arora, 2004)	70.050000	0.315900	14.250000	0.0128334	13
RO (Kaveh & Khayatazad, 2012)	0.051370	0.349096	11.76279	0.0126788	8
HS (Mahdavi et al., 2007)	0.051154	0.349871	12.076432	0.0126706	4
CSCA (Huang et al., 2007)	0.051609	0.354714	11.410831	0.0126702	3
GA (Coello, 2000)	0.051480	0.351661	11.632201	0.01270478	11
WOA (Mirjalili & Lewis, 2016)	0.051207	0.345215	12.004032	0.0126763	7
MVO (Mirjalili et al., 2016)	0.05251	0.37602	10.33513	0.012790	12
PSO (He & Wang, 2007a)	0.051728	0.357644	11.244543	0.0126747	5
ES (Mezura-Montes & Coello, 2008)	0.051643	0.355360	11.397926	0.012698	9
AO	0.0502439	0.35262	10.5425	**0.011165**	1

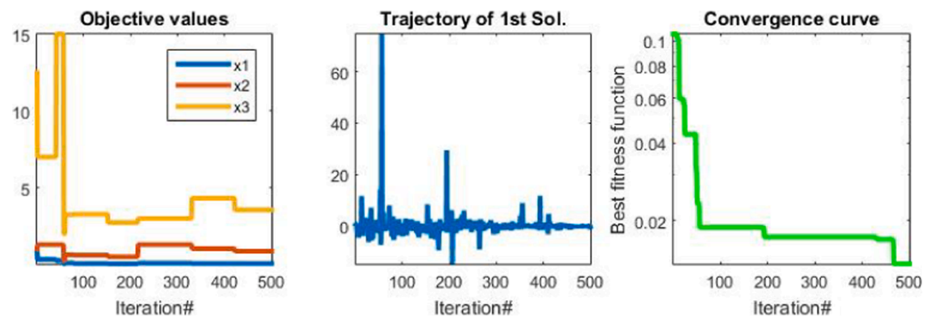

Fig. 13. Qualitative results for the tension/compression spring design problem.

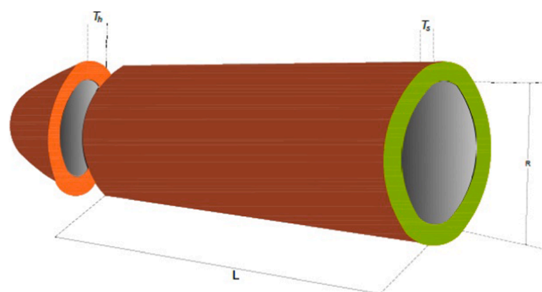

Fig. 14. Pressure vessel design problem.

Table 18
Results of the comparative algorithms for solving the pressure vessel design problem.

Algorithm	**Optimal values for variables**				Optimal	Ranking
	T_s	T_h	R	L	cost	
Branch-bound (Sandgren, 1990)	1.125	0.625	48.97	106.72	7982.5	15
HS (Mahdavi et al., 2007)	1.125000	0.625000	58.29015	43.69268	7197.730	14
MVO (Mirjalili et al., 2016)	0.8125	0.4375	42.090738	176.73869	6060.8066	12
WOA (Mirjalili & Lewis, 2016)	0.812500	0.437500	42.0982699	176.638998	6059.7410	9
ACO (Kaveh & Talatahari, 2010a)	0.812500	0.437500	42.098353	176.637751	6059.7258	7
ES (Mezura-Montes & Coello, 2008)	0.8125	0.4375	42.098087	176.640518	6059.74560	10
HPSO (He & Wang, 2007b)	0.8125	0.4375	42.0984	176.6366	6059.7143	5
CSS (Kaveh & Talatahari, 2010b)	0.8125	0.4375	42.1036	176.5727	6059.0888	4
CSCA (Huang et al., 2007)	0.8125	0.4375	42.098411	176.63769	6059.7340	8
PSO-SCA (Liu et al., 2010)	0.8125	0.4375	42.098446	176.6366	6059.71433	6
GWO (Mirjalili et al., 2014)	0.8125	0.4345	42.0892	176.7587	6051.5639	3
GA (Coello, 2000)	0.81250	0.43750	42.097398	176.65405	6059.94634	11
CPSO (He & Wang, 2007a)	0.8125	0.4375	42.091266	176.7465	6061.0777	13
GSA (Rashedi et al., 2009)	1.125	0.625	55.9886598	84.4542025	8538.8359	6
SMA (Zhao et al., 2020)	0.7931	0.3932	40.6711	196.2178	5994.1857	2
AO	1.0540	0.182806	59.6219	38.8050	**5949.2258**	1

