KIIT Deemed to be University
Online End Semester Examination(Autumn Semester-2021)

Subject Name & Code: Probability and Statistics & MA2011
Applicable to Courses: CS, IT, CSCE(2019-20 Admitted Batch & Back)

Full Marks=50 **Time:2 Hours**

SECTION-A(Answer All Questions. Each question carries 2 Marks)

Time:30 Minutes **(7×2=14 Marks)**

Question No	Question Type(MCQ/ SAT)	Question	CO Mapping	Answer Key (For MCQ Questions only)
Q.No:1	MCQ	For any two events A and B a. $P(A \cup B) = P(A) + P(B)$ b. $P(A \cup B') = P(B') + P(A \cap B)$ c. $P(A - B) = P(A) - P(B)$ d. $P(A \cup B) = 1$ and $P(A \cap B) = 0$	CO-1	b
	MCQ	A single card is drawn at random from an ordinary deck of cards. The probability that it is either a 2 or a 9 or a heart is a. 0.404 b. 0.365 c. 0.318 d. 0.414	CO-1	b
	MCQ	If the events A and B are independent, then a. A' and B' are independent b. A and B' are independent c. A' and B are independent d. All the above	CO-1	d
	MCQ	For any events A and B with $P(B) > 0$ a) $P(A\|B) + P(A'\|B)$ =1 b) $P(A\|B) + P(A'\|B)$ =0 c) $P(A\|B) + P(A'\|B)$ =0.75 d) $P(A\|B) + P(A'\|B)$ =0.5	CO-1	a

Q.No:2	**MCQ**	Let A, B and C be three events with probabilities $P(A) = 0.4, P(B) = 0.55$, $P(C) = 0.7$, $P(A \cap B) = 0.32$, $P(A \cap C) = 0.33$, $P(B \cap C) = 0.45$ and $(A \cap B \cap C) = 0.28$. What is the value of $(A \cup B \cup C)$? a) 0.81 b) 0.76 c) 0.78 d) 0.83	CO-2	d
	MCQ	Let A, B and C be three events with probabilities $P(A) = 0.42, P(B) = 0.51$, $P(C) = 0.67$, $P(A \cap B) = 0.29$, $P(A \cap C) = 0.33$, $P(B \cap C) = 0.41$ and $(A \cap B \cap C) = 0.22$. What is the value of $(A \cup B \cup C)$? a. 0.73 b. 0.79 c. 0.77 d. 0.81	CO-2	b
	MCQ	Let A, B and C be three events with probabilities $P(A) = 0.43, P(B) = 0.53$, $P(C) = 0.69$, $P(A \cap B) = 0.32$, $P(A \cap C) = 0.33$, $P(B \cap C) = 0.45$ and $(A \cap B \cap C) = 0.26$. What is the value of $(A \cup B \cup C)$? a. 0.86 b. 0.76 c. 0.81 d. 0.80	CO-2	c
	MCQ	Let A, B and C be three events with probabilities $P(A) = 0.39, P(B) = 0.52$, $P(C) = 0.71$, $P(A \cap B) = 0.30$, $P(A \cap C) = 0.31$, $P(B \cap C) = 0.45$ and $(A \cap B \cap C) = 0.23$. What is the value of $(A \cup B \cup C)$? a. 0.79 b. 0.78 c. 0.83 d. 0.81	CO-2	a

Q.No:3	**MCQ**	The cdf of the rv X is as follows: $$F(x) = \begin{cases} 0 & x < 0 \\ 0.06 & 0 \le x < 1 \\ 0.19 & 1 \le x < 2 \\ 0.39 & 2 \le x < 3 \\ 0.67 & 3 \le x < 4 \\ 0.92 & 4 \le x < 5 \\ 0.95 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$ Then $P(2 < X < 6)$ is a. 0.81 b. 0.76 c. 0.73 d. 0.56	CO-3	d
	MCQ	The cdf of the rv X is as follows: $$F(x) = \begin{cases} 0 & x < 0 \\ 0.06 & 0 \le x < 1 \\ 0.19 & 1 \le x < 2 \\ 0.39 & 2 \le x < 3 \\ 0.67 & 3 \le x < 4 \\ 0.92 & 4 \le x < 5 \\ 0.95 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$ Then $P(3 < X < 6)$ is a. 0.61 b. 0.76 c. 0.28 d. 0.56	CO-3	c
	MCQ	The cdf of the rv X is as follows: $$F(x) = \begin{cases} 0 & x < 0 \\ 0.06 & 0 \le x < 1 \\ 0.19 & 1 \le x < 2 \\ 0.39 & 2 \le x < 3 \\ 0.67 & 3 \le x < 4 \\ 0.92 & 4 \le x < 5 \\ 0.95 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$ Then $P(2 \le X < 5)$ is a. 0.76 b. 0.28 c. 0.56 d. 0.73	CO-3	d

