# EE650 MINI PROJECT

- **SUBJECT:** Autonomous Vehicle Steering
- **Tools:** SIMULINK
- **REFERENCE:** K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Princeton University Press, 2009.


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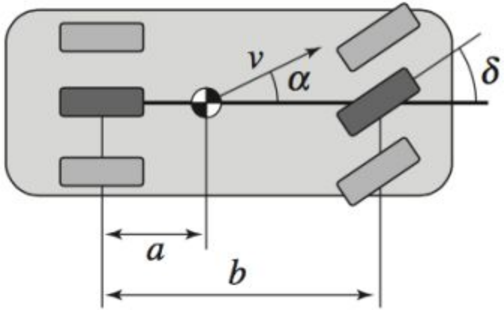
## Introduction:

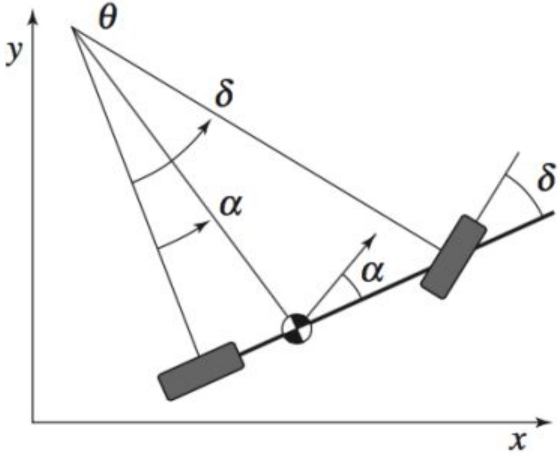
- A common problem in the motion control is to control the trajectory of vehicle through an *actuator that causes a change in the orientation.
- We can understand the basic behavior of these systems through the use of a simple model that captures the basic kinematics of the system.
- A steering wheel on an automobile and the front wheel of a bicycle are two examples, but similar dynamics occur in the steering of ships or control of the pitch dynamics of an aircraft.
- Motion control systems involve the use of computation and feedback to control the movement of a mechanical system.

*Actuator: A device that alters a physical quantity

# Vehicle steering dynamics:




View of a vehicle with four wheels.




By approximating the motion,we obtain a bicycle model.

# Mathematical Modeling:

$$\alpha(\theta) = arc\tan\left(\frac{\alpha\tan(\theta)}{b}\right)$$

$$\frac{dx}{dt} = v\cos((\alpha+\theta)) = v_o\frac{\cos((\alpha+\theta))}{\cos(\alpha)}$$

$$\frac{dy}{dt} = v\sin(\alpha+\theta) = v_o\frac{\sin(\alpha+\theta)}{\cos(\alpha)}$$

$$\frac{d\theta}{dt} = \frac{v_o}{r_a} = \frac{v_o}{b}\tan(\delta)$$

δ= steering angle/angle of the front wheel
$\theta$ = heading angle/orientation of COM
**α**= angle between the velocity vector v and centerline of the vehicle
a= distance from the rear wheel
b=wheelbase
x,y coordinates of center of mass/ position
vo=velocity of rear wheel

$$b = \mathrm{r}_a\tan(\delta)\ ,\ a = \mathrm{r}_a\tan(\alpha)\ ,\ it\ follows\ \tan(\alpha) = \left(\frac{a}{b}\right)\tan(\delta)$$

# State-space model:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} v\cos(\alpha(\delta) + \theta) \\ v\sin(\alpha(\delta) + \theta) \\ \frac{v_o}{b}\tan(\delta) \end{bmatrix}, \quad \alpha(\delta) = arc\tan\left(\frac{a\tan(\delta)}{b}\right)$$

The Non-linear equations of system

- ❑ Suppose instead that we are concerned with the lateral deviation of the vehicle from a straight line.
- ❑ For simplicity, we let θe = 0, which corresponds to driving along the x axis. We can then focus on the equations of motion in the y and θ directions.
- ❑ We introduce the state x = (y, θ) and u = δ.

  - ❑ **State Variables (x):** In the case of autonomous vehicle steering is x = (y, θ)
  - ❑ **Input Variables (u):** The input is the desired steering angle ,u = δ.
  - ❑ **Output Variables (y):** lateral deviation of the vehicle from a straight line.