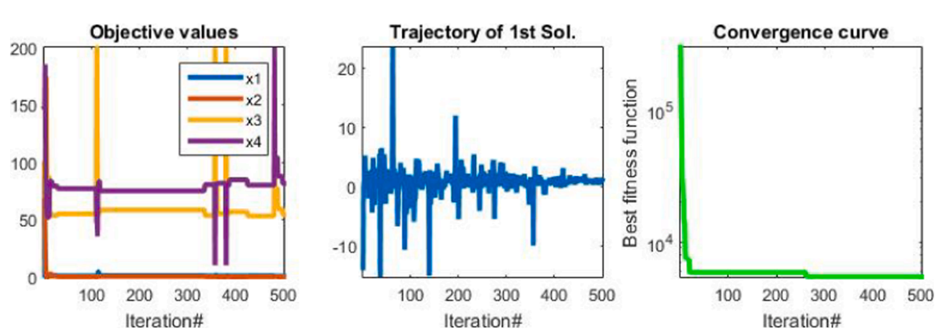

Fig. 15. Qualitative results for the pressure vessel design problem.

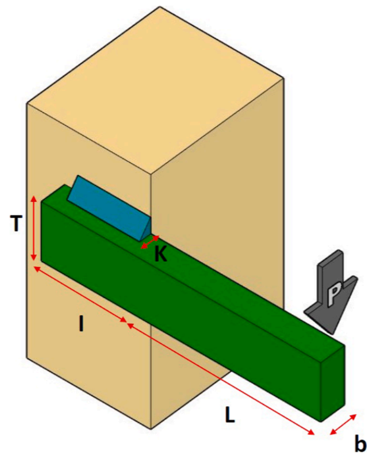

Fig. 16. Welded beam design problem.

Table 19
Results of the comparative algorithms for solving the welded beam design problem.

Algorithm	Optimal values for variables				Optimal	Ranking
	h	l	t	b	cost	
OBSCA (Elaziz et al., 2017)	0.230824	3.069152	8.988479	0.208795	1.722315	3
GSA (Saremi et al., 2017)	0.182129	3.856979	10.000	0.202376	1.87995	11
RO (Kaveh & Khayatazad, 2012)	0.203687	3.528467	9.004233	0.207241	1.735344	10
CSCA (Huang et al., 2007)	0.203137	3.542998	9.033498	0.206179	1.733461	9
GA (Deb, 1991)	0.2489	6.1730	8.1789	0.2533	2.4300	15
DAVID (Ragsdell & Phillips, 1976)	0.2434	6.2552	8.2915	0.2444	2.3841	14
SIMPLEX (Ragsdell & Phillips, 1976)	0.2792	5.6256	7.7512	0.2796	2.5307	16
APPROX (Ragsdell & Phillips, 1976)	0.2444	6.2189	8.2915	0.2444	2.3815	13
HS (Lee & Geem, 2005)	0.2442	6.2231	8.2915	0.2400	2.3807	12
SSA (Mirjalili et al., 2017)	0.2057	3.4714	9.0366	0.2057	1.72491	5
CPSO (He & Wang, 2007a)	0.202369	3.544214	9.04821	0.205723	1.72802	7
WOA (Mirjalili & Lewis, 2016)	0.205396	3.484293	9.037426	0.206276	1.730499	8
MVO (Mirjalili et al., 2016)	0.205463	3.473193	9.044502	0.205695	1.72645	6
MPA (Faramarzi, Heidarinejad, Mirjalili, et al., 2020)	0.205728	3.470509	9.036624	0.205730	1.724853	4
SMA (Zhao et al., 2020)	0.2054	3.2589	9.0384	0.2058	1.69604	2
AO	0.1631	3.3652	9.0202	0.2067	**1.6566**	1

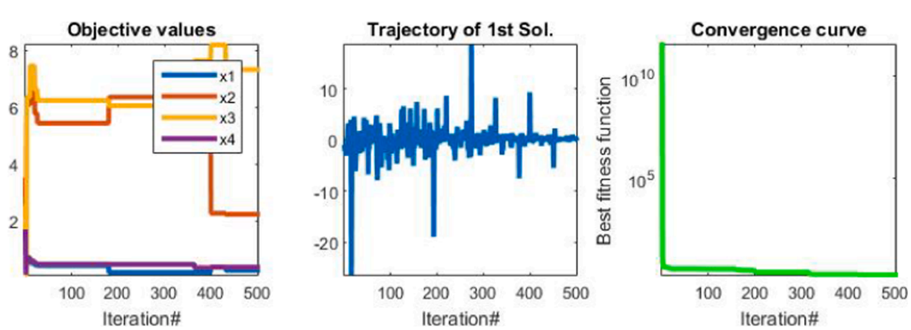

Fig. 17. Qualitative results for the welded beam design problem.

