INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

Time-series/Sequential Models

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Outline

- Introduction
- Auto regressive (AR)
- Moving Average (MA)
- ARMA
- ARIMA
- Conclusion

- **Introduction**
- Auto regressive (AR)
- Moving Average (MA)
- ARMA
- ARIMA
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Introduction

- A time series is a sequence of observations taken sequentially in time.
- Exemplary time series:
  - A monthly sequence of the quantity of goods shipped from a factory.
  - Yield of a product.
  - Temperature variations.
  - Speech
  - Weather forecast
  - Stock variations
- Observed in economics, business, engineering, natural sciences, and social sciences

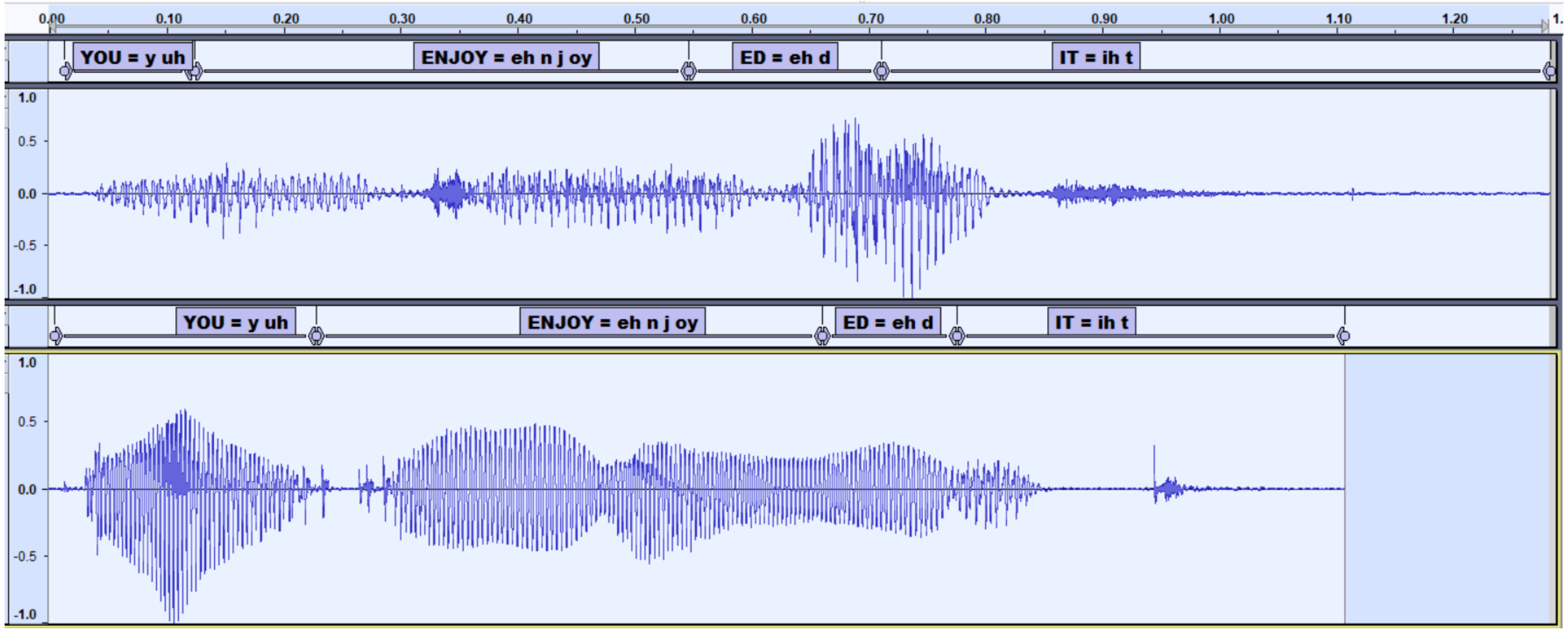
Introduction

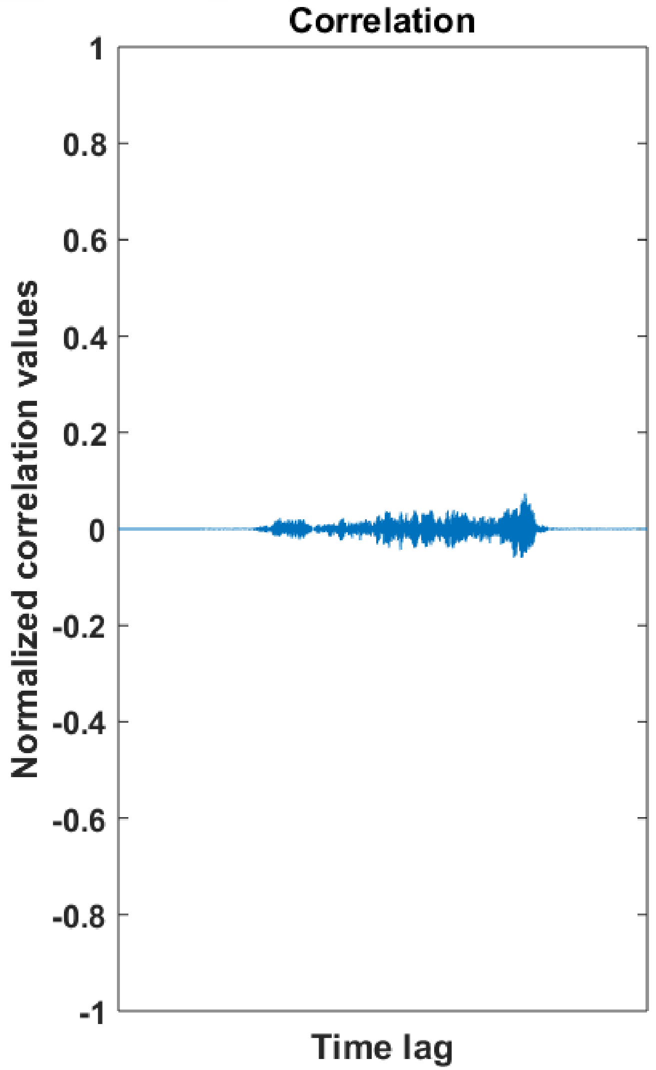
- In a time-series, adjacent observations are dependant.
- The interest is to analyse the nature of these dependencies.
- Time series analysis is concerned with the techniques that analyse these dependencies.
- Consider development of
  - Stochastic models
  - Dynamic models
- Use of these models in the respective area of applications.