After the assumptions, the non-linear equations can converge into linear equations

$$f(x,u) = \begin{bmatrix} v\sin(\alpha(u) + x2) \\ \frac{v_o}{b}\tan(u) \end{bmatrix}, \ \alpha(u) = arc\tan\left(\frac{a\tan(u)}{b}\right), \ h(x,u) = x_1$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=0,\,u=0} = \begin{bmatrix} 0 & v_0 \\ 0 & 0 \end{bmatrix}, \ B = \left.\frac{\partial f}{\partial u}\right|_{x=0,\,u=0} = \begin{bmatrix} \frac{av_o}{b} \\ \frac{v_o}{b} \end{bmatrix}$$

$$C = \left.\frac{\partial h}{\partial x}\right|_{x=0,\,u=0} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ A = \left.\frac{\partial h}{\partial u}\right|_{x=0,\,u=0} = 0$$

The linear form of the system is: $\frac{dx}{dt} = Ax + Bu, \ y = Cx + Du$

## Objective of the Controller:

We need to design the controller for our system to serve the following necessities:

- **Trajectory Tracking:** The controller in autonomous vehicle steering generates steering commands based on the deviation between the desired trajectory and the vehicle's current state to ensure accurate path tracking.

- **Stability and Performance:** Controllers enhance system stability and performance by appropriately responding to disturbances like wind or road conditions, and can be tuned to optimize vehicle performance, minimizing overshooting during turns.

- **Adaptability to Changing Conditions:** Controllers can be designed to adapt to varying conditions, such as changes in road friction, vehicle dynamics, or external disturbances. This adaptability is crucial for the reliable and safe operation of autonomous vehicles in real-world scenarios.

## Design of the state- feedback controller:

*General form of Linear Systems,*

$$\frac{dx}{dt} = Ax + Bu,\ y = Cx$$

*The closed loop system,*

$$\frac{dx}{dt} = (A - BK)\,x + BK_r$$

*At the equilibrium point,* $$\frac{dx}{dt} = 0$$

$$x_e = -\frac{(A - BK)^{-1}BK}{r}$$

*The output should be follow the reference , so* $r = Cx$

*This gives us*

$$K_r = \frac{1}{C(A - BK)^{-1}B}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\quad B = \begin{bmatrix} \gamma \\ 1 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},\quad D = 0$$

$$W_r = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} \gamma & 1 \\ 1 & 0 \end{bmatrix}$$

$$u = -Kx + K_r r = -k_1 x_1 - k_2 x_2 + k_r r$$

$$\frac{dx}{dt} = (A - BK)x + BK_r r = \begin{bmatrix} -\gamma k_1 & 1 - \gamma k_2 \\ -k_1 & -\gamma k_2 \end{bmatrix} x + \begin{bmatrix} \gamma k_r \\ k_r \end{bmatrix} r$$

$$y = Cx + Du = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

The closed loop system has the characteristic polynomial

$$\det(sI - A + BK) = \det \begin{bmatrix} s + \gamma k_1 & \gamma k_2 - 1 \\ k_1 & s + k_2 \end{bmatrix} = s^2 + (\gamma k_1 + k_2)s + k_1.$$

Suppose that we would like to use feedback to design the dynamics of the system to have the characteristic polynomial

$$p(s) = s^2 + 2\zeta_c\omega_c s + \omega_c^2.$$

Comparing this polynomial with the characteristic polynomial of the closed loop system, we see that the feedback gains should be chosen as

