

MANTRA : A Scalable Approach to Mining Temporally Anomalous Sub-trajectories

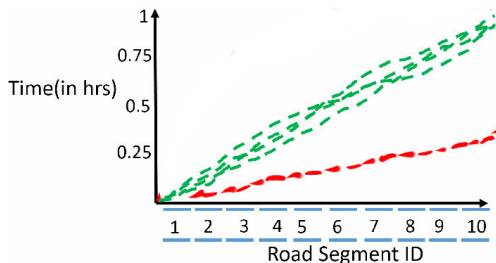
ACM SIGKDD Conference on Knowledge Discovery and Data Mining (KDD 2016)

Prithu Banerjee, Pranali Yawalkar, Sayan Ranu

17 June 2016

Introduction : Mining Temporally Anomalous Sub-Trajectories

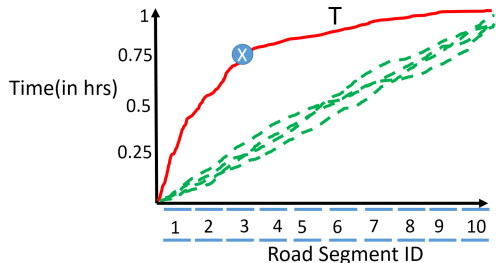
Temporally Anomalous



- Time taken to cover the trajectory deviates **significantly** from the remaining population
- Both **over-speeding** and **under-speeding** are anomalous

Introduction : Mining Temporally Anomalous Sub-Trajectories

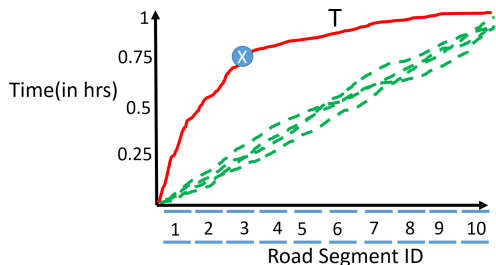
Why mine temporally anomalous *sub-trajectories*?



A non-anomalous trajectory may contain temporally anomalous sub-trajectories

Introduction : Mining Temporally Anomalous Sub-Trajectories

Why mine *maximal* anomalies ?



- No extra information provided by $T[1 : 2]$, $T[5 : 9]$ over $T[1 : 3]$ and $T[4 : 10]$
- Identifying *longest stretches* of anomalous driving

Applications

■ Real Time Vehicle Monitoring :

- Abundance of **GPS data** from smart devices ; MANTRA works **directly** on users' GPS data
- Identifying **anomalous drivers** (over-speeding & under-speeding) in real time ; < **25 ms**
- Robust against all year **weather & traffic conditions** ; useful in countries like **India**

Applications

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■ Identifying Bus Bunching

- A common phenomenon in countries like *India*
- Buses do not maintain the distance between the following ones *uniformly*
- *Undesirable* behaviour of *under-speeding* to pick up maximum passengers followed by *over-speeding* to maintain the distance from the following bus

Applications

■ Rating cab drivers :

- **GPS trackers** already installed in all cabs
- Identifying **how** anomalous and **where** was the anomalous driving exhibited
- **Real-time** application ; a pilot version already deployed by **Uber**

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■ Personalized Car Insurance, *Pay How You Drive (PHYD)*, *Usage Based Insurance (UBI)*

- Direct approach to *assess driving behaviour* from user's historical driving records
- *Mutually beneficial* scheme ; for the insurance company and the driver
- Efforts steered in *USA, Japan, Australia, EU* partnered with *Toyota*

Notations

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- **D** denote prevailing traffic conditions containing trajectories in history

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- **Trajectory**

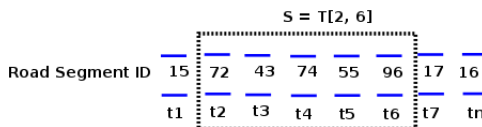
Road Segment ID	15	72	43	74	55	96	17	16
	t1	t2	t3	t4	t5	t6	t7	tn

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- **Sub-trajectory**



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- Time taken to traverse T times(T) = $\sum_{\forall e \in S} T.e_t$
- **Sub-trajectory**
- **Maximal Anomalous Sub-Trajectory (MAS) S** : no *anomalous super-trajectory* of S

PROBLEM STATEMENT

Given \mathbb{D} , the reference dataset of trajectories. For an input trajectory, identify all of its maximal temporally anomalous sub-trajectories under a user-provided threshold θ with respect to \mathbb{D} .

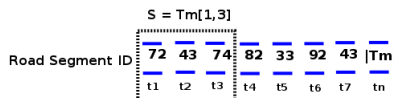
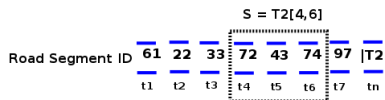
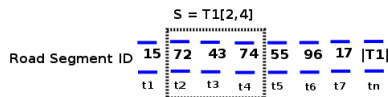
Anomaly Model

- Standard **z-score** based anomaly model with *Normal Distribution*
- Distribution of travel times along $S = N(\mu_S, \sigma_S^2)$, where μ_S is the **mean** and σ_S^2 is the **variance**