Introduction

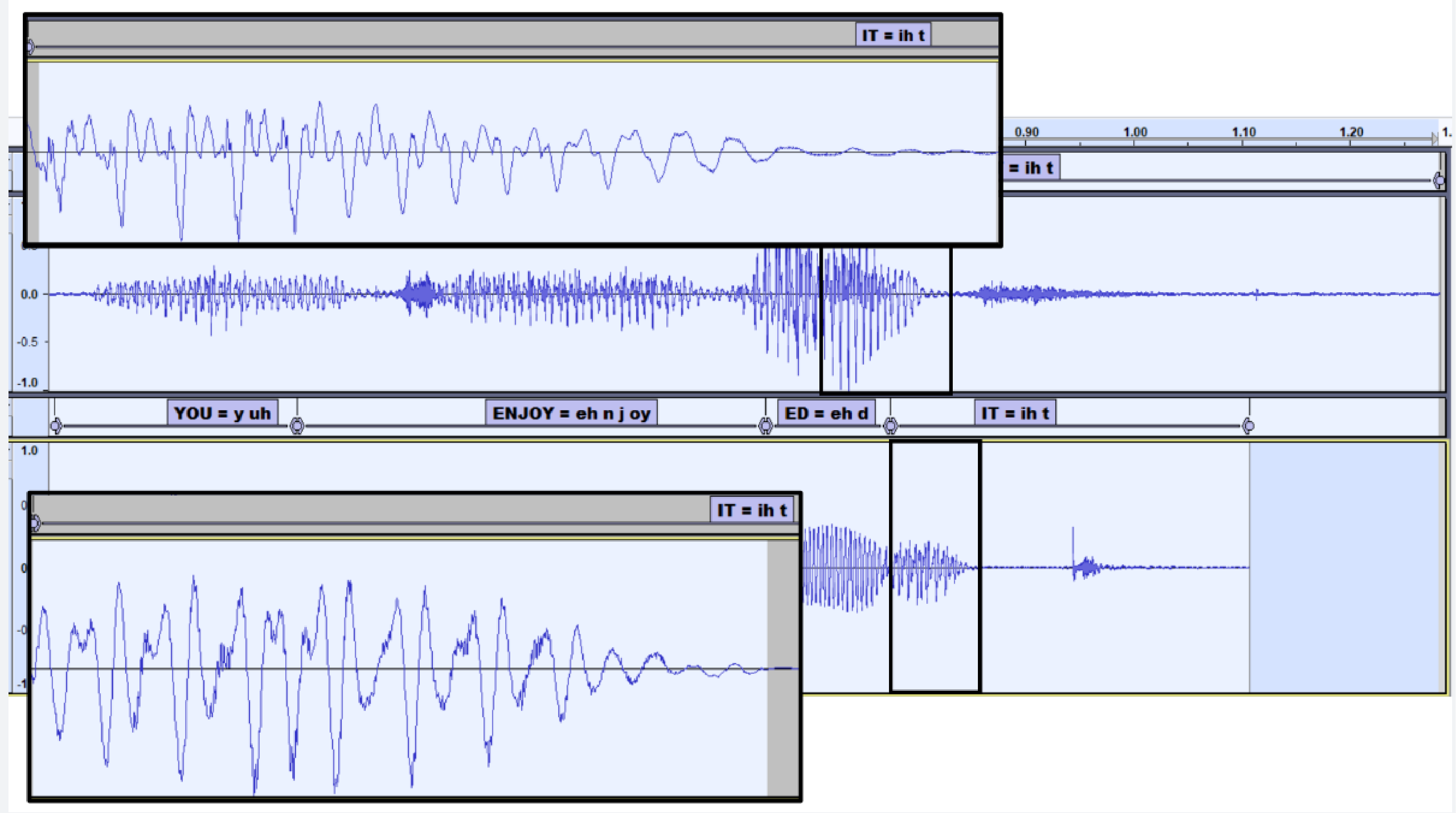
- Stochastic models are received great deal of attention in time-series analysis.
  - Stationary models.
  - Quasi-stationary models.
  - Non-stationary models.
- Stationary models
  - In statistical equilibrium: probabilistic properties do not change with time.
    - Strict sense stationary: distribution is same at two different points of time.
    - Wide sense stationary: constant mean and variance (till second order statistics)
- Many time series are non-stationary in nature.
  - Approximated and solve based on quasi-stationary models.

Introduction





Introduction



Introduction

- **Forecasting**: Predict the future values of the time-series from current and past values.
- **Determination of transfer function**: Identify the dynamic input-output model that can show the effect on the output of a system of any given series of inputs.
- **Analysis of unusual intervention** of events on the behaviour of a time-series.
- **Multivariate analysis**: Examination of interrelationships among several related time series variables

Introduction

- Linear models
  - Auto regressive (AR)
  - Moving average (MA)
  - Auto regressive and moving average (ARMA)
  - Auto regressive integrated moving average (ARIMA)
- Non-linear models
  - RNNs/LSTMs/GRUs
  - Encoder-Decoder

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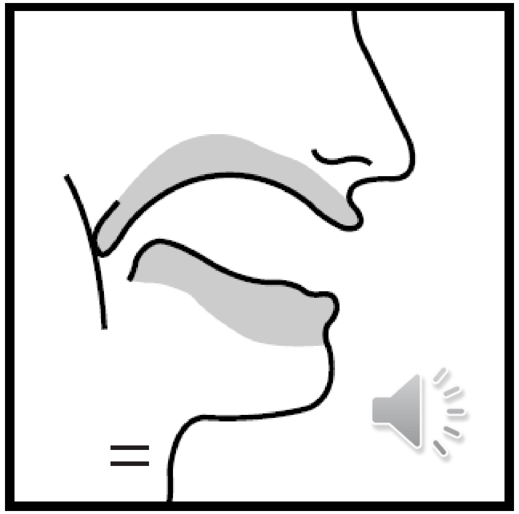
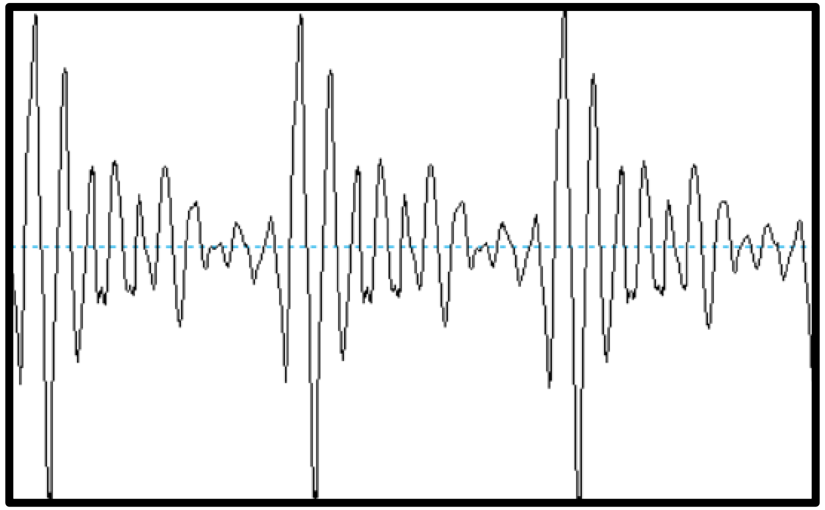
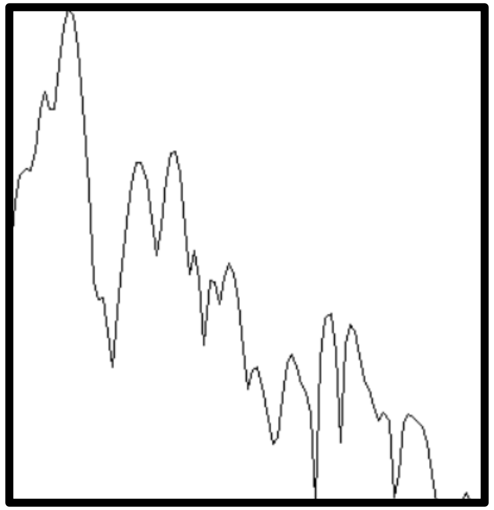
Auto regressive

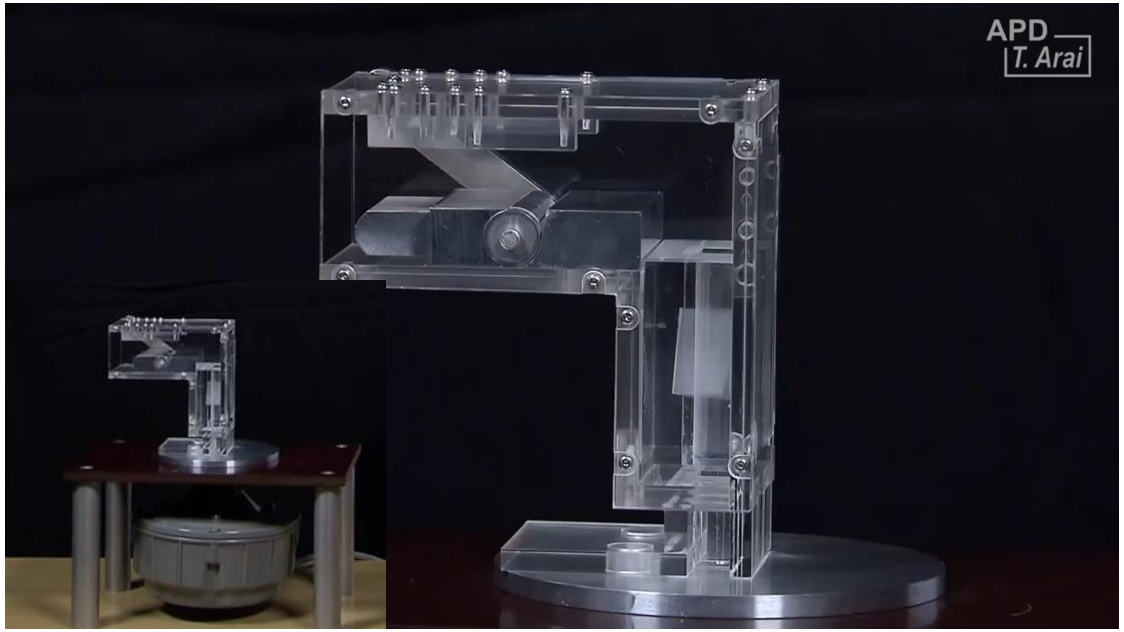
- Current time values depend on the present and the past values.
  - Infinite sum of input values.
  - Finite sum of past output values and current input value.
- Most cases the input values are assumed to be a random noise.