$$k_1 = \omega_c^2, \qquad k_2 = 2\zeta_c\omega_c - \gamma\omega_c^2.$$

gives $k_r = k_1 = \omega_c^2$, and the control law can be written as

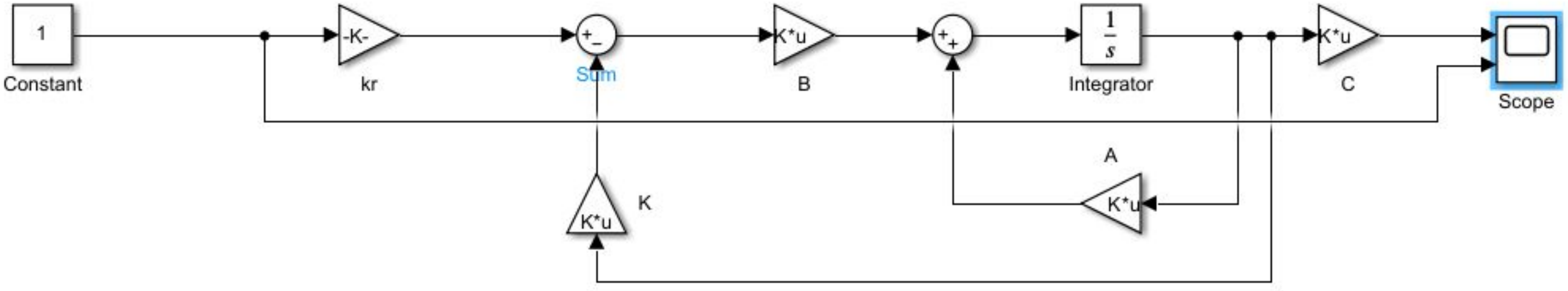
$$u = k_1(r - x_1) - k_2 x_2 = \omega_c^2(r - x_1) - (2\zeta_c\omega_c - \gamma\omega_c^2)x_2.$$

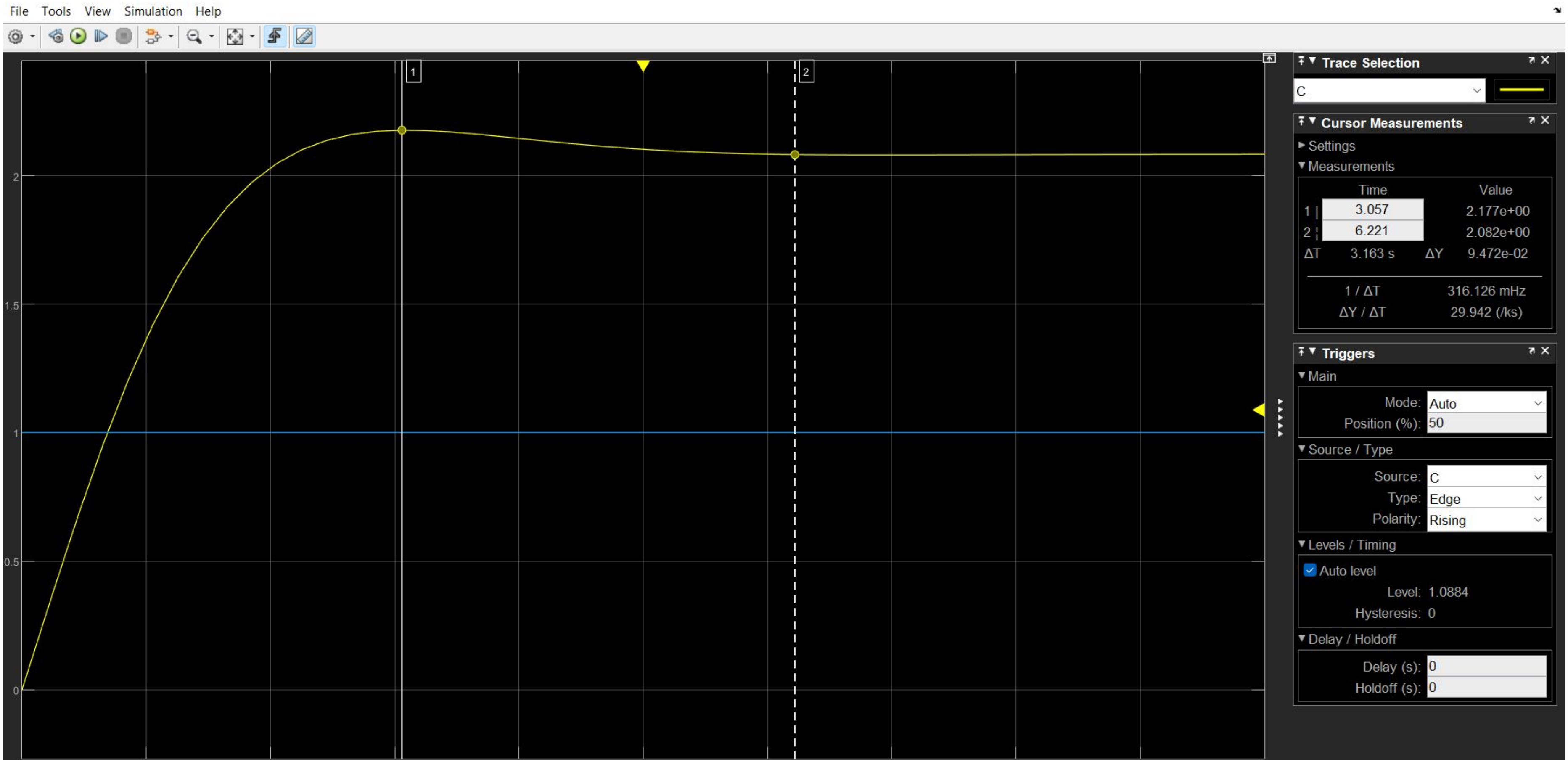
$Considering\ w_c = 1,\ \zeta_c = 0.7,\ v_0 = 1,\ a = 0.5,\ b = 1$

