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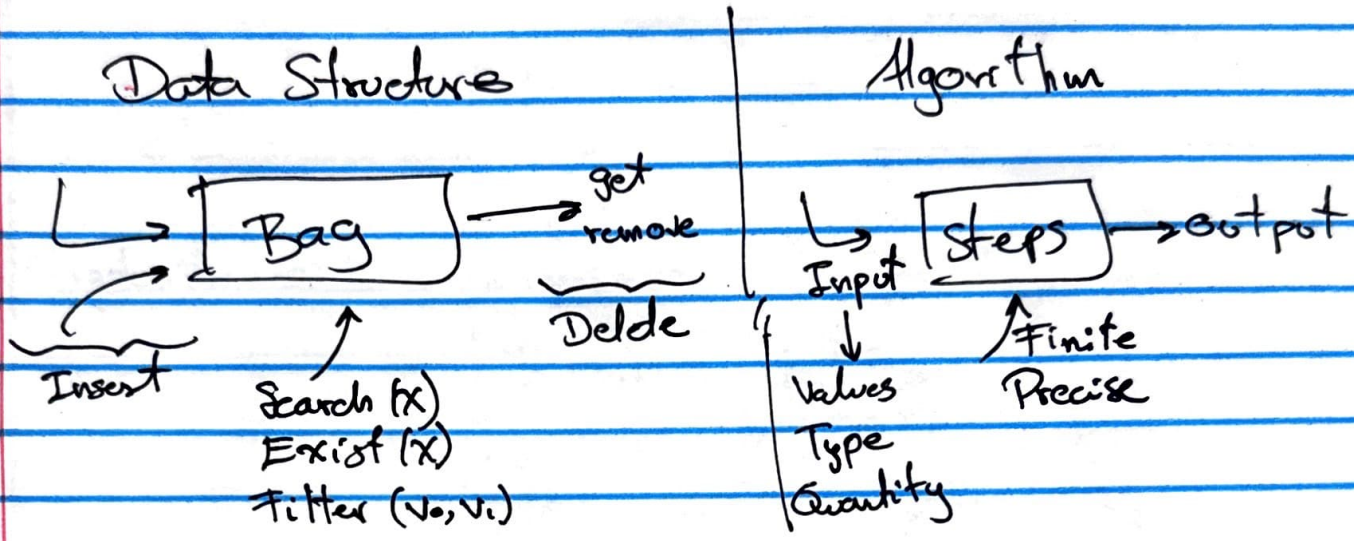
# CS251 — DS and Algo




## Primality Test: $f: \mathbb{N} \to \mathbb{B}$

→ Assumptions

```
Algorithm IsPrime(n: Z+)
    if n = 1 then
        return false
    end if
    for i from 2 to √n do
        if n mod i = 0 then
            return false
        end if
    end for
    return true
end algorithm
```

Correct?

Proof of Correctness

P(i): n is not divisible by i

(Proof by Invariant)

Runtime?

$$T(n) = \begin{cases} 3 & , n = 1 \\ 3 + \sum_{i=2}^{n-1} 3 & , \text{otherwise}^* \end{cases}$$

Linear function if n is prime

At most a linear amount in $n$ is not prime

36: 1 · 36, 2 · 18, 3 · 12, 4 · 9, 6 · 6, 9 · 4, 12 · 3, 18 · 2, 36 · 1

$\sqrt{36}$ = 6.6

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Eratosthenes:

| | | | | |
|---|---|---|---|---|
| ~~0~~ | ~~1~~ | (2) | (3) | ~~4~~ |
| (5) | ~~6~~ | 7 | ~~8~~ | ~~9~~ |
| ~~10~~ | 11 | ~~12~~ | 13 | ~~14~~ |
| ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 |
| ~~20~~ | ~~21~~ | ~~22~~ | 23 | ~~24~~ |

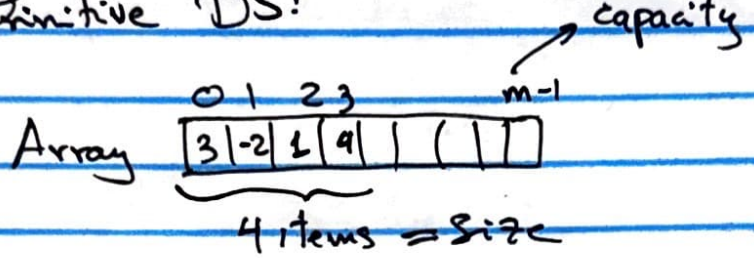
n=24

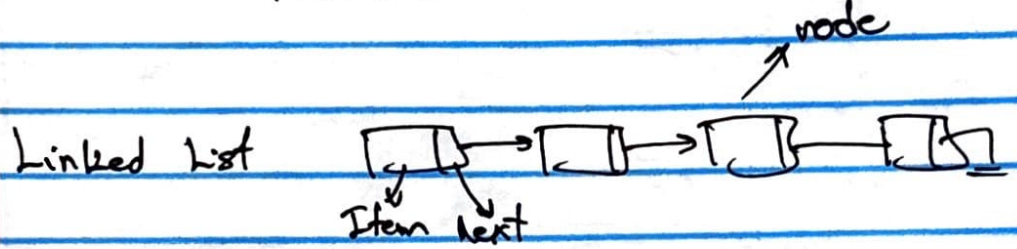
Algorithm Sieve ($n: \mathbb{Z}$)
  let S be an array of size n+1 initialized with true values
  S[0] ← false
  S[1] ← false
  for i from 2 to $\sqrt{n}$ do
    if S[i] then
      for j from $i^2$ to n by i do
        S[j] ← false
      end for
    end if
  end for
  return $\{i \in \{0,\cdots,n\} \mid S[i]\}$
end algorithm