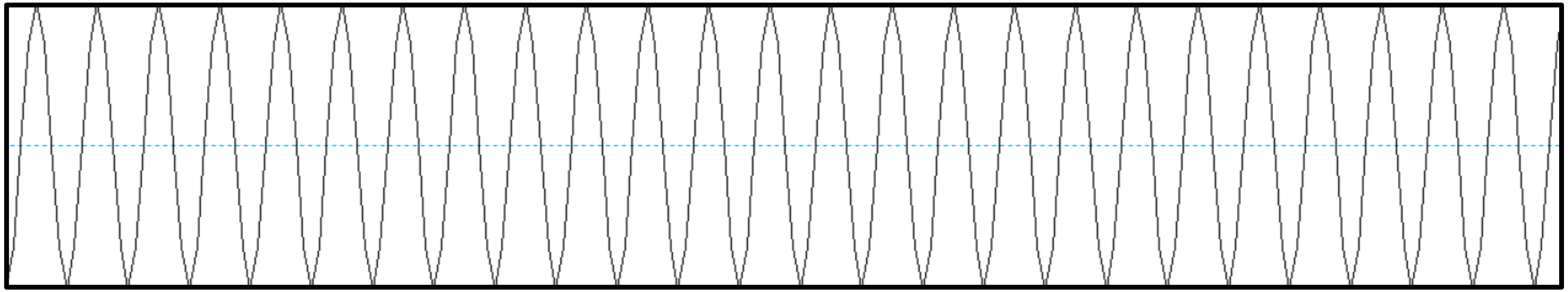




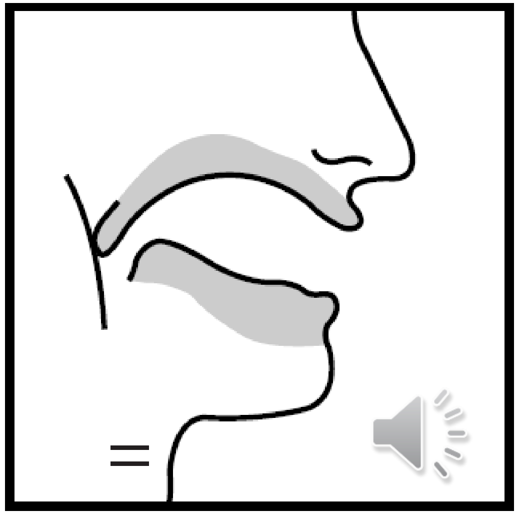


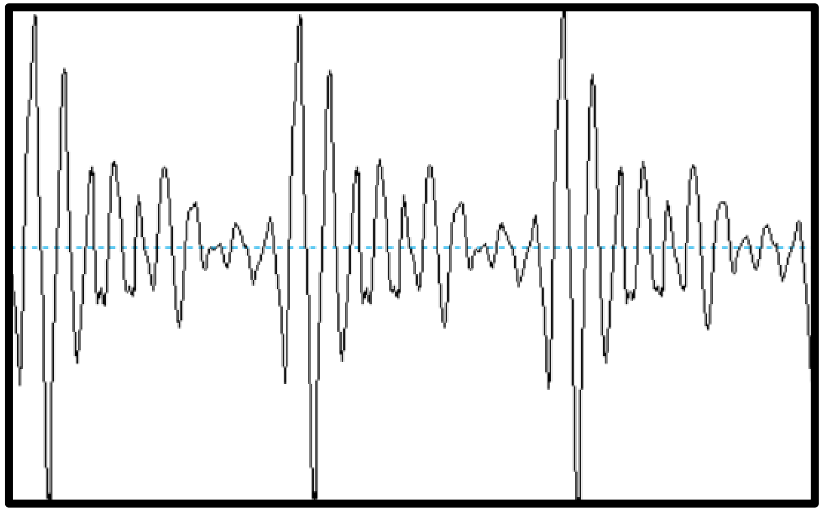
Auto regressive (contd..)

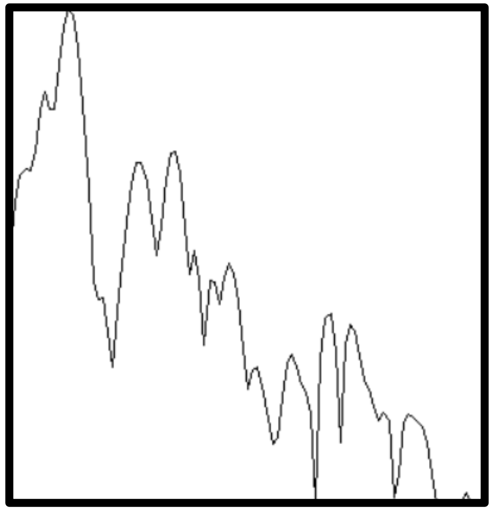




Auto regressive (contd..)