$$k_r = k_1 = \omega_c^{\ 2} = 1, \quad k_2 = 2\xi_c w_c - \gamma w_c^{\ 2} = 0.7$$

$$B = \begin{bmatrix} \gamma \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ K = \begin{bmatrix} 1 \\ 0.7 \end{bmatrix}$$

## System with only controller:





File Tools View Simulation Help
Trace Selection
C
Cursor Measurements
Settings
Measurements
Time Value
1 3.057 2.177e+00
2 6.221 2.082e+00
ΔT 3.163 s ΔY 9.472e-02
1 / ΔT 316.126 mHz
ΔY / ΔT 29.942 (/ks)
Triggers
Main
Mode: Auto
Position (%): 50
Source / Type
Source: C
Type: Edge
Polarity: Rising
Levels / Timing
Auto level
Level: 1.0884
Hysteresis: 0
Delay / Holdoff
Delay (s): 0
Holdoff (s): 0
2
1.5
1
0.5
0

## Justification for the performance of the controller:

The performance of a controller in an automation vehicle steering system is critical for ensuring the safety, stability, and efficiency of the vehicle's operation.

- Rise-time= 3.061s
- Peak overshoot=  1.77
- Settling time = 6.21

# Why do we need observer?

1. Sensor Limitations:

- Limitation: Sensors may not provide measurements for all system states.
- Observer Benefit: An observer can estimate unmeasured states based on available measurements, compensating for sensor limitations and providing a complete state estimate.

2. Cost and Implementation Challenges:

- Limitation: Adding more sensors to measure additional states can increase system cost and complexity.
- Observer Benefit: An observer provides a cost-effective solution by estimating unmeasured states using existing sensors, reducing the need for additional expensive hardware.

3. Noise and Measurement Inaccuracies:

- Limitation: Sensors may introduce noise or inaccuracies in measurements.
- Observer Benefit: An observer can filter out noise and enhance the accuracy of state estimates by incorporating a dynamic model of the system

4. Dynamic System Changes:

- Limitation: Changes in the system dynamics may not be immediately reflected in the controller's response.
- Observer Benefit: An observer, designed with appropriate dynamics, can adapt to changes in the system, ensuring the controller receives accurate and timely information about the state of the system.

# Observer design:

*Obeserver is then ,*

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$$

$$= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\hat{x} + \begin{pmatrix} r \\ 1 \end{pmatrix}u + \begin{pmatrix} l1 \\ l2 \end{pmatrix}(y - \widehat{x1})$$

$$\frac{d\tilde{x}}{dt} = (Ax + Bu) - (A\hat{x} + Bu + LC\tilde{x}) = (A - LC)\tilde{x}$$

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 2\zeta_c\omega_c \\ w_c{}^2 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 1 \end{bmatrix}$$

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}x + \begin{pmatrix} \gamma \\ 1 \end{pmatrix}u \ , \ y = \begin{pmatrix} 1 & 0 \end{pmatrix}x$$

*observability matrix*

$$\mathbf{W}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \textit{identity matrix}$$