Anomaly Model

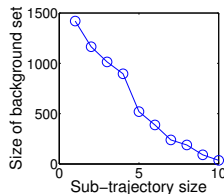
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- Compute μ_S and σ_S^2 from the background set of S in \mathbb{D}

$D = \{T1, T2, \dots, Tm\}$
 $S = [72, 43, 74]$



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- Compute μ_S and σ_S^2 from the background set of S in \mathbb{D}
- Issue of **data sparsity** ; non existent background set for sub-trajectory size > 10

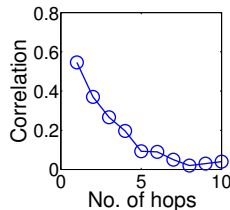


Managing Data Sparsity

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- 2 $\forall e, e' \in E, cov(e, e') \geq 0$

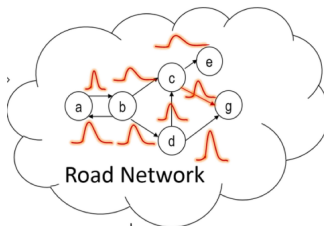
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- For $\forall e \in E$, fit a normal distribution $time(e) = \mathcal{N}(\mu_e, \sigma_e^2)$

- Modelling travel times along S as **multivariate distribution** of its edges

$$\sigma_S^2 = \sum_{\forall e \in S} \sigma_e^2 + 2 \sum_{\forall \{e, e'\} \in S} cov(e, e') \quad (1)$$

Anomaly Model

- **Deviation of S** as an *aggregate* of the deviation in its constituent edges

$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2	0.5	-1	-1.5	-2.5
Road Segment	1	2	3	4	5	6	7	8
		MAS 1				MAS 2		

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$$\text{dist}(S) = \sum_{\forall e \in S} \mathcal{I}(e)(\mu_e - S.t_e)^2 \quad (2)$$

$$\mathcal{I}(e) = \begin{cases} 1 & \text{if } S.t_e \geq \mu_e : \text{Over-speeding} \\ -1 & \text{if } S.t_e < \mu_e : \text{Under-speeding} \end{cases} \quad (3)$$

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- S is **anomalous** if

$$\frac{|\text{dist}(S)|}{\sigma_S^2} > \theta \quad (4)$$

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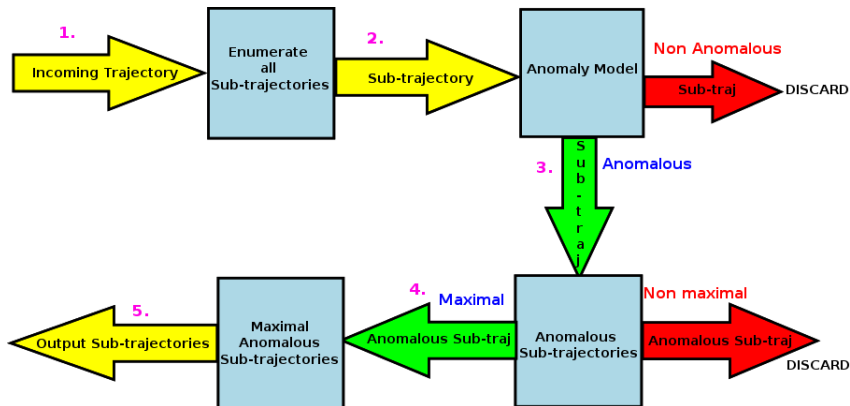
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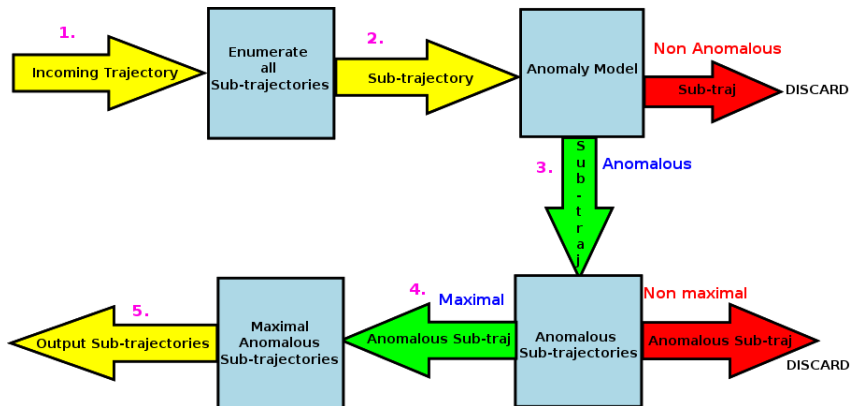
$$\frac{|\text{dist}(S)|}{\sigma_S^2} > \theta \quad (4)$$

- Anomalous sub-trajectory **can** contain **non-anomalous edges**.
- Anomalous sub-trajectory **must** contain at least one **anomalous edge**.

