

Ques1 $O(n^2)$ for $n=10 \rightarrow 5$ sec

$$100 \rightarrow 5$$
$$2500 \rightarrow \frac{5}{100} \times 2500 = 125 \text{ sec}$$

Ques2

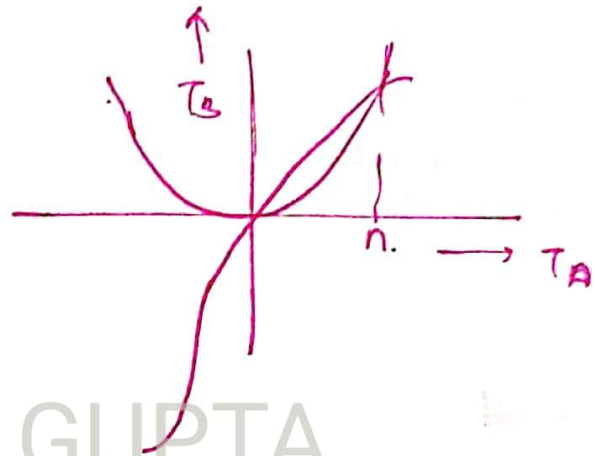
$$T_A(n) = n^3$$

$$T_B(n) = 2n^2$$

$$n^3 - 2n^2 = 0$$

$$n^2(n-2) = 0$$

$$\boxed{n=2}$$



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Ques3

$$f(n) = n2^n \quad O(4^n)$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{n2^n}{4^n} = n\left(\frac{1}{2}\right)^n$$

$$\lim_{n \rightarrow \infty} \frac{n}{2^n} \Rightarrow \lim_{n \rightarrow \infty} = 0$$

$$O(f(n)) < O(g(n))$$

so, the $n2^n$ is $O(4^n)$.

Ques4

$$O(\log_n n)$$

An algorithm's running time depends on its input-sizes

$$\text{ex, } \log_3 27 \text{ is } 3$$

logarithmic function gets slightly slower as n grows where n doubles the running time covered by constant

Ques 5

(a) Worst case (O) is the funcⁿ which performs the maximum numbers of steps on input data of size n .

ex - If we want to count an array of integers in ascending order then the worst case would be that array is already sorted in descending order.

Average case (Θ) is used when we refer to average no. of input to test the algorithm. It is the average resources used taking into all possible inputs.

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(b) Big-oh notation is an asymptotic notation for the asymptotic upper bound for the growth rate of the running of an algorithm. (O)

Ω - Asymptotic notation for best case. It provides us with an asymptotic lower bound for the growth rate of the running algorithm.

$$\textcircled{6} \quad n^4 + \log(n) + 17$$

growth rate of constant—

$$O(n^4) > O(\log n) > O(1)$$

$$\text{So, } n^4 + \log n$$

$$= n^4 O(n^4)$$

As we consider the input size & increase it the growth rate of logarithm function is lesser than the polynomial function.

$$\textcircled{7} \quad k=1 \longrightarrow O(1)$$

$$\text{while } (k \leq n) \longrightarrow O(n)$$

$$\{ \quad k = k+1; \longrightarrow O(n)$$

}

$$\boxed{N=5}$$

$$\text{for } (i=1 \text{ to } n-1) \longrightarrow O(n)$$

{

$$\text{for } (j=i+1 \text{ to } n) \longrightarrow O(n-1)$$

{

$$\text{swap} \longrightarrow O(n-1)$$

}

}

}

$$\text{Time complexity} \Rightarrow O(n) + O(n-1) + O(n-1) \\ = O(n)$$

$$\textcircled{8} \quad f(n) = n^2 + 2n + 4$$

$$O(n^2) > O(n) > O(1)$$

$$f(n) = O(n^2)$$

As increase the input size double then the running time will become larger



As the graph shows as the input increases time also increases.

⑨

$$T_A \approx \log n$$

$$T_B \approx n$$

$$\lim_{n \rightarrow \infty} \frac{T_A}{T_B} = \frac{\log n}{n^4}$$

$$T_B(n) < T_A(n)$$

$$n^4 < \log n$$

$$\lim_{n \rightarrow \infty} \frac{\log n}{n^4} = 0$$

thus, $f(n)$ is in $O(g(n))$ thus n^4 is a upper bound of $(100)^n$.

as $n \rightarrow \infty$, T_A will grow faster than T_B function.

⑩

$$n \log n \in O(\log n!)$$

as n increases

$$\log(n!) = \log(1) + \log 2 + \dots + \log(n-1) + \log(n)$$

$$n \log n \rightarrow \log 1 + 2 \log 2 + \dots + n \log n$$

⑪

$$2^{n+1} + 4^{n+1}$$

$$T_A = n^2$$

$$T_B = n+2$$

$$n^2 = n+2$$

$$n^2 - n - 2 = 0$$

$$\boxed{n=2} \rightarrow \text{breaking point}$$

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14

total = 0 $\rightarrow O(1)$

for (i = 0 to n-1) $\rightarrow O(n-1)$

{

sum = sum + a[i] $\rightarrow C(O(n))$

return sum $\rightarrow C(O(n))$

}

$$T_{\text{sum}} = O(1) + O(n-1) + C(O(n-1)) + \dots$$
$$\rightarrow O(n)$$

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15

int count = 0; $\rightarrow O(1)$

for (i = 0; i < N; i++) $\rightarrow O(N)$

{

for (j = 0; j < i; j++)

{

count++ $\rightarrow C(O(n))$

}