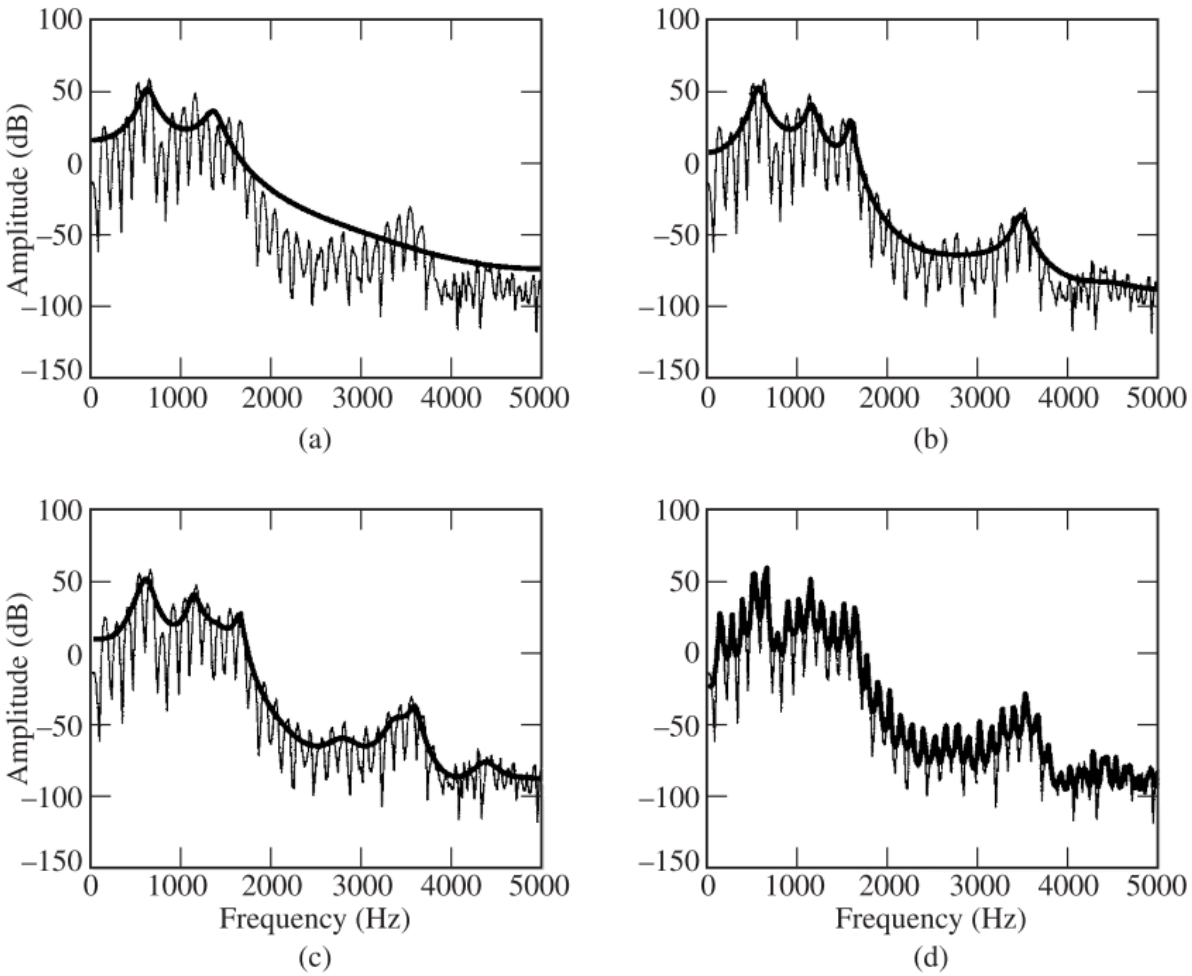


- The oscillatory behaviour.
  - 2nd order approximation: $y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 \delta(n)$
  - In general: $y(n) = \sum_{k=1}^{p} a_k y(n-k) + b_0 x(n)$
- Mostly $x(n)$ has autocorrelation of $\delta(l)$, where l is time lag.
  - Example: $\delta(n)$, random white noise

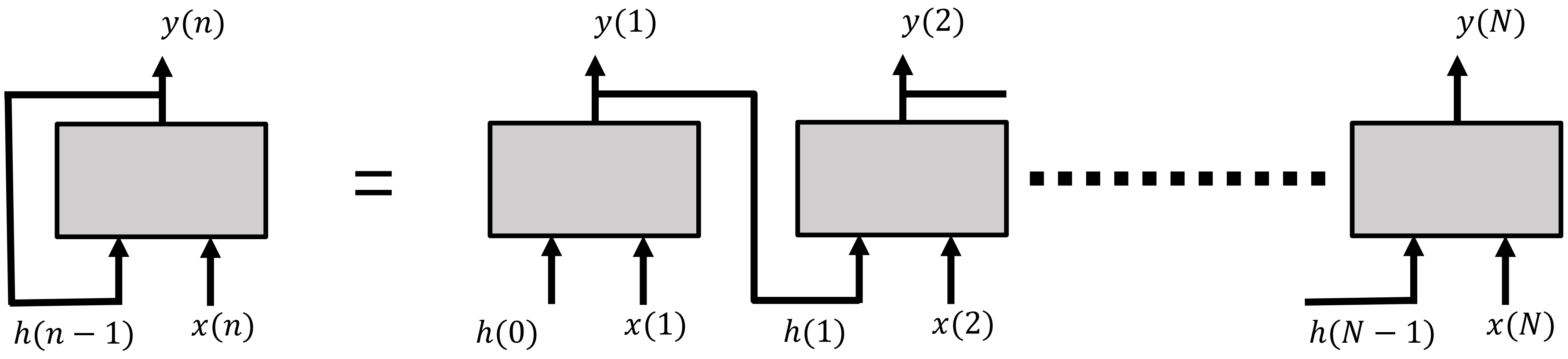
Auto regressive (contd..)

- The AR formulation:
  - $y(n) = \sum_{k=1}^{p} a_k y(n-k) + b_0 x(n)$, where $x(n)$ is white noise and $a_k < 0$.
- Solution:
  - $\rho_l = \sum_{k=1}^{p} a_k \rho_{l-k}$, where $\rho_{l-k} = E(y(n-l)y(n-k))$
  - Yule-Walker equations:
    - Substitute $l = 1, 2, 3, \dots, p$
    - Results p equations.
    - Solving these p equations all a_k values are obtained.

Auto regressive (contd..)

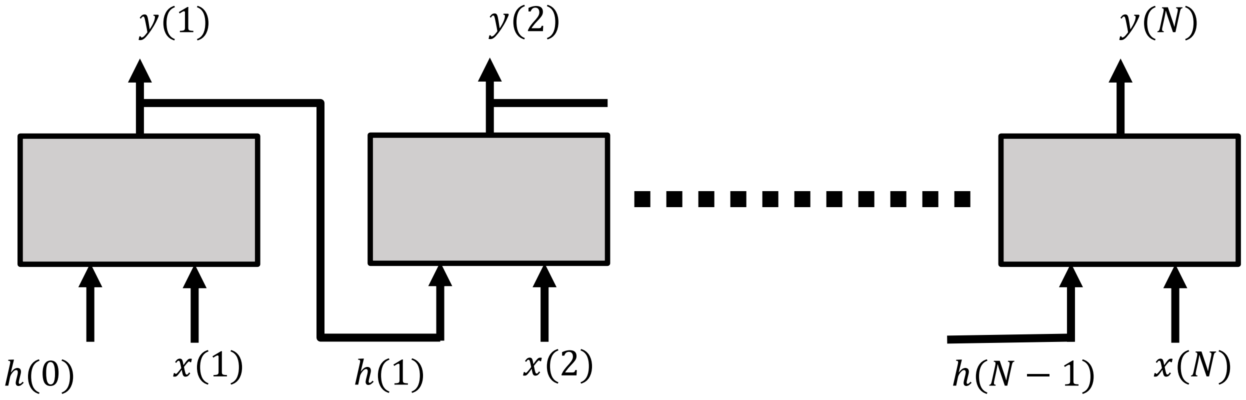


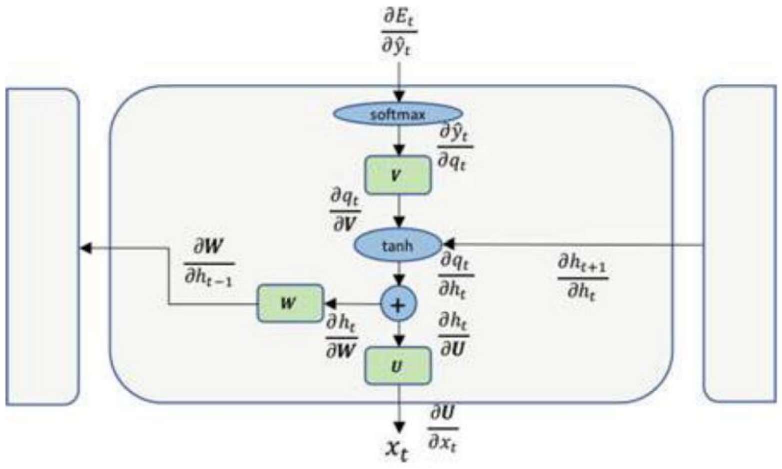
Recurrent Neural Networks (RNNs)



- Relation: $y(n) = h(n) = f(x(n), h(n-1))$
- Preciously: $y(n) = h(n) = f(Ux(n) + Wh(n-1))$
- First order recursive (Markov) process, but non-linear.