Approach 1 : The Naïve Approach



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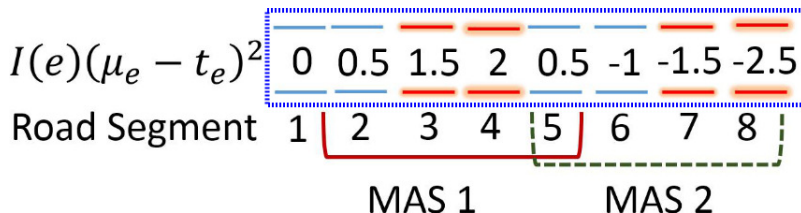
- Computation complexity of T with n edges = $\mathcal{O}(n^2)$; not scalable

Approach 2 : Bi-Directional Sliding Window

- **Avoid** evaluating **non-maximal** anomalous sub-trajectories
- Evaluating **longer sub-trajectories first**

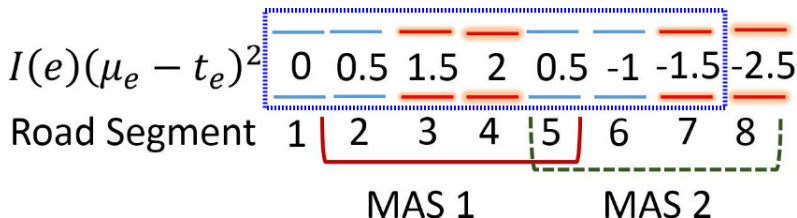
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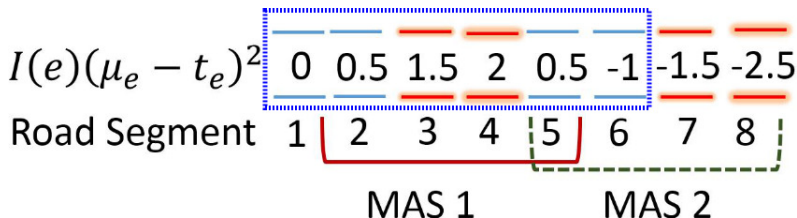
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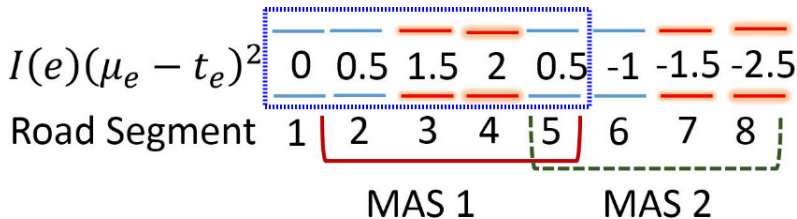
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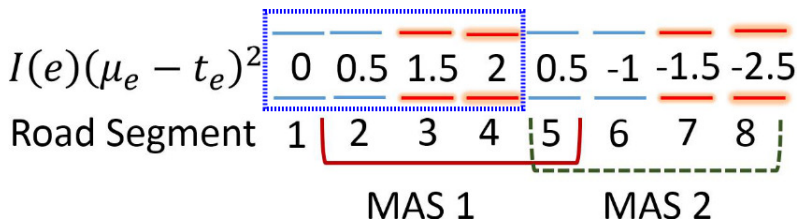
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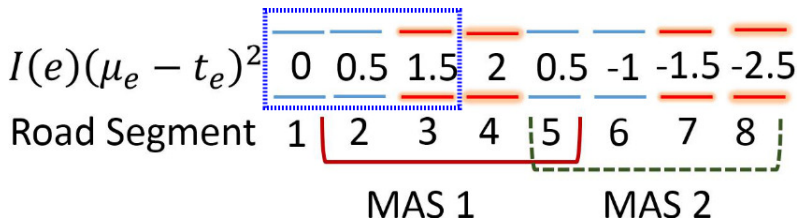
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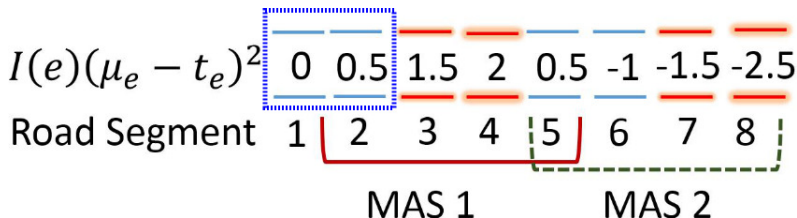
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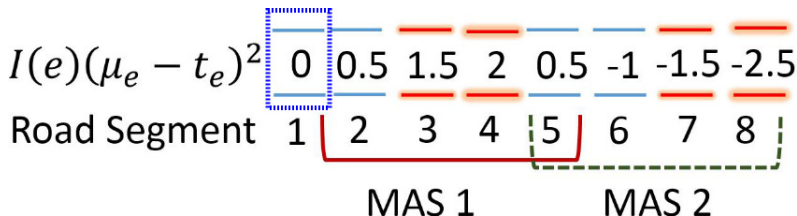
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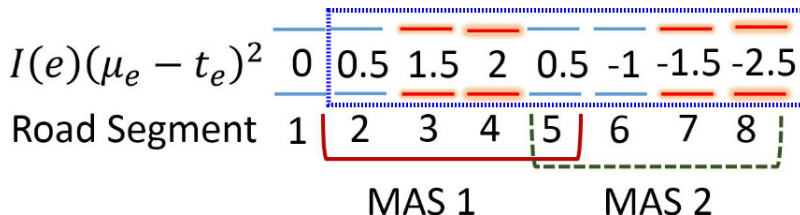
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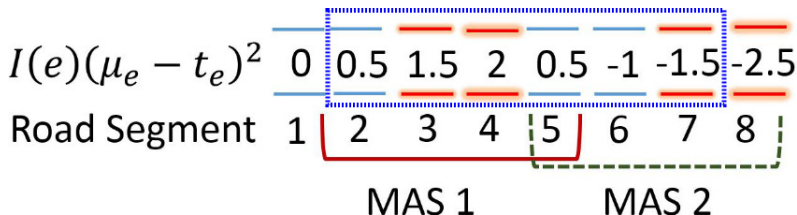