$T(n) \approx \sqrt{n}\,(\log\log(n))$

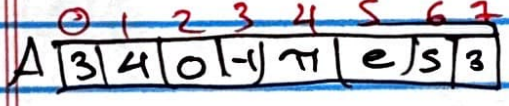
Prime Harmonic series

Primitive DS:




Size = Capacity




m=8
n=8

where -1 ?

Algorithm linear Search (A:array, x:item)

1: Let n be the size of A
2: for i from 0 to n-1 do
3: if A[i] = x then
4: return i → Successful Search
5: end if
6: end for
7: return -1 → Unsuccessful Search
end algorithm

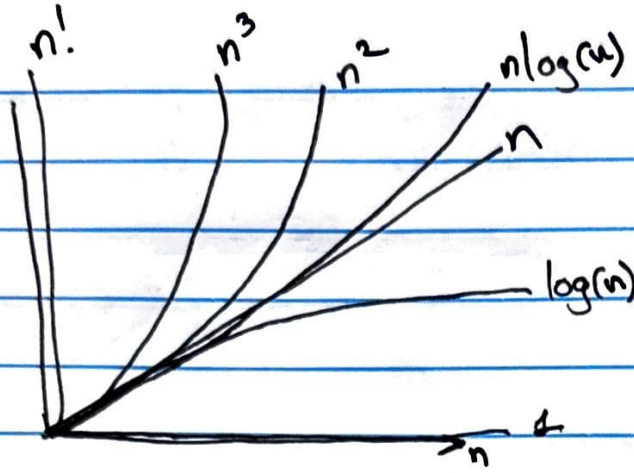
Runtime T(n)

1: Var init ($C_{vi}$)
2: Var init + Assign + Comp ($C_{as}$, Comp)
3: Array access + comp + comp ($C_{AA}$)
4:
5: ($C_{add}$)
6: Add + Assign + comp
7: return ($C_{ret}$)

$$\Rightarrow T(n) = \underbrace{C_{vi}}_{\text{Unsuccessful Search}} + \underbrace{C_{vi} + C_{as} + C_{comp}}_{2} + \underbrace{\sum_{i=0}^{n-1} C_{AA} + C_{comp} + C_{comp}}_{3}$$

$$+ \underbrace{\sum_{i=0}^{n-1} C_{add} + C_{as} + C_{comp}}_{} + C_{ret}$$

$$T(n) = 2C_{vi} + C_{as} + C_{comp} + n(C_{AA} + C_{comp} + C_{comp}) + n(C_{add} + C_{as} + C_{comp}) + C_{ret}$$

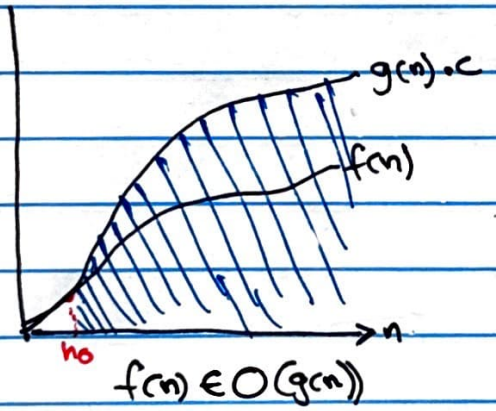



big-Oh: $f(n) \in O(g(n))$, $\exists c>0, n_0>0$ s.t. $f(n) \leq c \cdot g(n)$, $\forall n > n_0$

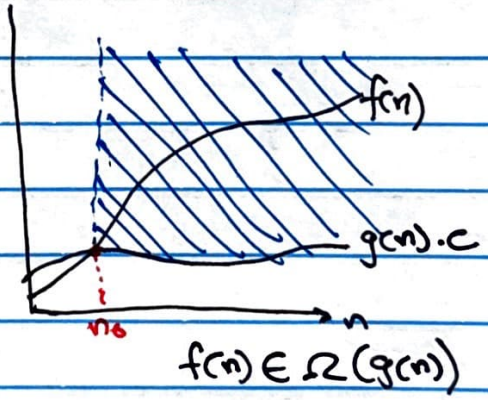
big-$\Omega$: $f(n) \in \Omega(g(n))$, $\exists c>0, n_0>0$ s.t. $c \cdot g(n) \leq f(n)$, $\forall n > n_0$

big-$\Theta$: $f(n) \in \Theta(g(n))$, $\exists c_1>0, c_2>0, n_0>0$ s.t.

$c_1 \cdot g(n) < f(n) \leq c_2 \cdot g(n)$ $\forall n > n_0$




$f(n) \in O(g(n))$




$f(n) \in \Omega(g(n))$