Hubble Constant with the Pantheon+SH0ES Supernova Sample

Pranauv C.H

Abstract

We present a cosmological analysis of the Pantheon+SH0ES Type Ia supernova dataset within the flat ΛCDM framework. Using numerical integration of the luminosity distance and χ^2 minimization, we obtain best-fit parameters of

$$H_0 = 72.97 \pm 0.26 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \quad \Omega_m = 0.351 \pm 0.019.$$

These values are consistent with local Cepheid-calibrated measurements from the SH0ES program but remain in significant tension with the Planck 2018 cosmic microwave background determination of $H_0 = 67.4 \pm 0.5 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The resulting age of the Universe is estimated as $t_0 \approx 12.36$ Gyr, notably lower than the 13.8 Gyr inferred from CMB-based ΛCDM cosmology. Subsample tests (low-z vs. high-z supernovae) and fixed-parameter fits demonstrate the robustness of the results across redshift ranges and cosmological priors. These findings reinforce the persistence of the Hubble tension and highlight either residual systematics or physics beyond the standard model.

Contents

1 Introduction

The discovery of the relation between galaxy redshifts and their distances by Edwin Hubble in 1929 established the expanding–Universe paradigm and laid the foundation for modern observational cosmology. Subsequent decades have been dedicated to refining the measurement of the Hubble constant (H_0), which quantifies the present–day expansion rate of the Universe.

Type Ia supernovae (SNe Ia) have become essential probes in this effort. Their remarkable uniformity in peak luminosities, combined with empirical corrections, makes them powerful standardizable candles for measuring extragalactic distances. Large–scale compilations such as the Pantheon+ sample have provided unprecedented statistical precision, enabling cosmological analyses that rival those of other probes. The Pantheon+SH0ES dataset, in particular, combines high–redshift supernovae with nearby Cepheid–calibrated SNe Ia to anchor the cosmic distance ladder.

In the framework of the ΛCDM model, the Universe is described as spatially flat, dominated by cold dark matter (Ω_m) and a cosmological constant (Ω_Λ). Within this model, the Hubble parameter evolves with redshift according to the Friedmann equation, allowing theoretical predictions of distance measures that can be directly compared with supernova observations.

A key focus of contemporary cosmology is the so–called "Hubble tension"—the discrepancy between early–Universe inferences of H_0 from cosmic microwave background measurements (Planck 2018) and late–Universe direct determinations from Cepheid–calibrated SNe Ia (SH0ES). While Planck data favors a value near $67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, SH0ES measurements indicate a significantly higher value around $73\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Resolving this discrepancy remains one of the most pressing challenges in cosmology.

The objective of this study is to analyze the Pantheon+SH0ES dataset under the flat ΛCDM framework. By fitting theoretical distance moduli to observed supernova data, we aim to obtain best–fit values of H_0 and Ω_m, estimate the corresponding age of the Universe, and assess the results in the context of the Hubble tension.

2 Data Selection & Preprocessing

2.1 Data Source

The data used in this study are drawn from the Pantheon+SH0ES compilation of Type Ia supernovae, one of the most comprehensive SN Ia datasets currently available. It includes more than 1,500 supernovae across a redshift range extending from the nearby Universe ($z \sim 0.01$) to high redshift ($z \sim 2.3$). The SH0ES component provides an absolute calibration through Cepheid–variable distance measurements in nearby galaxies hosting SNe Ia. The dataset contains observed redshifts, measured distance moduli, and associated uncertainties, which together provide the basis for cosmological parameter estimation.

2.2 Data Cleaning and Preparation

The raw dataset was inspected for missing or invalid entries, and non–physical values were excluded from further analysis. Each supernova entry provides a redshift z, an observed distance modulus μ_{obs}, and an uncertainty σ_μ. To ensure reliability, only supernovae with well–defined uncertainties were retained. The resulting dataset forms a robust basis for χ^2 minimization against theoretical cosmological predictions.