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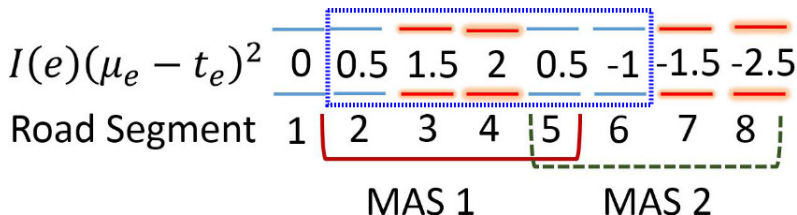
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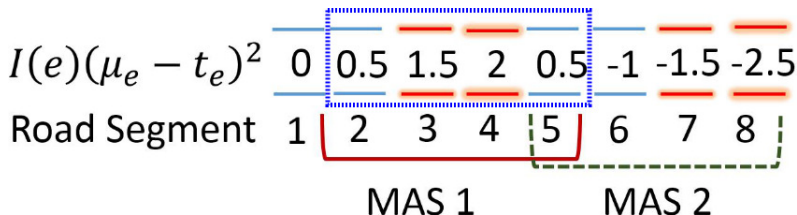
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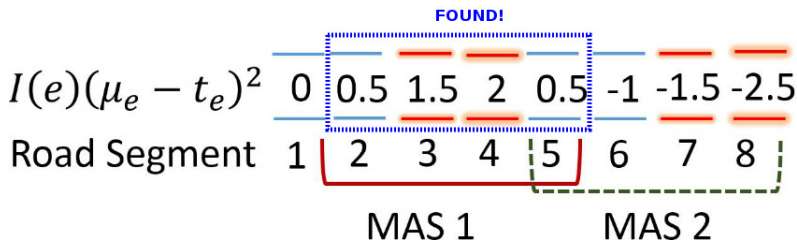
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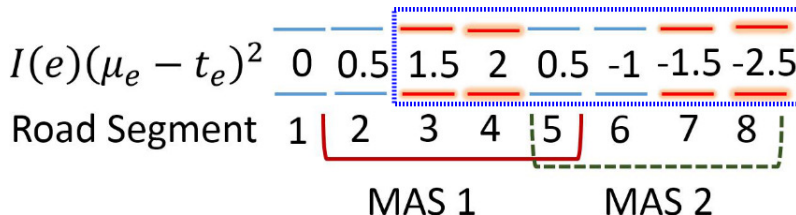
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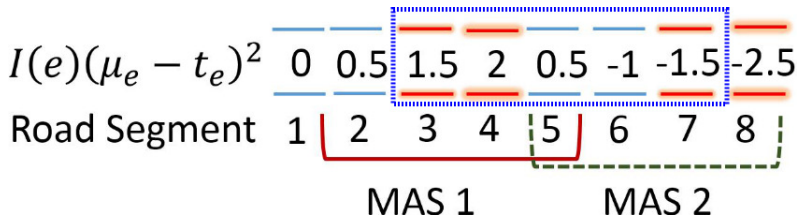
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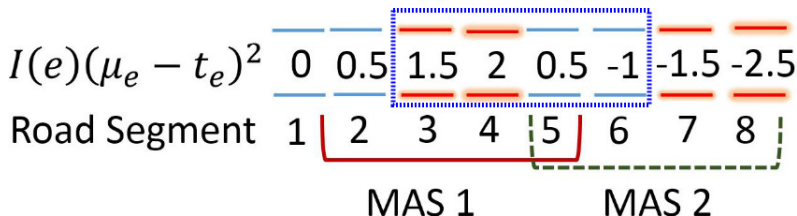
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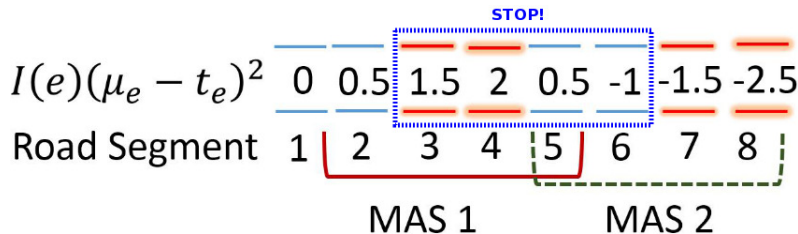
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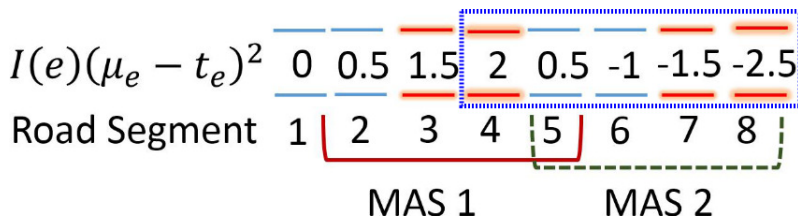
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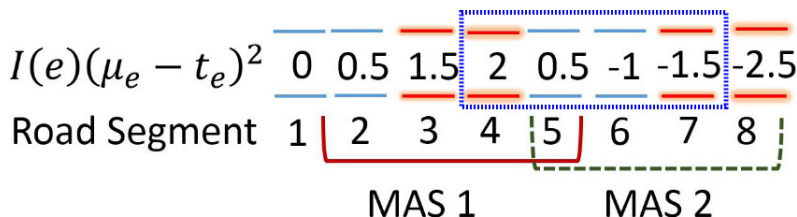
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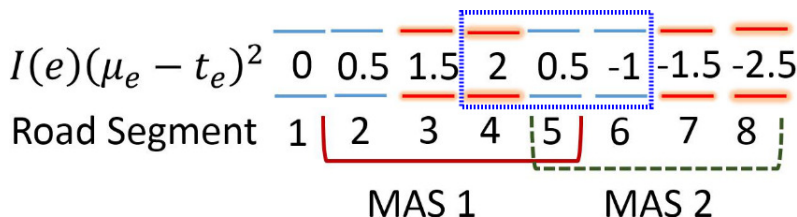
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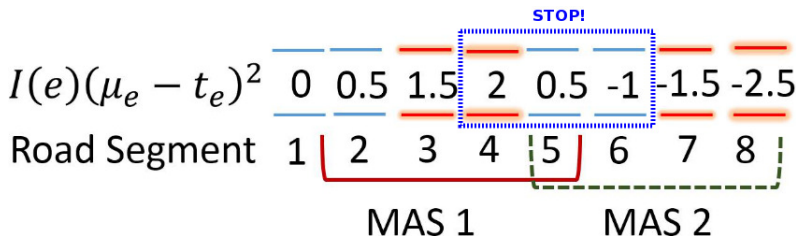
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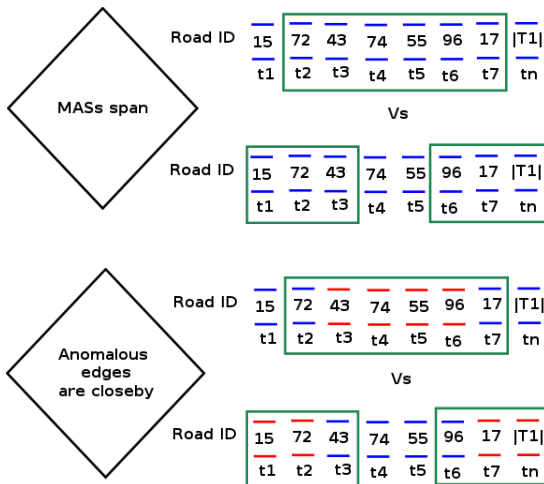