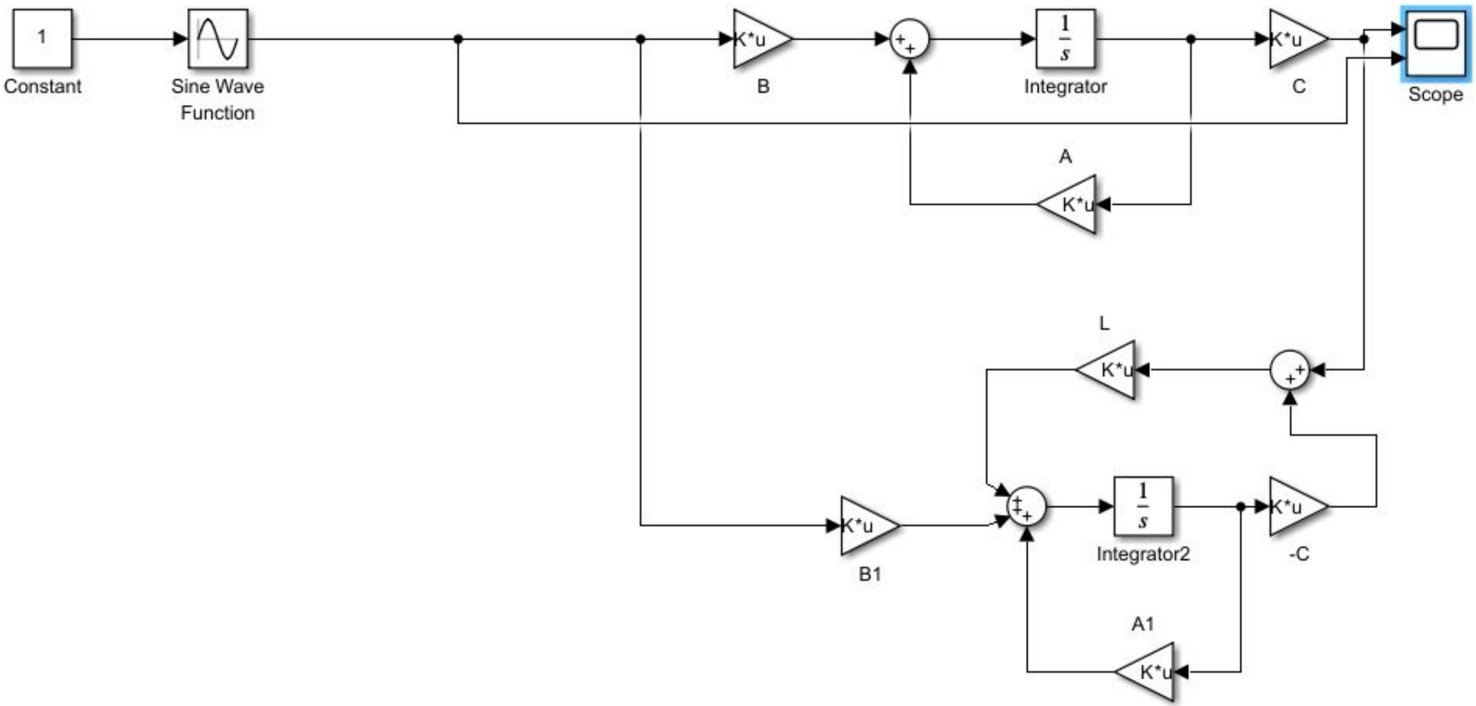
$$A - LC = \begin{pmatrix} -l1 & 1 \\ -l2 & 0 \end{pmatrix}$$

$$det(sI - A + LC) = det\left(\begin{pmatrix} s + l1 & -1 \\ l2 & s \end{pmatrix}\right) = s^2 + l1s + l2$$