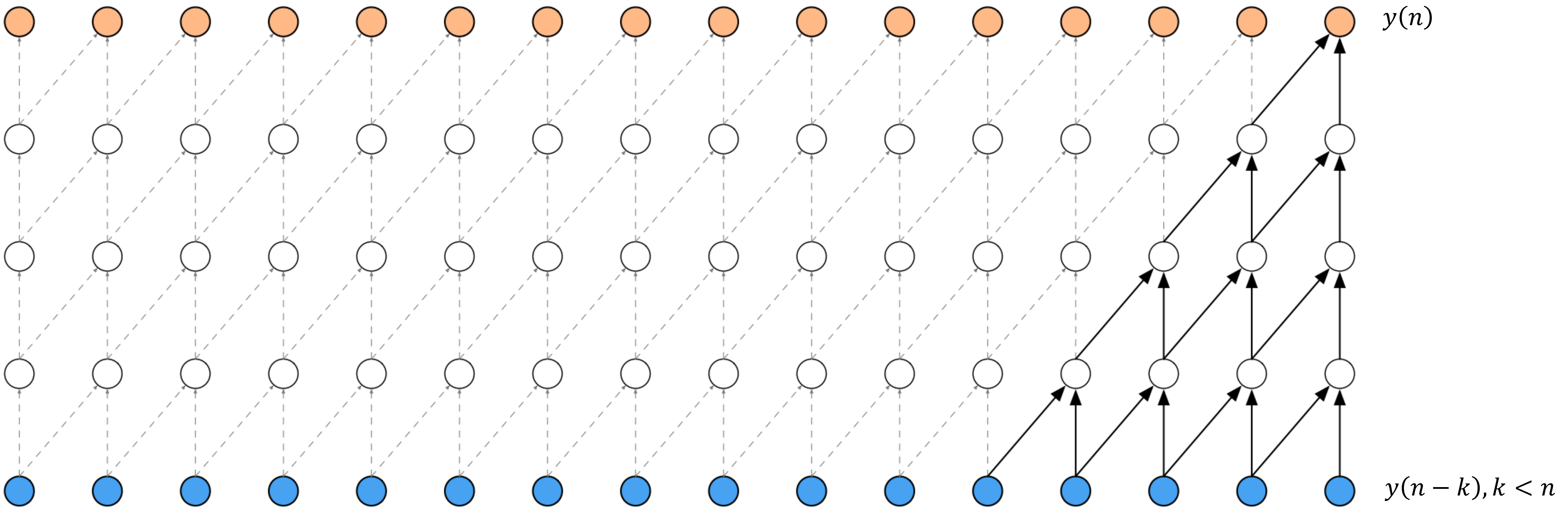
RNN variants





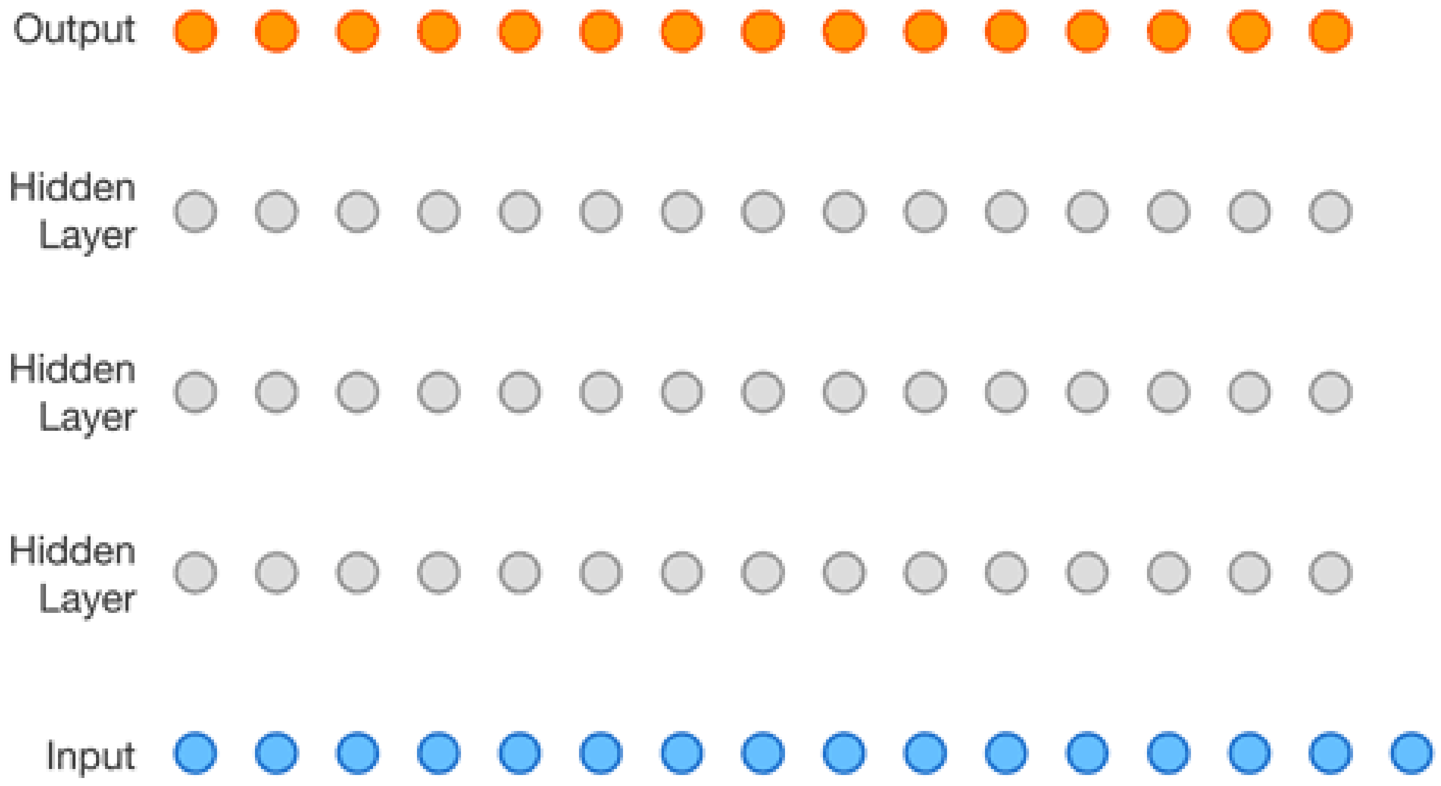
- Vanishing/exploding gradient problems.
- Overcome by
  - Long Short term memory (LSTM) networks, Gated recurrent unit (GRU)
  - Gradient clipping

Wavenet



- $y(n) = f(y(n-1) \ldots y(n-k), x(n))$, obtained by maximizing $p(\boldsymbol{y}) = \prod_{k=1}^{N} p(y(k)|y(1) \ldots y(k-1), x(k))$

Wavenet



- $y(n) = f(y(n-1) \dots y(n-k), x(n))$, obtained by maximizing $p(\mathbf{y}) = \prod_{k=1}^{N} p(y(k)|y(1) \dots y(k-1), x(k))$