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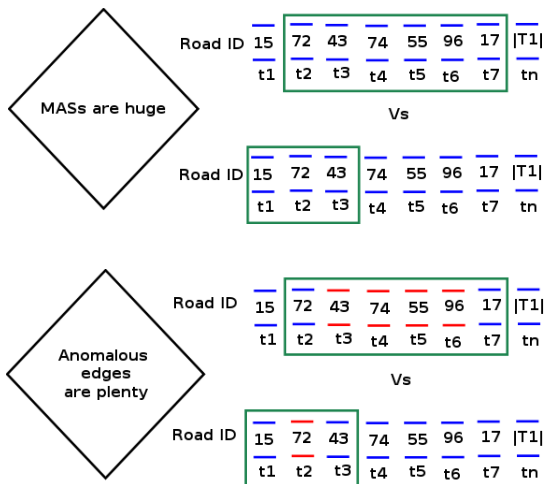
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1. Best scenario for Bi-Directional Sliding Window



2. Best scenario for Bi-Directional Sliding Window



MANTRA's Mantra

MANTRA identifies segments of input trajectory which are best suited for Bi Directional Sliding Window.

MANTRA applies BSW on these special segments, called *Islands*.

Impact Regions

- Seeds (contiguous anomalous edges) : $S_1 = T[3 : 4]$ and $S_2 = T[7 : 8]$

$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2	0.5	-1	-1.5	-2.5
Road Segment	1	2	3	4	5	6	7	8

Impact Regions

- Seeds (contiguous anomalous edges) : $S_1 = T[3 : 4]$ and $S_2 = T[7 : 8]$

- Left Boundary :

		LB(S1)				LB(S2)			
$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2		0.5	-1	-1.5	-2.5
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Impact Regions

- Seeds (contiguous anomalous edges) : $S_1 = T[3 : 4]$ and $S_2 = T[7 : 8]$

- Left Boundary :

- Right Boundary :

	RB(S1)					RB(S2)		
$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2	0.5	-1	-1.5	-2.5
Road Segment	1	2	3	4	5	6	7	8

Continued

■ Impact Region :

■ S_1

		LB(S_1)		RB(S_1)				
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Continued

■ Impact Region :

■ S_1

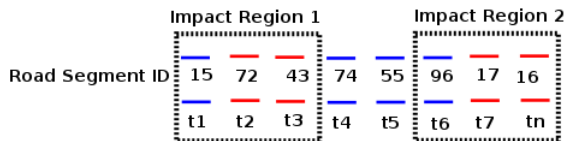
■ S_2

					LB(S_2)		RB(S_2)	
$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2	0.5	-1	-1.5	-2.5
Road Segment	1	2	3	4	5	6	7	8

					LB(S_2)		RB(S_2)	
$I(e)(\mu_e - t_e)^2$	0	0.5	1.5	2	0.5	-1	-1.5	-2.5
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Impact Regions

Impact Regions separated by more than one non-anomalous edges **DO NOT** interact.