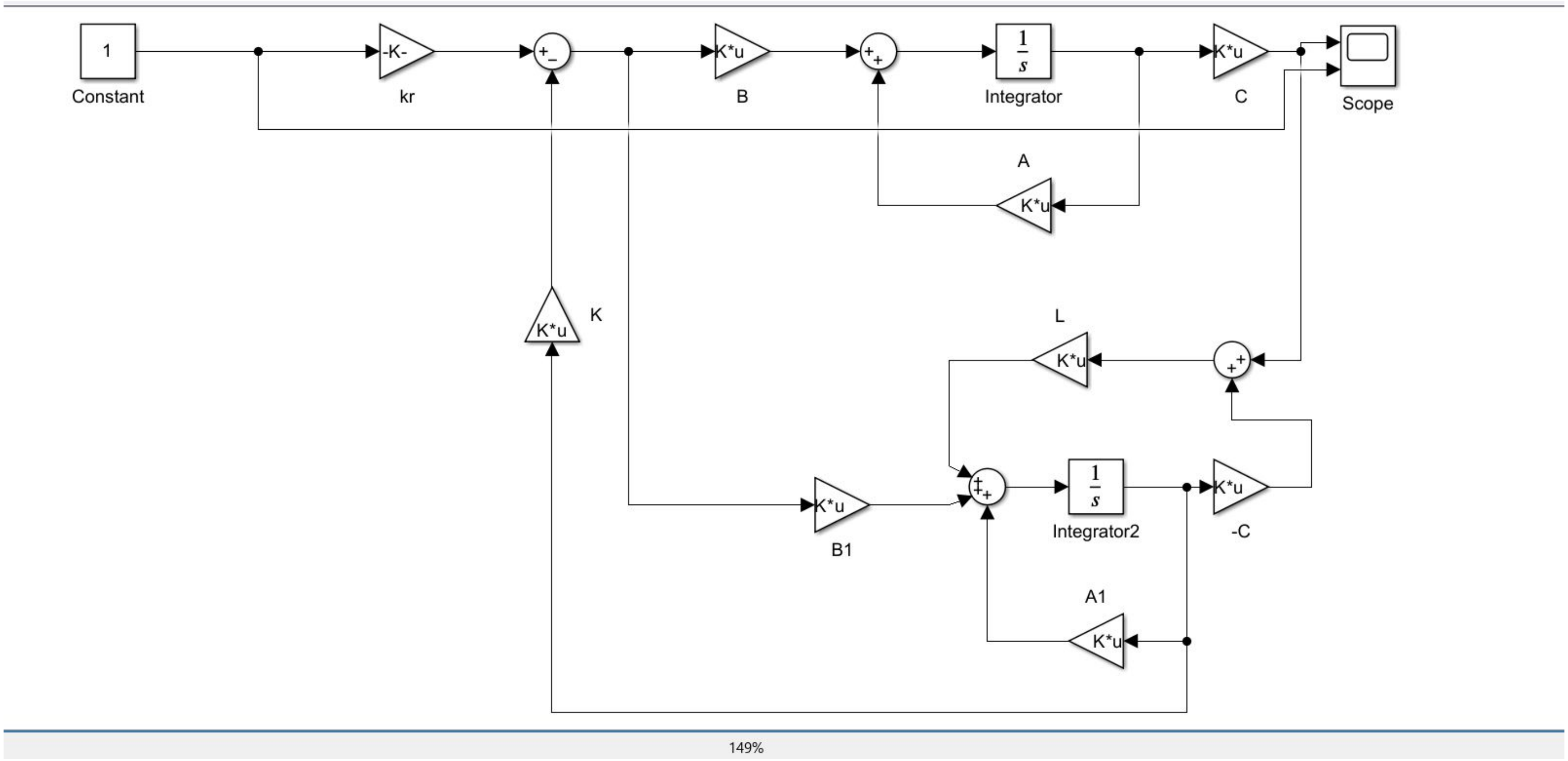
$$s^2 + p1s + p2 = s^2 + 2\zeta_0\mathbf{W}_0 s + \mathbf{W}_0^2$$