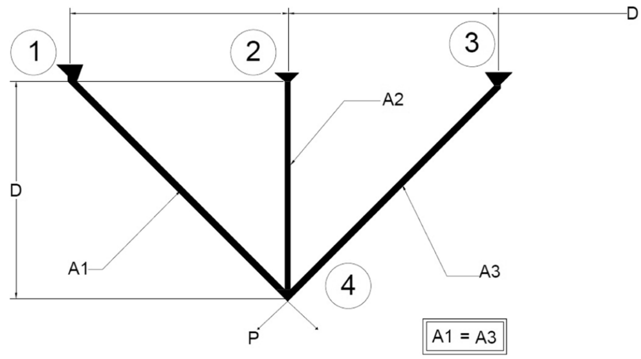

Fig. 18. 3-bar truss design problem.

Table 20
Results of the comparative algorithms for solving the 3-bar truss design problem.

Algorithm	**Optimal values for variables**		Optimal weight	Ranking
	x_1	x_2		
Ray and Sain (Ray & Saini, 2001)	0.795	0.395	264.3	9
AAA (Yildirim & Karci, 2018)	0.7887354	0.408078	263.895880	6
SSA (Mirjalili et al., 2017)	0.78866541	0.408275784	263.89584	2
MBA (Sadollah et al., 2013)	0.7885650	0.4085597	263.89585	5
DEDS (Zhang et al., 2008)	0.78867513	0.40824828	263.89584	2
GOA (Saremi et al., 2017)	0.78889755557	0.40761957011	263.89588149	7
PSO-DE (Liu et al., 2010)	0.7886751	0.4082482	263.89584	2
CS (Gandomi et al., 2013)	0.78867	0.40902	263.9716	8
AO	0.7926	0.3966	263.8684	1

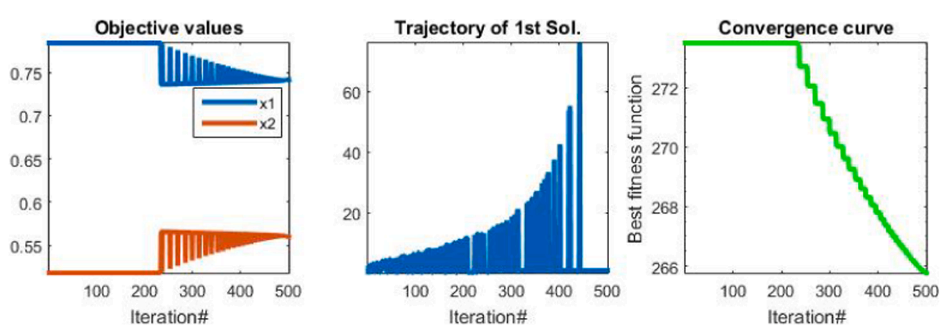

Fig. 19. Qualitative results for the 3-bar truss design problem.

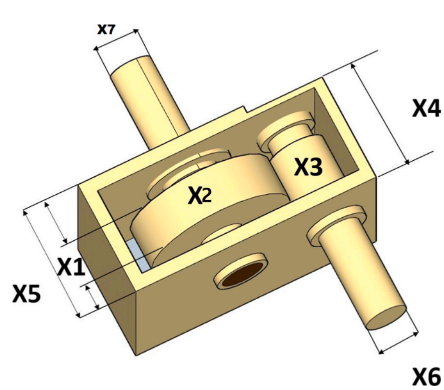

Fig. 20. Speed reducer problem.

Table 21
Results of the comparative algorithms for solving the speed reducer design problem.