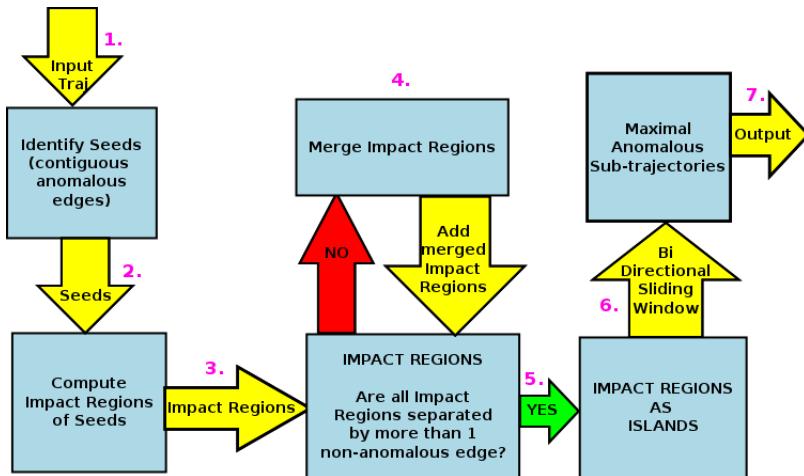
Such Impact Regions are called *Islands*.

All **MASs** are contained within *Islands* and do not span across them.

Islands are **best suited** for Bi Directional Sliding Window.

MANTRA Pipeline

APPROACH 3. MANTRA



MANTRA Example

Road Segment ID

1	2	3	4	5	6	7	8	9	10	11	12	13	14
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MANTRA Example

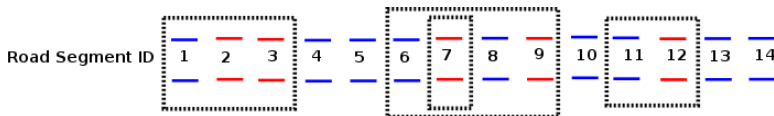
■ Seed identification

Road Segment ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—	—

MANTRA Example

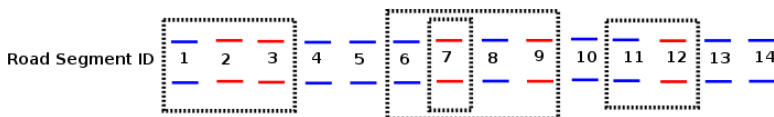
Seed identification

Compute impact region of the seeds



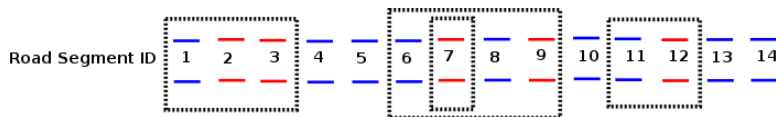
Continued

■ Merge interacting impact regions of seeds till convergence

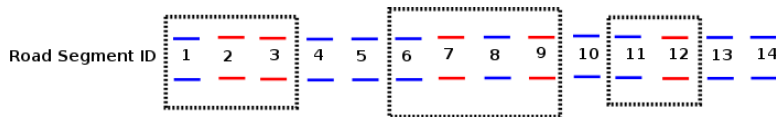


Continued

■ Merge interacting impact regions of seeds till convergence

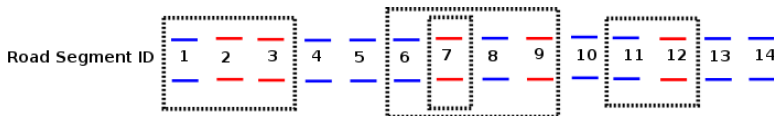


■ 1st iteration :

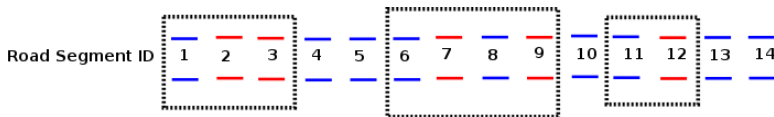


Continued

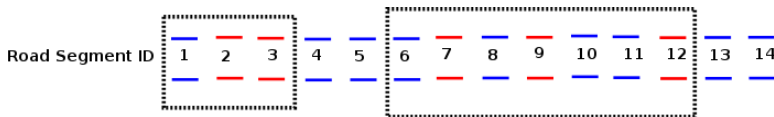
■ Merge interacting impact regions of seeds till convergence



■ 1st iteration :



■ 2nd iteration :

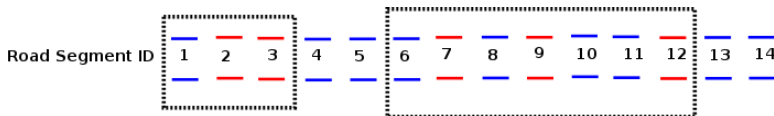


Continued

■ Merge interacting impact regions of seeds till convergence

■ 1st iteration :