2.3 Dataset Overview

The Pantheon+SH0ES sample spans a wide redshift range, enabling both low–redshift calibration and high–redshift cosmological testing. The inclusion of SH0ES calibration ensures sensitivity to the present–day expansion rate (H_0), while the broad redshift coverage constrains the evolution of the Hubble parameter through the matter density parameter (Ω_m).

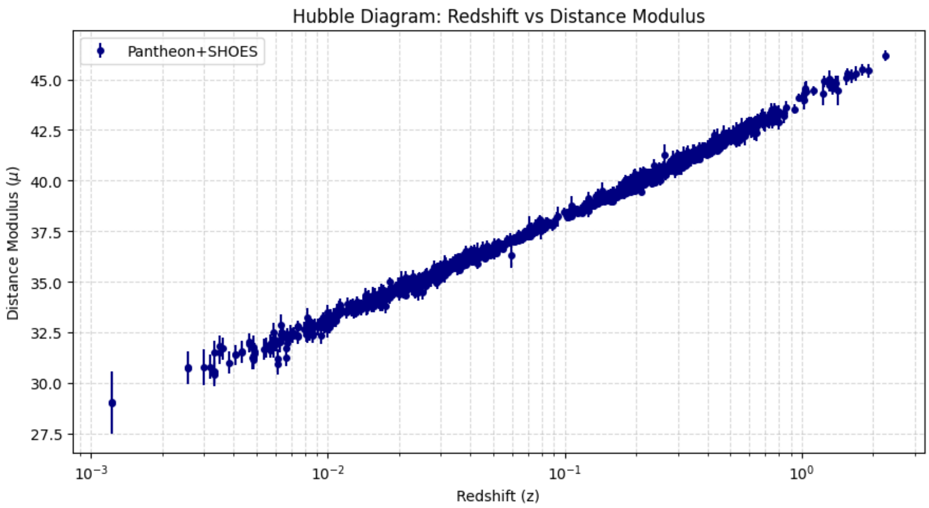

Figure 1: Hubble diagram of Pantheon+SH0ES supernovae showing distance modulus μ versus redshift z. The monotonic rise reflects cosmic expansion and illustrates the wide redshift coverage of the sample.

3 Methodology

3.1 Theoretical Framework

Assuming flat ΛCDM, the Hubble parameter evolves as

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}.$$

The luminosity distance and distance modulus are

$$d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + (1-\Omega_m)}}, \qquad \mu_{\rm th}(z) = 5\log_{10}\left(\frac{d_L(z)}{10\,\mathrm{pc}}\right).$$

3.2 Parameter Estimation

We minimize

$$\chi^2(H_0,\Omega_m) = \sum_i \frac{\left[\mu_{\mathrm{obs},i} - \mu_{\rm th}(z_i; H_0, \Omega_m)\right]^2}{\sigma^2_{\mu,i}}$$

to obtain best-fit parameters and uncertainties (Gaussian approximation from the fit covariance). We also report a fixed-Ω_m variant and redshift-split consistency checks.