	MCQ	The cdf of the rv X is as follows: $$F(x)=\begin{cases}0 & x<0\\ 0.06 & 0\le x<1\\ 0.19 & 1\le x<2\\ 0.39 & 2\le x<3\\ 0.67 & 3\le x<4\\ 0.92 & 4\le x<5\\ 0.95 & 5\le x<6\\ 1 & 6\le x\end{cases}$$ Then $P(3<X\le 9)$ is a. 0.33 b. 0.61 c. 0.73 d. 0.56	CO-3	a
Q.No:4	**MCQ**	If probability density function of the random variable X is $f(x)=k(x-1)(2-x)$ for $1<x<2$ and 0 otherwise then find the value of k and $p=P(X<1.1)$ a. k =3 and p $=0.029$ b. k =5 and p $=0.031$ c. k =4 and p $=0.026$ d. k =6 and p $=0.028$	CO-4	d
	MCQ	If probability density function of the random variable X is $f(x)=k(x-1)(2-x)$ for $1<x<2$ and 0 otherwise then find the value of k and $p=P(X<1.2)$ a) k =3 and p $=0.109$ b) k =5 and p $=0.102$ c) k =6 and p $=0.104$ d) k =6 and p $=0.107$	CO-4	c
	MCQ	If probability density function of the random variable X is $f(x)=k(x-1)(2-x)$ for $1<x<2$ and 0 otherwise then find the value of k and $p=P(X<1.3)$ a) k =3 and p $=0.239$ b) k =6 and p $=0.216$ c) k =4 and p $=0.226$ d) k =6 and p $=0.208$	CO-4	b

	MCQ	If probability density function of the random variable X is $f(x) = k(x-1)(2-x)$ for $1 < x < 2$ and 0 otherwise then find the value of k and $p = P(X < 1.4)$ a) k =6 and p = 0.352 b) k =5 and p = 0.364 c) k =4 and p = 0.344 d) k =6 and p = 0.342	CO-4	a
Q.No:5	**MCQ**	The probability that an individual is left-handed is 0.16. In a class of 10 students, what is the mean and standard deviation of the number of left-handed students? a. $\mu = 1.5;\ \sigma = 1.19$ b. $\mu = 1.6;\ \sigma = 1.26$ c. $\mu = 1.5;\ \sigma = 1.21$ d. $\mu = 1.6;\ \sigma = 1.16$	CO 5	d
	MCQ	The probability that an individual is left-handed is 0.21. In a class of 10 students, what is the mean and standard deviation of the number of left-handed students? a. $\mu = 2.5;\ \sigma = 1.29$ b. $\mu = 2.1;\ \sigma = 1.29$ c. $\mu = 1.9;\ \sigma = 1.26$ d. $\mu = 2.2;\ \sigma = 1.21$	CO 5	b
	MCQ	The probability that an individual is left-handed is 0.26. In a class of 10 students, what is the mean and standard deviation of the number of left-handed students? a. $\mu = 2.5;\ \sigma = 1.36$ b. $\mu = 2.8;\ \sigma = 1.36$ c. $\mu = 2.6;\ \sigma = 1.39$ d. $\mu = 3.1;\ \sigma = 1.26$	CO 5	c
	MCQ	The probability that an individual is left-handed is 0.23. In a class of 10 students, what is the mean and standard deviation of the number of left-handed students? a. $\mu = 2.3;\ \sigma = 1.33$ b. $\mu = 2.3;\ \sigma = 1.31$ c. $\mu = 2.9;\ \sigma = 1.36$ d. $\mu = 2.6;\ \sigma = 1.26$	CO 5	a
Q.No:6	**MCQ**	If the mean and variance of a random variable X are 0.5 and 3 respectively then the mean and	CO2	**c**