$$l1 = p1 = 2\zeta_0\mathbf{W}_0 \qquad l2 = p2 = \mathbf{W}_0^2$$

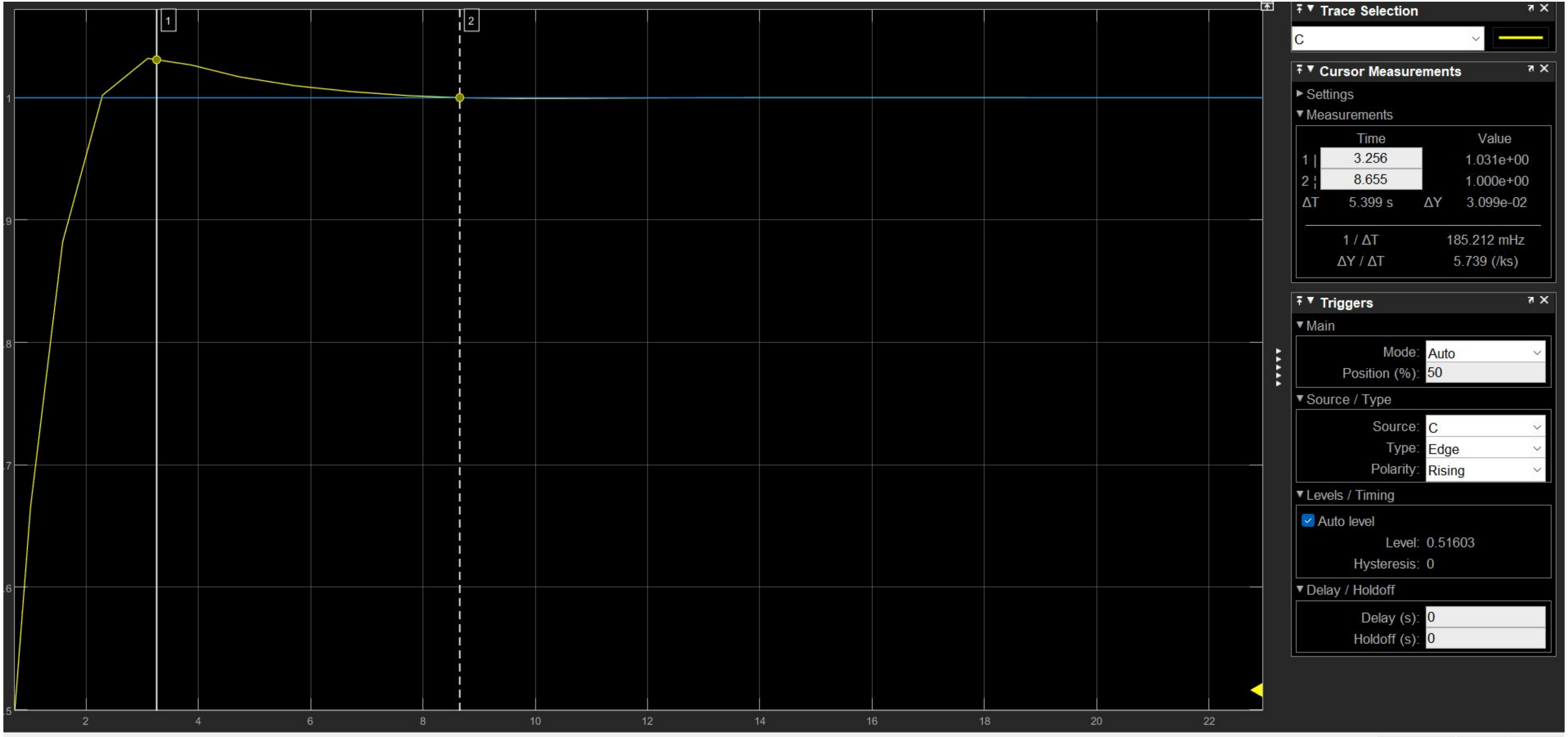
# Only with Observer:



# Block Diagram of the complete system:



$$
\begin{aligned}
K_r &= 3 \\
K_B &= [0.44, 1] \\
K_A &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\
K_C &= [1, 0] \\
K_L &= \begin{bmatrix} 1.4 \\ 1 \end{bmatrix} \\
K_{A1} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\
K_k &= [1.44, 1.0464]
\end{aligned}
$$


Trace Selection
C
Cursor Measurements
Settings
Measurements
Time Value
1 3.256 1.031e+00
2 8.655 1.000e+00
ΔT 5.399 s ΔY 3.099e-02
1 / ΔT 185.212 mHz
ΔY / ΔT 5.739 (/ks)
Triggers
Main
Mode: Auto
Position (%): 50
Source / Type
Source: C
Type: Edge
Polarity: Rising
Levels / Timing
Auto level
Level: 0.51603
Hysteresis: 0
Delay / Holdoff
Delay (s): 0
Holdoff (s): 0

# Justification for the performance of the observer:

We calculated the $K_r$ values as 3 but for optimized results we decreased the $K_r$ to 1.6

- Rise-time= 3.15s
- Peak overshoot= 0.332
- Settling time = 8.11
- Steady state value = 1

# THANK YOU