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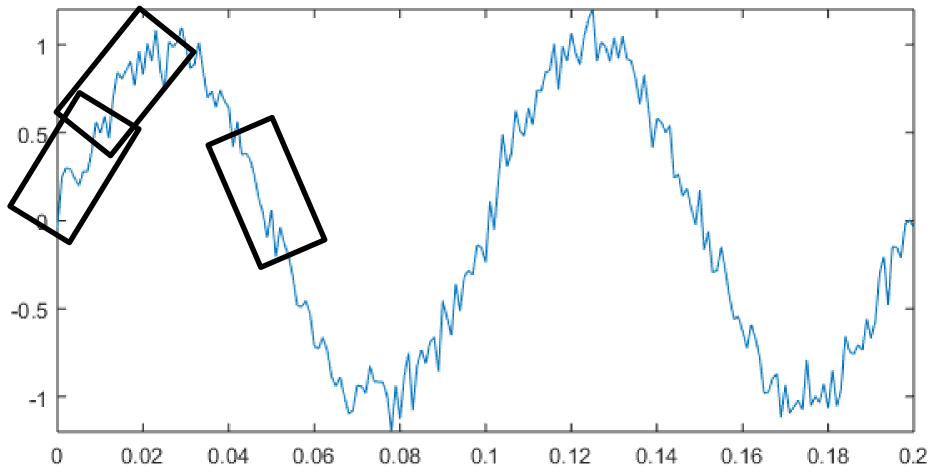
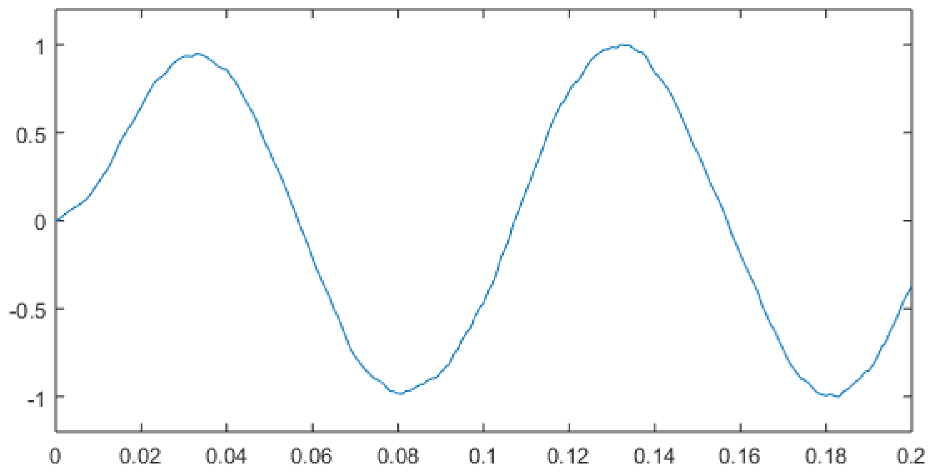
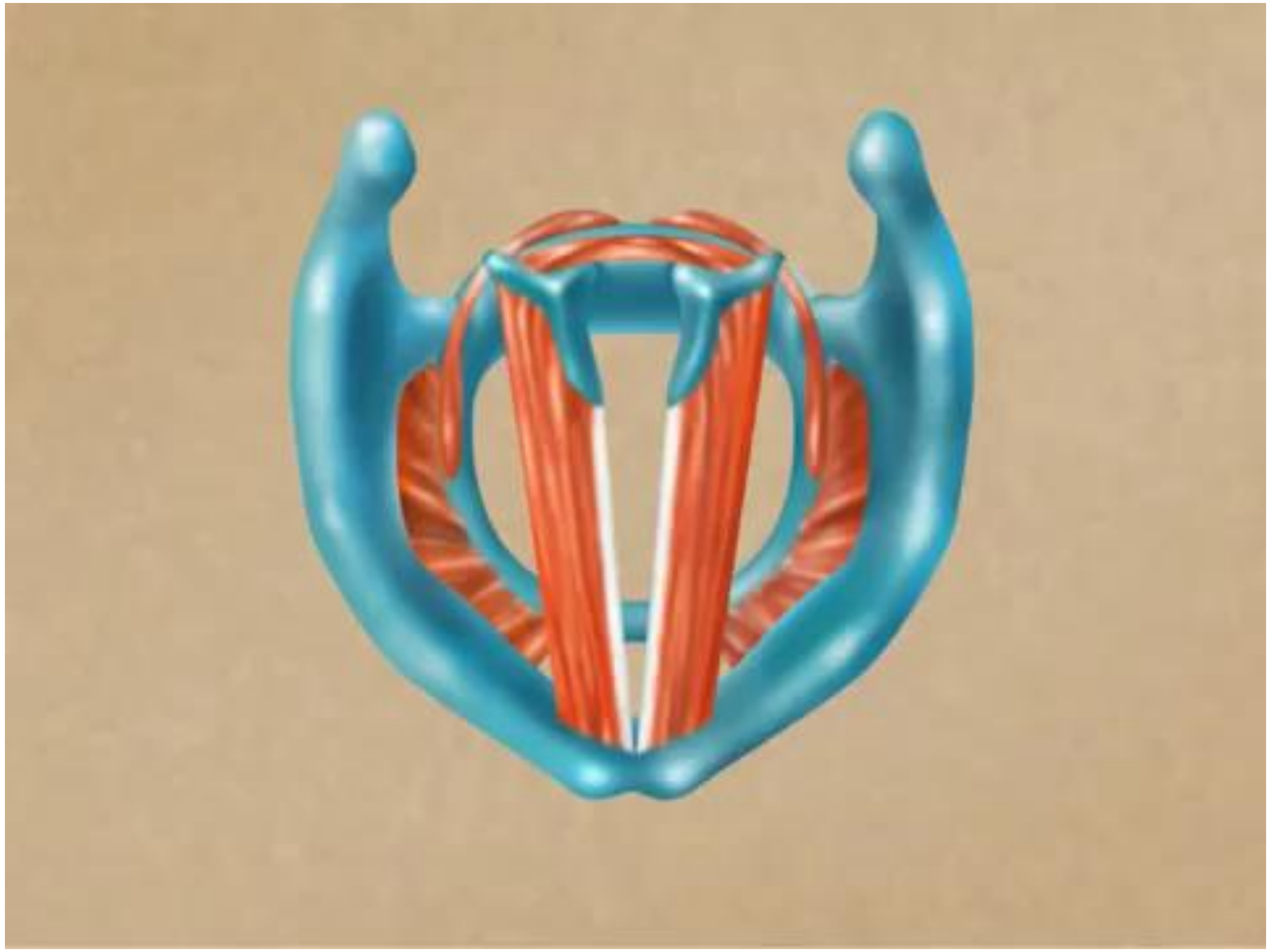
Moving average

- Current time output depends on the current and few past input values.
  - Finite sum of input values.
  - No delayed output is involved.





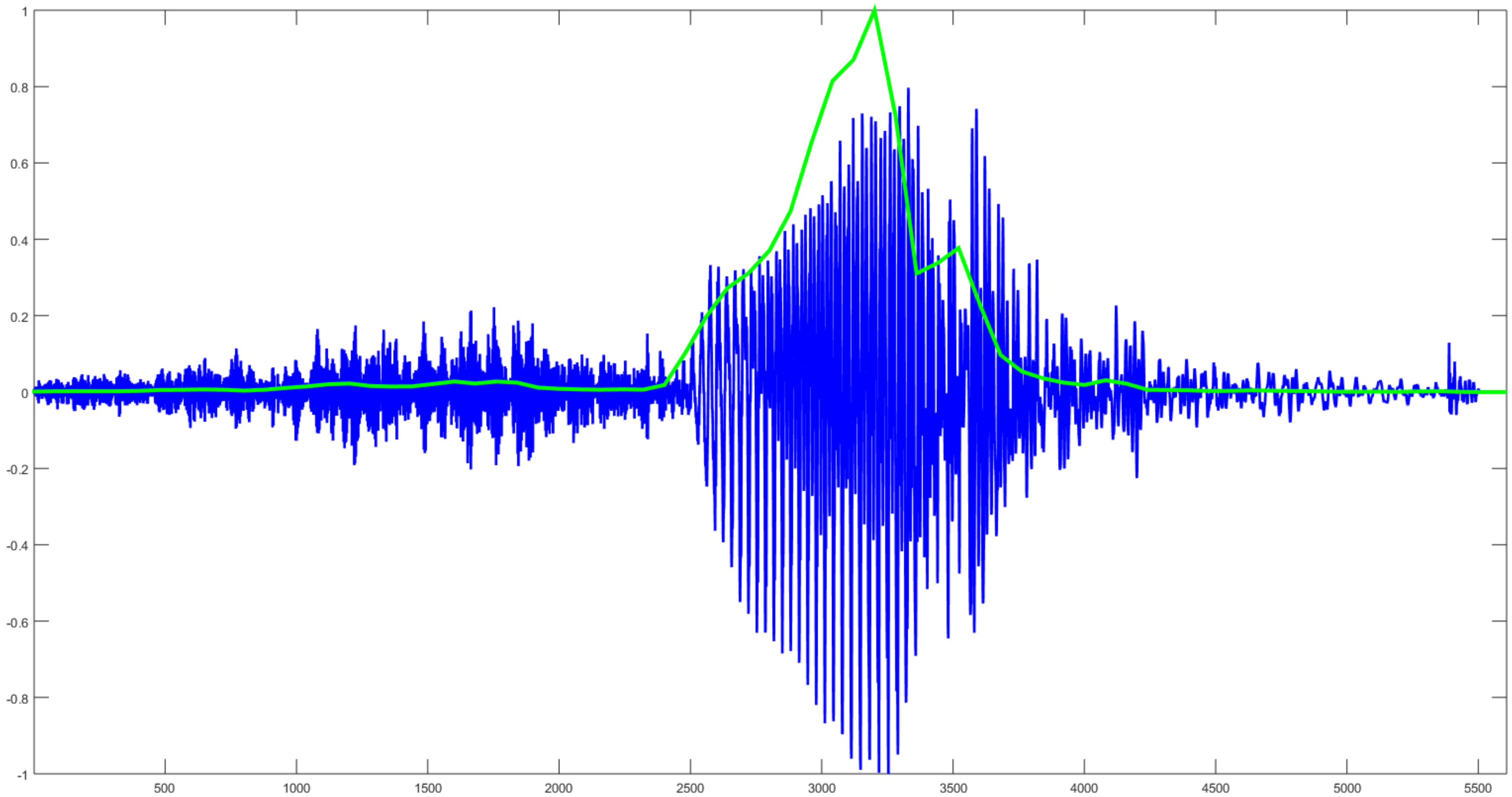
Moving average (contd..)