Algorithm	Optimal values for variables							Optimal weight	Ranking
	x_1	x_2	x_3	x_4	x_5	x_6	x_7		
GA (Saruhan & Uygur, 2003)	3.510253	0.7	17	8.35	7.8	3.362201	5.287723	3067.561	10
SES (Mezura-Montes et al., 2003)	3.506163	0.700831	17	7.460181	7.962143	3.362900	5.308949	3025.005127	5
PSO (Stephen et al., 2018)	3.5001	0.7000	17.0002	7.5177	7.7832	3.3508	5.2867	3145.922	11
GSA (Rashedi et al., 2009)	3.600000	0.7	17	8.3	7.8	3.369658	5.289224	3051.120	9
hHHO-SCA (Kamboj et al., 2020)	3.506119	0.7	17	7.3	7.99141	3.452569	5.286749	3029.873076	7
MDA (Lu & Kim, 2010)	3.5	0.7	17	7.3	7.670396	3.542421	5.245814	3019.583365	4
SCA (Mirjalili, 2016a)	3.508755	0.7	17	7.3	7.8	3.461020	5.289213	3030.563	8
HS (Geem et al., 2001)	3.520124	0.7	17	8.37	7.8	3.366970	5.288719	3029.002	6
FA (Baykasoğlu & Ozsoydan, 2015)	3.507495	0.7001	17	7.719674	8.080854	3.351512	5.287051	3010.137492	3
SBSM (Akhtar et al., 2002)	3.506122	0.700006	17	7.549126	7.859330	3.365576	5.289773	3008.08	2
AO	3.5021	0.7000	17.0000	7.3099	7.7476	3.3641	5.2994	3007.7328	1

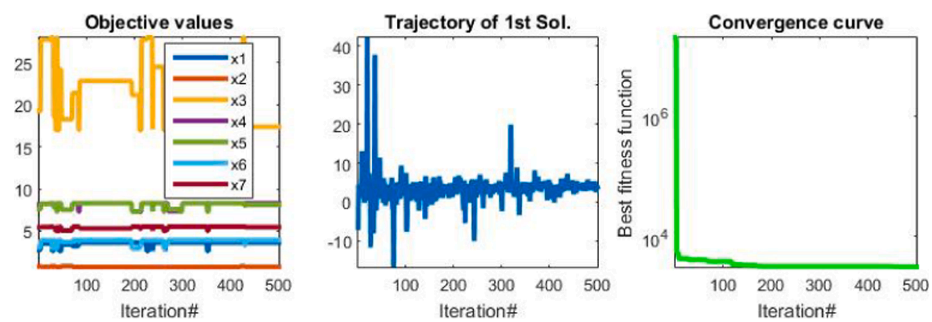

Fig. 21. Qualitative results for the speed reducer design problem.

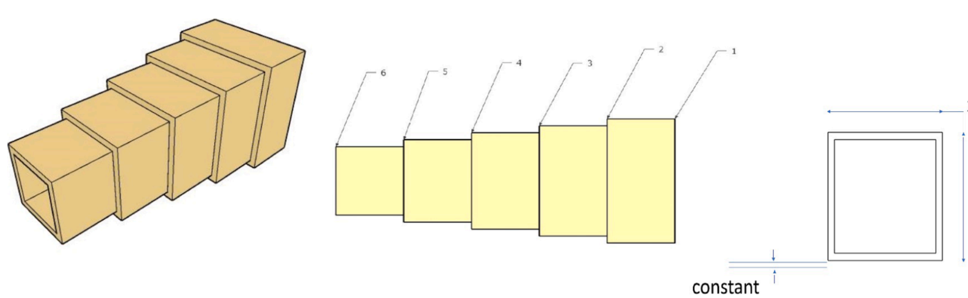

Fig. 22. Cantilever beam design problem.

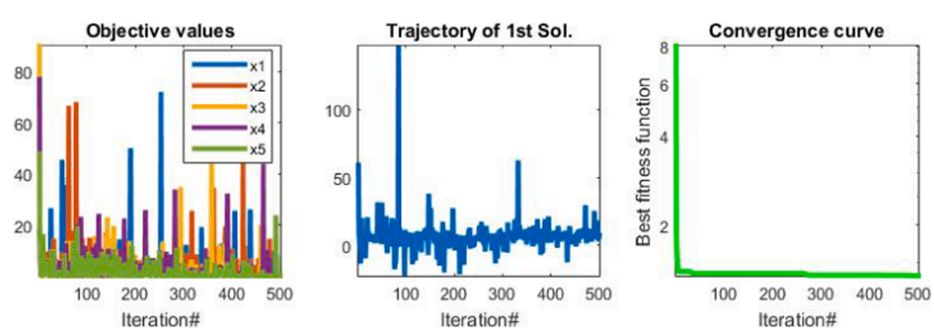

Fig. 23. Qualitative results for the cantilever beam design problem.

Table 22
Results of the comparative algorithms for solving the cantilever beam design problem.