■ 2nd iteration :



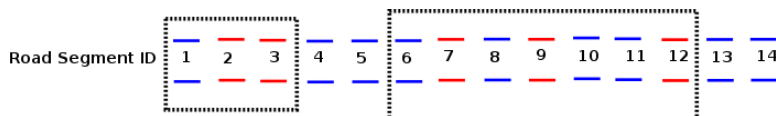
■ **Final seeds** : $ST1[1 : 3]$ and $ST5[6 : 12]$

Continued

■ Merge interacting impact regions of seeds till convergence

■ 1st iteration :

■ 2nd iteration :



■ **Final seeds** : $ST1[1 : 3]$ and $ST5[6 : 12]$

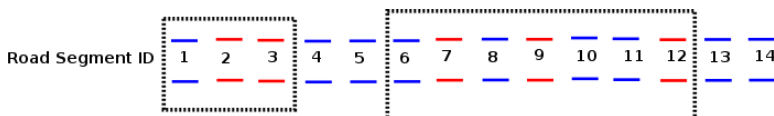
■ **Islands** : non-interacting impact regions

Continued

■ Merge interacting impact regions of seeds till convergence

■ 1st iteration :

■ 2nd iteration :



■ **Final seeds** : $ST1[1 : 3]$ and $ST5[6 : 12]$

■ **Islands** : non-interacting impact regions

■ **Perform Bi-Directional Sliding Window on the islands**

Set Up and Datasets

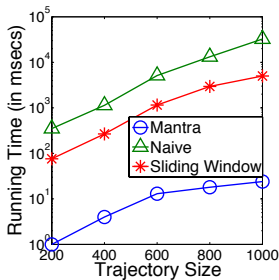
EXPERIMENTATION SET UP

- Java JDK 1.7.0
- 12GB memory
- Intel i5 2.60GHz quad core processor
- Ubuntu 13.04

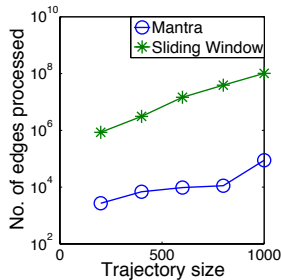
DATASETS FROM BEIJING

- T-drive dataset
 - Largest publicly available trajectory dataset
 - 136,759 trajectories
- Geolife dataset
 - 18760 trajectories
 - Vehicle annotated trajectories : car, walk, bus

Effect of Trajectory Size



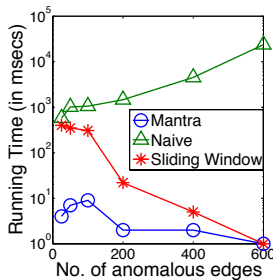
(a)



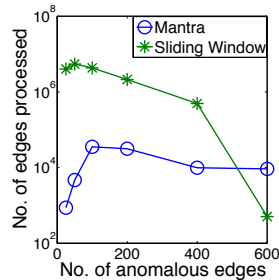
(b)

- 1 MANTRA is upto **3 orders of magnitude faster** ; **less** number of edges processed
- 2 For longer trajectories, MANTRA **consumes** < **25 ms**

Effect of Number of Anomalous Edges



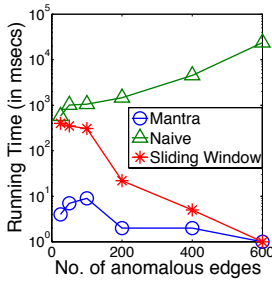
(c)



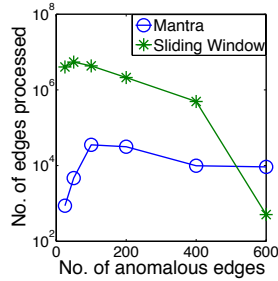
(d)

- 1 For sliding window, **number of anomalous edges** $\uparrow \Rightarrow$ **running time** \downarrow , **# edges processed** \downarrow
- 2 For MANTRA, **hump** $\hat{=}$ **more iterations to converge** islands formation

Effect of Number of Anomalous Edges



(e)



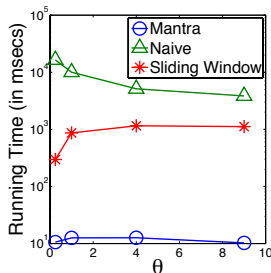
(f)

1 For sliding window, number of anomalous edges $\uparrow \Rightarrow$ running time \downarrow , # edges processed \downarrow

2 For MANTRA, hump $\hat{=}$ more iterations to converge islands formation

3 Sliding Window overtakes MANTRA ; islands formation redundant with \uparrow in anomalous edges

Effect of Anomaly Threshold on Runtime



- \uparrow threshold \Rightarrow \downarrow anomalous edges
- Running time for Naive \downarrow with \uparrow in threshold ; less anomalous sub-trajectories
- Running time Sliding Window \uparrow with \uparrow in threshold ; less anomalous edges
- Hump $\hat{=}$ convergence of island formation

Efficacy and Applications

- *Are we able to identify sub-trajectories that would be considered anomalous by humans ?*