		variance of the random variable $Y = 3X + 2$ are a. 0 and 1 b. 3.5 and 0 c. 3.5 and 27 d. 0 and 27		
	MCQ	If the mean and variance of a random variable X are 1 and 4 respectively then the mean and variance of the random variable $Y = 3X + 2$ are a. 3 and 1 b. 5 and 36 c. 5 and 10 d. 3 and 27	CO2	**b**
	MCQ	If the mean and variance of a random variable X are 2 and 4 respectively then the mean and variance of the random variable $Y =- 3X + 2$ are a. -4 and 36 b. -4 and 12 c. -6 and 36 d. -6 and 12	CO2	**a**
	MCQ	If the mean and variance of a random variable X are 0.6 and 2 respectively then the mean and variance of the random variable $Y = 3X + 2$ are a. 1.8 and 18 b. 1.8 and 6 c. 3.8 and 18 d. 3.8 and 6	CO2	**c**
Q.No:7	**MCQ**	Sample mean and sample variance of the data 2, 7,3, 4,1 are a. $\bar{x} = 3.2, s^2 = 6.52$ b. $\bar{x} = 3.4, s^2 = 4.24$ c. $\bar{x} = 3.4, s^2 = 5.3$ d. $\bar{x} = 2.6, s^2 = 3.56$	CO5	**c**
	MCQ	Sample mean and sample variance of the data 2, 7,3, 4, 5 are a. $\bar{x} = 4.2, s^2 = 3.7$ b. $\bar{x} = 4.2, s^2 = 2.96$ c. $\bar{x} = 3.4, s^2 = 5.3$ d. $\bar{x} = 2.6, s^2 = 3.56$	CO5	a
	MCQ	Sample mean and sample variance of the data 3, 7,3, 4, 5 are a. $\bar{x} = 4.4, s^2 = 2.24$ b. $\bar{x} = 4.4, s^2 = 2.8$ c. $\bar{x} = 4.4, s^2 = 2.9$ d. $\bar{x} = 4.2, s^2 = 3.56$	CO5	b
	MCQ	Sample mean and sample variance of the data 2, 7,6, 4,1 are a. $\bar{x} = 4, s^2 = 6.5$ b. $\bar{x} = 4, s^2 = 6.2$ c. $\bar{x} = 3, s^2 = 5.3$ d. $\bar{x} = 3.6, s^2 = 6.16$	CO5	a

SECTION-B(Answer Any Three Questions. Each Question carries 12 Marks)

Time: 1 Hour and 30 Minutes **(3×12=36 Marks)**

Question No	**Question**	**CO Mapping (Each question should be from the same CO(s))**
Q.No:8	A computer consulting firm presently has bids out on three projects. Let A_i = {awarded project i}, for i =1,2,3, and suppose that $P(A_1) = 0.28$, $P(A_2) = 0.23$, $P(A_3) = 0.32$, $P(A_1 \cap A_2) = 0.16$, $P(A_1 \cap A_3) = 0.08$, $P(A_2 \cap A_3) = 0.09$, $P(A_1 \cap A_2 \cap A_3) = 0.03$. Compute the probability of each event: a) $A_1 \cup A_2$, b) $A'_1 \cap A'_2$, c) $A_1 \cup A_2 \cup A_3$, d) $A'_1 \cap A'_2 \cap A'_3$, e) $A'_1 \cap A'_2 \cap A_3$, f) $(A'_1 \cap A'_2) \cup A_3$.	**CO1**
	A computer consulting firm presently has bids out on three projects. Let A_i = {awarded project i}, for i =1,2,3, and suppose that $P(A_1) = 0.29$, $P(A_2) = 0.21$, $P(A_3) = 0.34$, $P(A_1 \cap A_2) = 0.18$, $P(A_1 \cap A_3) = 0.11$, $P(A_2 \cap A_3) = 0.12$, $P(A_1 \cap A_2 \cap A_3) = 0.06$. Compute the probability of each event: a) $A_1 \cup A_2$, b) $A'_1 \cap A'_2$, c) $A_1 \cup A_2 \cup A_3$, d) $A'_1 \cap A'_2 \cap A'_3$, e) $A'_1 \cap A'_2 \cap A_3$ f) $(A'_1 \cap A'_2) \cup A_3$.	**CO1**
	A computer consulting firm presently has bids out on three projects. Let A_i = {awarded project i}, for i =1,2,3, and suppose that $P(A_1) = 0.23$, $P(A_2) = 0.33$, $P(A_3) = 0.26$, $P(A_1 \cap A_2) = 0.15$, $P(A_1 \cap A_3) = 0.14$, $P(A_2 \cap A_3) = 0.18$, $P(A_1 \cap A_2 \cap A_3) = 0.09$. Compute the probability of each event: a) $A_1 \cup A_2$, b) $A'_1 \cap A'_2$, c) $A_1 \cup A_2 \cup A_3$, d) $A'_1 \cap A'_2 \cap A'_3$, e) $A'_1 \cap A'_2 \cap A_3$, f) $(A'_1 \cap A'_2) \cup A_3$.	**CO1**
Q.No:9	(i) Define Poisson distribution and show that mean and variance of Poisson distribution are same. (ii) Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour, so that the number of arrivals during a time period of t hours is a Poisson rv with parameter $= 8t$. a. What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 3? At least 5? b. What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period? c. What is the probability that at least 15 small aircraft arrive during a 2.5-hour period? That at most 8 arrive during this period?	CO2
	(i) Define geometric distribution. Find the moment generating function of geometric distribution, then find the mean and variance using it. (ii) A company that produces fine crystal knows from experience that 15% of its goblets have cosmetic flaws and must be classified as "seconds."	