Algorithm	Optimal values for variables					Optimal weight	Ranking
	x_1	x_2	x_3	x_4	x_5		
GCA_I (Chickermane & Gea, 1996)	6.0100	5.30400	4.4900	3.4980	2.1500	1.3400	7
ALO (Mirjalili, 2015b)	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995	3
GCA_II (Chickermane & Gea, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400	7
CS (Gandomi et al., 2013)	6.0089	5.3049	4.5023	3.5077	2.1504	1.3399	2
MMA (Chickermane & Gea, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400	7
SOS (Cheng & Prayogo, 2014)	6.01878	5.30344	4.49587	3.49896	2.15564	1.33996	5
SMA (Zhao et al., 2020)	6.017757	5.310892	4.493758	3.501106	2.150159	1.33996	4
MFO (Mirjalili, 2015a)	5.9830	5.3167	4.4973	3.5136	2.1616	1.33998	6
AO	5.8881	5.5451	4.3798	3.5973	2.1026	1.3390	1

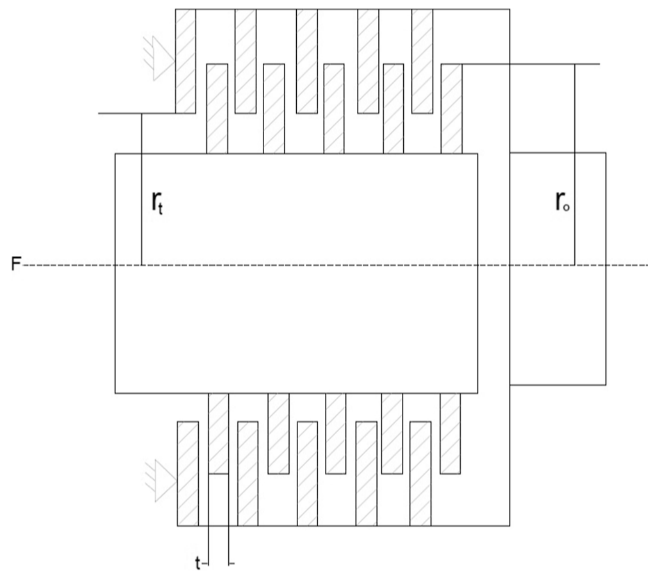

Fig. 24. Multiple disc clutch brake problem.

Table 23
Results of the comparative algorithms for solving the multiple disc clutch brake problem.

Algorithm	Optimal values for variables					Optimal weight	Ranking
	x_1	x_2	x_3	x_4	x_5		
TLBO (Rao et al., 2011)	70	90	1	810	3	0.313657	6
MFO (Bhesdadiya et al., 2018)	70	90	1	910	3	0.313656	2
NSGA-II (Deb & Srinivasan, 2008)	70	90	1.5	1000	3	0.470400	7
MVO (Sayed et al., 2018)	70	90	1	910	3	0.313656	2
WCA (Eskandar et al., 2012)	70	90	1	910	3	0.313656	2
CMVO (Sayed et al., 2018)	70	90	1	910	3	0.313656	2
AO	78.4228	98.5674	1	846.8894	2.5294	0.30835	1

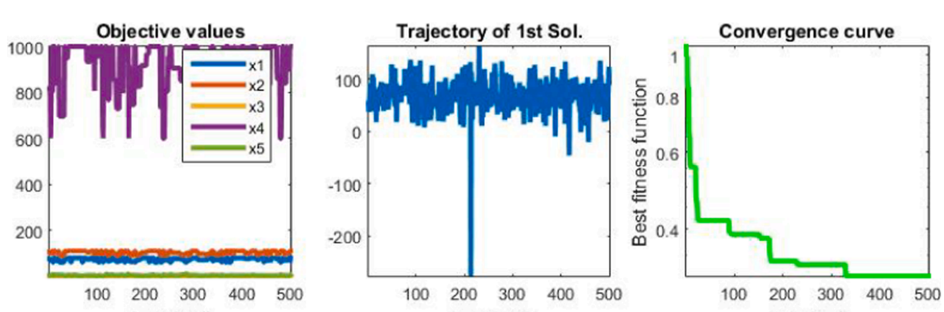

Fig. 25. Qualitative results for the multiple disc clutch brake problem.

CRediT authorship contribution statement

Laith Abualigah: Supervision, Conceptualization, Methodology, Software, Investigation, Validation, Writing - original draft. **Dalia Yousri:** Writing - original draft, Visualization, Investigation. **Mohamed Abd Elaziz:** Writing - original draft, Visualization, Investigation. **Mohammed A.A. Al-qaness:** Writing - original draft, Visualization, Investigation. **Amir H. Gandomi:** Writing - review & editing.

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