MA1002 — STATISTICAL AVERAGES

UNIT 2

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CONTENTS

For more visit `https://github.com/pranaovs/college-notes`.

1 EXPECTATION

Expectation talks about the long term.

1.1 *Mathematical Expectation*

If X is a random variable which can assume any one of the values $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_1, p_2, p_3, \ldots, p_n$, then the expectation of X is defined as:

$$\mathrm{E}(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n = \sum x_ip_i$$

If X is a continuous random variable with probability density function $f(x)$, then the expectation of X is defined as:

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} xf(x)dx$$

1.2 *Properties*

1. $\mathrm{E}(c) = c$ where c is a constant
2. $\mathrm{E}(cX) = c\mathrm{E}(X)$ where c is a constant
3. $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ where a and b are constants
4. $\mathrm{E}(X + Y) = \mathrm{E}(X) + \mathrm{E}(Y)$
5. $\mathrm{E}(XY) = \mathrm{E}(X) \cdot \mathrm{E}(Y)$, if X and Y are **independent** random variables

Note

- For a random variable, expectation is the average value of the variable.
- $\mathrm{E}(X^2) = \sum x_i^2 p_i$ or $\int_{-\infty}^{\infty} x^2 f(x)dx$

2 VARIANCE

Variance tells us how much the values of a random variable differ from the mean.

$$Var(x) = \sigma^2 = \mathrm{E}[X - \mathrm{E}(X)]^2 = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$$

$$= \sum x_i^2 p_i - [\sum x_ip_i]^2$$

$$\sigma = \sqrt{Var(X)}$$

2.1 *Properties of Variance*

1. $Var(c) = 0$ where c is a constant
2. $Var(X + c) = Var(X)$ where c is a constant
3. $Var(aX) = a^2 Var(X)$ where a is a constant
4. $Var(aX + b) = a^2 Var(X)$ where a and b are constants

Note

- $Var(X)$ is always non-negative.
- Standard deviation is practically more useful than variance.

2.2 *Moments*

Moments of a random variable are the expected values of powers of the random variable.

2.2.1 Moments about the mean

Mean is the first moment of a random variable.

$$\mu = E(X) = \sum x_i p_i$$

2.2.2 Moments about any point a

The r^{th} moment of a random variable about any point a is defined as:

$$\mu = E[(X - a)^r] = \sum (x_i - a)^r p_i$$

2.2.3 Moments about the origin

The r^{th} moment of a random variable about the origin is defined as:

$$\mu = E(X^r) = \sum x_i^r p_i$$

2.3 *Relation between moments*

- $\mu_2 = \mu'_2 - \mu'^2_1$
- $\mu_3 = \mu'_3 - 3\mu_1\mu'_2 + 2\mu^3_1$
- $\mu_4 = \mu'_4 - 4\mu_1\mu'_3 + 6\mu^2_1\mu'_2 - 3\mu^4_1$

where $\mu_1 = E(X)$, $\mu_2 = E(X^2)$, $\mu_3 = E(X^3)$, $\mu_4 = E(X^4)$ and μ'_1, μ'_2, μ'_3, μ'_4 are the moments about the origin.

Moment	**Discrete**	**Continuous**
About the mean	$E[(X-\mu)^r] = \sum(x_i - \mu)^r p_i$	$E[(X-\mu)^r] = \int_{-\infty}^{\infty} (x-\mu)^r f(x)dx$
About any point a	$E[(X-a)^r] = \sum(x_i - a)^r p_i$	$E[(X-a)^r] = \int_{-\infty}^{\infty} (x-a)^r f(x)dx$
About the origin	$E(X^r) = \sum x_i^r p_i$	$E(X^r) = \int_{-\infty}^{\infty} x^r f(x)dx$

Table 1: Moments of Discrete and Continuous Random Variables

3 SKEWNESS

Skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean.

Coefficient of skewness is defined as:

$$\gamma_1 = \frac{\mu_3}{\sigma^3}$$

where μ_3 is the third moment about the mean and σ is the standard deviation.

$\mu_3 = E[(X-\mu)^3] = \sum(x_i - \mu)^3 p_i$

4 KURTOSIS

Kurtosis is a measure of the "tailedness" of the probability distribution of a real-valued random variable.

Coefficient of kurtosis is defined as:

$$\gamma_2 = \frac{\mu_4}{\sigma^4}$$

where μ_4 is the fourth moment about the mean and σ is the standard deviation. σ can be calculated as $\sqrt{Var(X)}$.