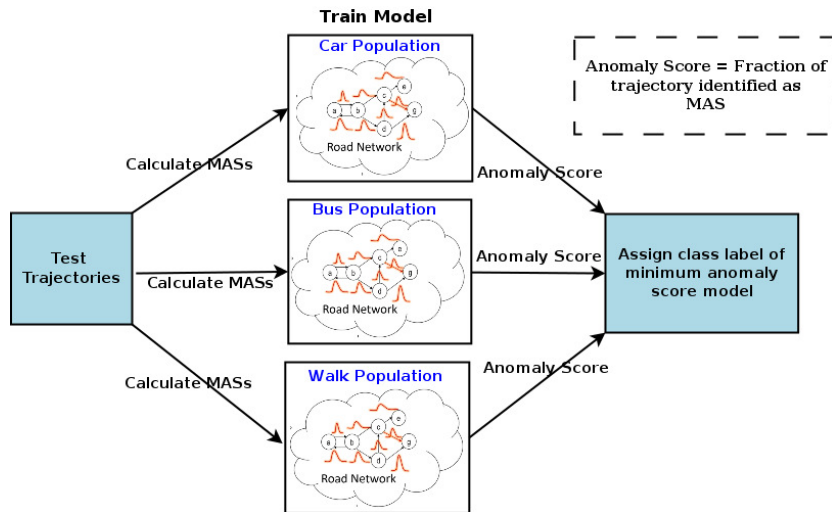
Efficacy and Applications

■ *Are we able to identify sub-trajectories that would be considered anomalous by humans ?*

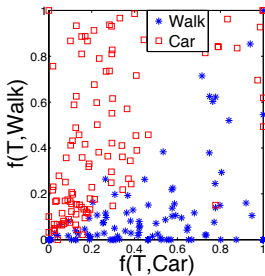
■ **Intuition :**

- Car trajectory **least** anomalous against Car population
- Car trajectory **more** anomalous against Walk and Bus population

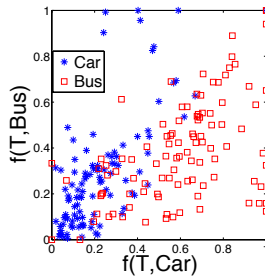
Trajectory Classification : Classification Model



Trajectory Classification

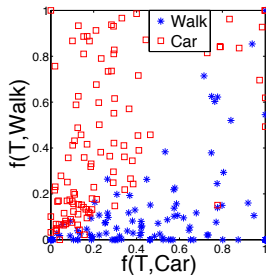


(g) Walk and Car

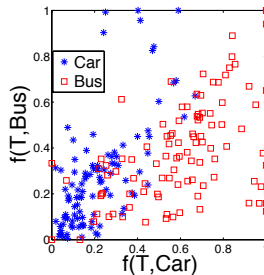


(h) Bus and Car

Trajectory Classification



(i) Walk and Car



(j) Bus and Car

- Majority *Walk* trajectories have **higher anomaly score** against *Car*
- *Car* vs *Walk* is **more drastic** than *Car* vs *Bus*

Trajectory Classification



(kϕl)
Walks
around
Car

- Majority *Walk* trajectories have **higher anomaly score** against *Car*
- Car* vs *Walk* is **more drastic** than *Car* vs *Bus*
- Two class classification f-score

Class label	Walk	Bus
Car	0.85	0.74
Walk	-	0.75

Trajectory Classification



(n)(h)
Walks
around
Car

- Majority *Walk* trajectories have **higher anomaly score** against *Car*
- Car* vs *Walk* is **more drastic** than *Car* vs *Bus*
- Two class classification f-score

Class label	Walk	Bus
Car	0.85	0.74
Walk	-	0.75

- Three class classification f-score

Class combination	car	walk	bus
Car-Walk-Bus	0.62	0.69	0.47

Trajectory Segmentation

	Walk		Car		Bus		Walk	
Road Segment ID	15	72	43	74	55	96	17	T1
	t1	t2	t3	t4	t5	t6	t7	tn

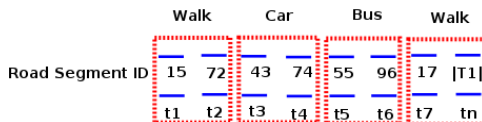
Trajectory Segmentation

	Walk		Car		Bus		Walk	
Road Segment ID	15	72	43	74	55	96	17	T1
	t1	t2	t3	t4	t5	t6	t7	tn

■ Segmentation model :

- Identify MASs on test trajectory against each of *Walk*, *Car*, *Bus* model
- Assign class labels to **MASs** edges to the **closest class** ; i.e the model it is **least anomalous against**

Trajectory Segmentation



■ Segmentation model :

- Identify MASs on test trajectory against each of *Walk*, *Car*, *Bus* model
- Assign class labels to **MASs** edges to the **closest class** ; i.e the model it is **least anomalous against**

■ F-score based on number of edges identified correctly

Class label	Walk	Bus
Car	0.80	0.65
Walk -		0.76

Conclusions

- Unique problem of mining **Maximal Anomalous Sub-trajectories**
- MANTRA refines the search space and identifies **islands** where all the MASs are present
- MANTRA is observed to be **3** orders of magnitude faster than baseline
- MANTRA conforms to human intuition of anomaly demonstrated through **classification** and **segmentation**
- MANTRA facilitates a **unique tool to classify segments of trajectories** based on vehicle type from their GPS traces