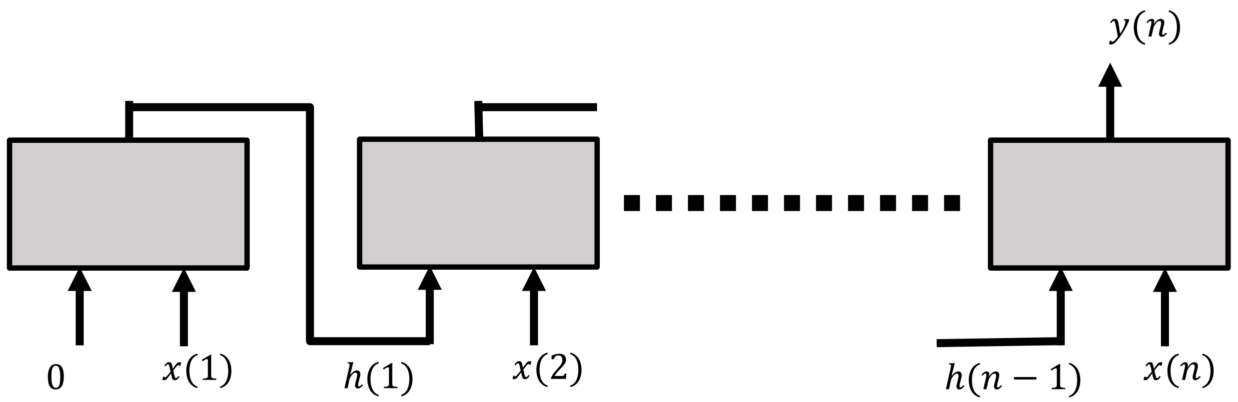
Moving average (contd..)

- The MA formulation:
  - $y(n) = \sum_{k=0}^{q} b_k x(n-k)$, where $x(n)$ is random noise.
- Solution:
  - $\rho_l = \sum_{k=0}^{q} b_k b_{k-l}, l = 1,2, \dots, q$, where $\rho_l = E(y(n-l)y(n))$
  - Number of unknowns and number of equations are the same, however, the equations are non-linear
  - Recursive solution exists.

Moving average (contd..)

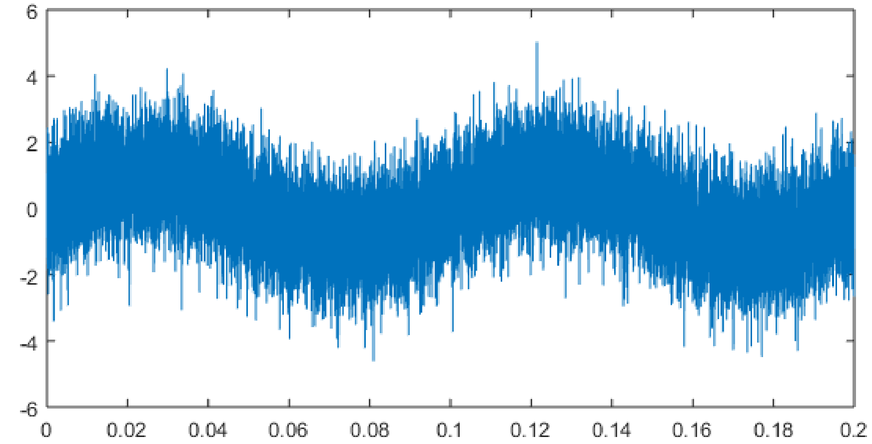


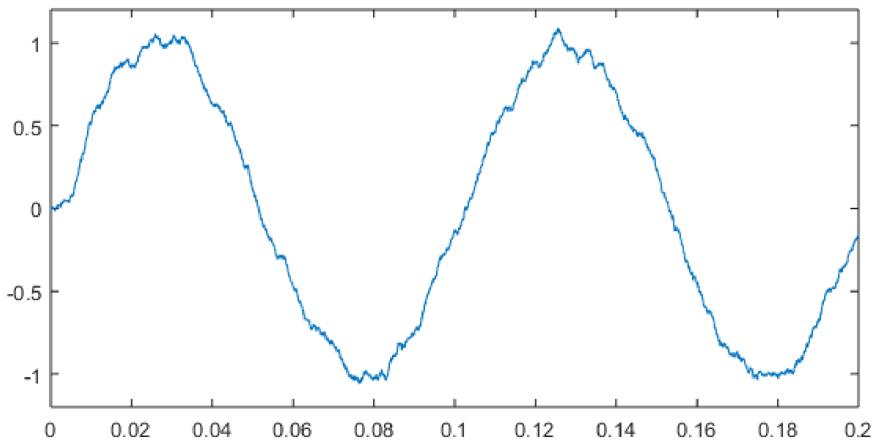
Using RNN for MA

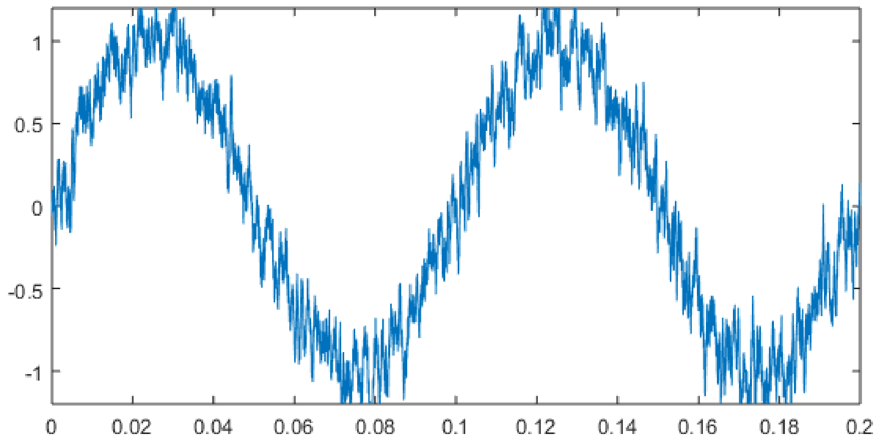


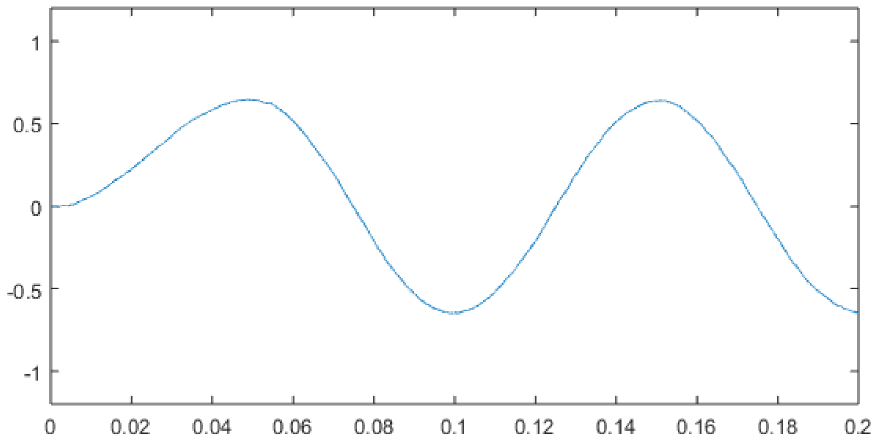
- Relation: $y(n) = f(x(n), x(n-1), \ldots, x(1))$
- Preciously: $y(n) = f(Ux(n) + Wh(n-1))$

Moving average (contd..)