	a. Among six randomly selected goblets, how likely is it that only 2 are seconds? b. Among six randomly selected goblets, what is the probability that at least two are seconds? c. If goblets are examined one by one, what is the probability that at most five must be selected to find four that are not seconds?	
	i. Define the pmf $h(x;\, n, M, N)$ of the hypergeometric distribution where the random variable X is the number of S's in a completely random sample of size n drawn from a population consisting of M S's and F's where S denotes success and F denotes failure. Write the cdf of hypergeometric distribution. Write the formula for mean and variance of hypergeometric distribution. Under which condition mean and variance of hypergeometric distribution converges to mean and variance of binomial distribution ii. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 8 examined that have a defective compressor. Compute the following: a. $P(X=5)$ b. $P(X \le 4)$ c. The probability that X exceeds its mean value by more than 2 standard deviation. d. Consider a large shipment of 400 refrigerators, of which 40 have defective compressors. If X is the number among 15 randomly selected refrigerators that have defective compressors, describe a less tedious way to calculate (at least approximately) $P(X \le 5)$ than to use the hypergeometric pmf.	
Q.No:10	The actual tracking weight of a stereo cartridge that is set to track at 3 g on a particular changer can be regarded as continuous rv X with pdf $$f(x) = \begin{cases} k[1-(x-3)^2] & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$ a. Find the value of k. b. Find the cumulative distribution function $F(x)$. c. What is the probability that the actual tracking weight is within 0.25 g of the prescribed weight? d. Find the median of X.	CO3
	Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$. a. What is the probability that the diameter of a randomly selected tree	

	will be at least 10 in.? b. What is the probability that the diameter of a randomly selected tree is within one standard deviation of its mean value? c. What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.? d. What value c is such that the interval includes 98% of all diameter values?	
	Two components of a mini computer have the following joint pdf for their useful lifetimes X and Y $f(x,y)=\begin{cases} xe^{-x(1+y)} & x\geq 0 \text{ and } y\geq 0 \\ 0 & \text{otherwise.}\end{cases}$ a. What is the probability that the lifetime X of the first component exceeds 3? b. What are the marginal pdf's of X and Y? c. Are the two lifetimes X and Y independent? Explain. d. What is the probability that the lifetime of at least one component exceeds 3?	
Q.No:11	a. Define covariance of two random variables X and Y and show that $\text{Cov}(aX+b, cY+d) = ac\text{Cov}(X,Y)$. b. Find the maximum likelihood estimate of θ if the density $f(x)=\begin{cases} \theta e^{-\theta x} & if \;\; x\geq 0 \\ 0 & if \;\; x<0\end{cases}$ c. Find a 99% confidence interval for μ of a normal population with standard deviation $\sigma = 6$ from the sample 38, 49, 43, 36, 48, 51.	CO4,CO5, CO6
	a. Define correlation coefficient of two random variables X and Y and prove that $\text{Corr}(aX+b, cY+d) = \text{Corr}(X,Y)$ when a and c have the same sign. What happens if a and c have opposite sign? b. Find the maximum likelihood estimate of mean of Poisson distribution. c. Find a 95% confidence interval for σ^2 of a normal population from the sample 63, 69, 65, 74, 66, 70, 68, 65, 63, 64.	
	a. Define covariance of two random variables X and Y and show that $Cov(X,Y) = E(XY) - E(X)E(Y)$.	

	b. Find the maximum likelihood estimate of mean and variance of Normal distribution. d. Find a 90% confidence interval for μ of a normalpopulation with standard variance 0.25, using the sample of 100 with the mean 212.3.	