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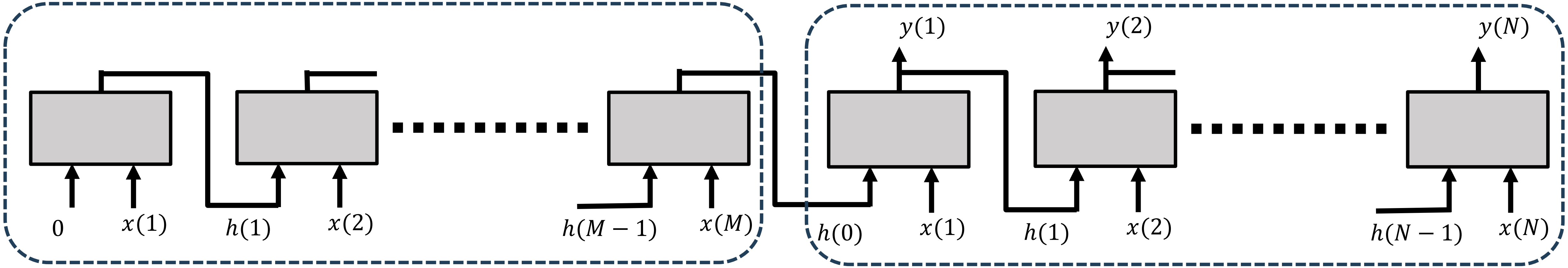
ARMA

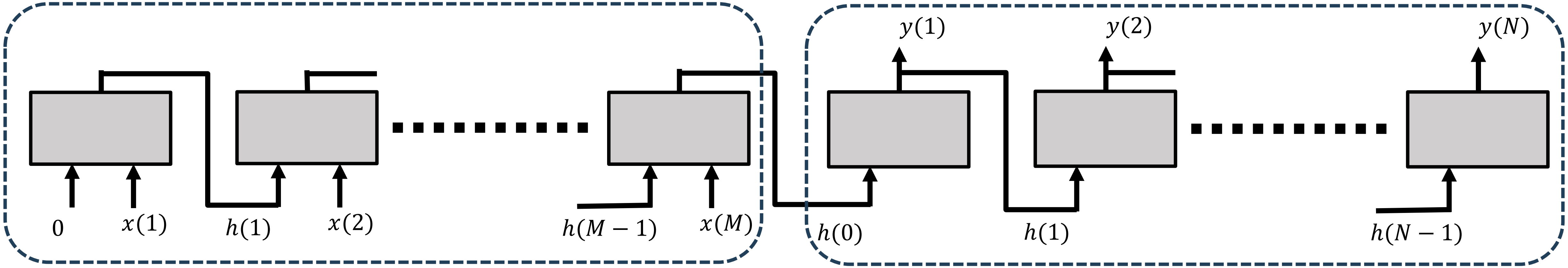
- Current time values depend on the present and the past values.
  - Finite sum of input values.
  - Finite sum of past output values and current input value.
- Most cases the input values are assumed to be a random noise.
- $y(n) = \sum_{k=1}^{p} a_k y(n-k) + \sum_{k=0}^{q} b_k \, x(n),$
  - where $x(n)$ is white noise and $a_k < 0$.
- Solution for this problem is based on Extended Yule-Walker equations, which is combination of AR and MA solution.

Using RNN for ARMA



- Relation: $y(n) = f\big(y(n-1), g\big(x(M), x(M-1), \ldots, x(1)\big), x(n)\big)$

Other RNN based models



- Relation: $y(n) = f(y(n-1), g(x(M), x(M-1), \dots, x(1)))$
- Maps variable length input sequence to variable length output sequence
- Known as Encoder-Decoder sequence model

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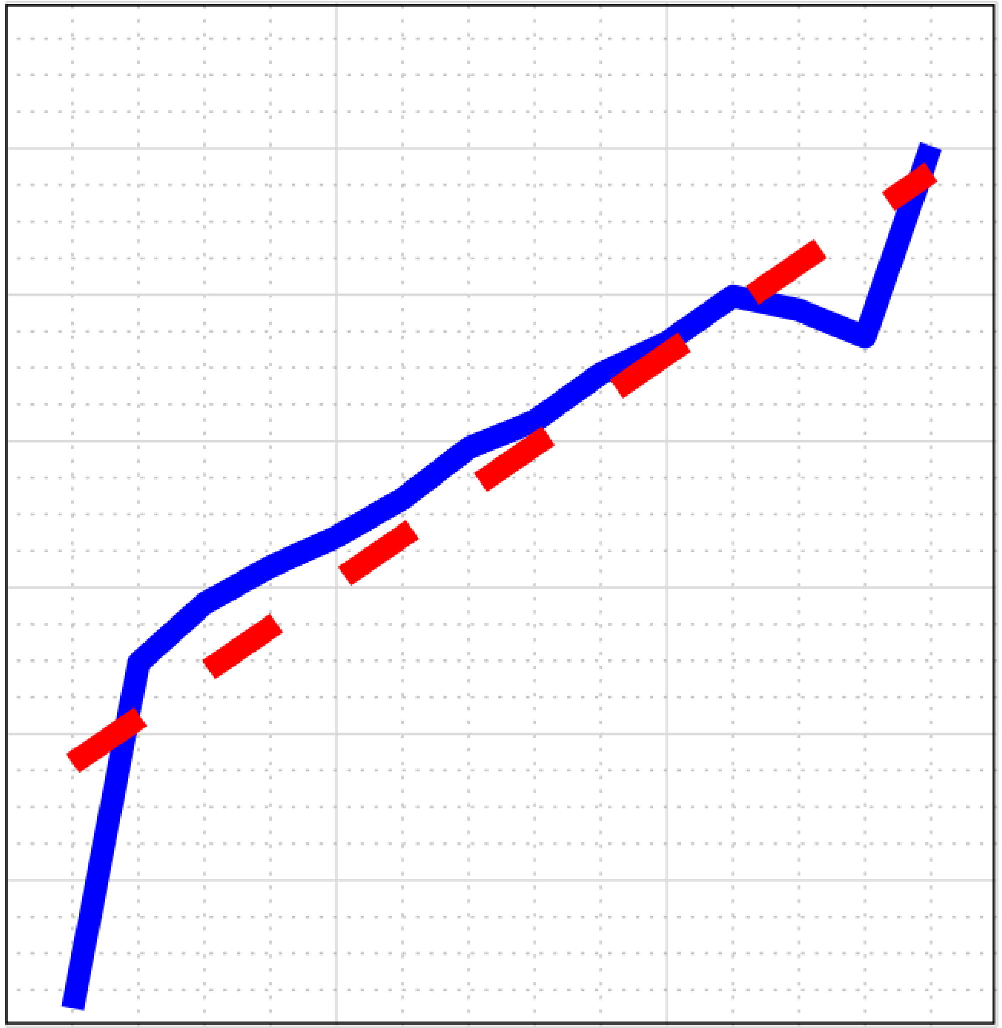
ARIMA



- $y(n) = mn + c;$
- $y(n-1) = m(n-1) + c$
- $y(n) - y(n-1) = m$
- $y(n+1) = y(n) + m$
- $y(n+1) - y(n) = y(n) - y(n-1)$
- $\Delta y(n+1) = \Delta y(n) + x(n)$
- In general
  - $\Delta y(n+1) = a_0 \Delta y(n) + b_0 x(n)$

ARIMA

- Current time values depend on the present and the past values.
  - Finite sum of input values.
  - Finite sum of past and present output differences.
- Most cases the input values are assumed to be a random noise.
- $\Delta y(n) = \sum_{k=1}^{p} a_k \Delta y(n-k) + \sum_{k=0}^{q} b_k\, x(n)$,
  - where $x(n)$ is white noise; $\Delta y(n) = y(n) - y(n-1)$ and $a_k < 0$.

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Conclusion

- Time series analysis is concerned with the techniques that analyse the dependencies in the time series.
- Non-stationary process can be solved by approximating it as quasi stationary process.
- Recursive relation among the time-series is obtained with AR for linear modelling and RNNs for non-linear modelling.
- Linear trend can be captured with ARIMA
- Encoder-decoder models can represent the ARMA and ARIMA process.

References

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Thank you