3.3 Model Fit and Diagnostics

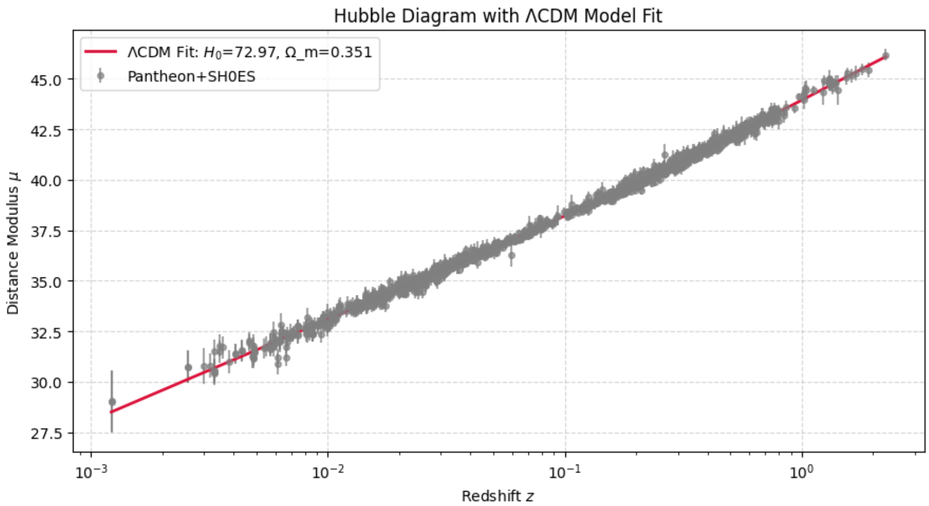

Figure 2: Hubble diagram with the best-fit flat ΛCDM model overlaid. Best-fit parameters: $H_0 \approx 72.97\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m \approx 0.351$.

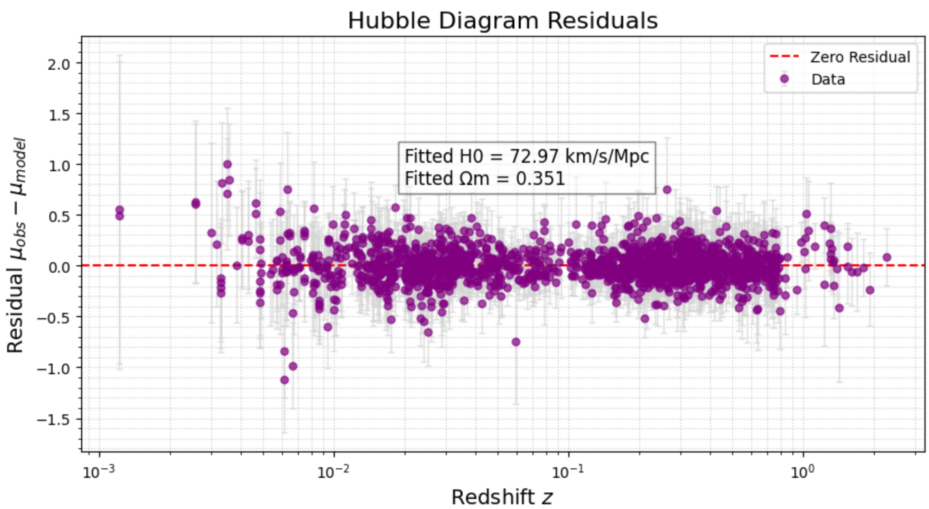

Figure 3: Residuals $\Delta\mu = \mu_{\rm obs} - \mu_{\rm th}$ versus redshift. The scatter is centered near zero with no strong redshift trend.

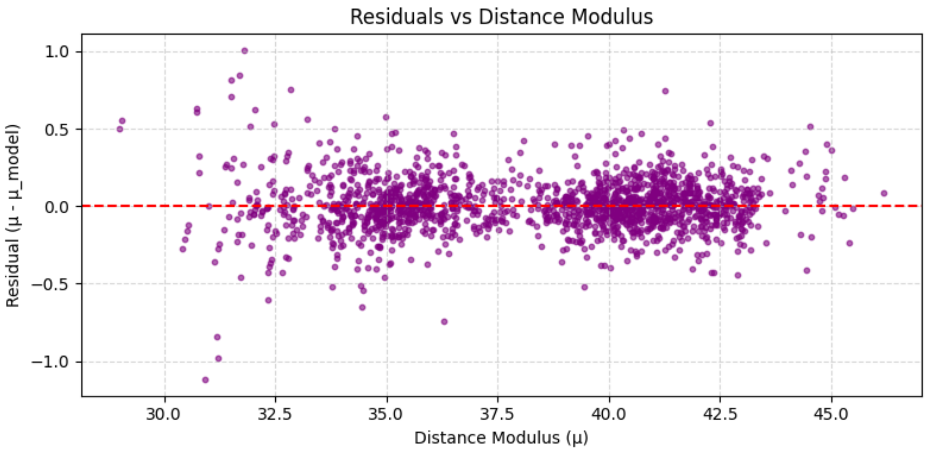

Figure 4: Residuals plotted against distance modulus. The absence of structure indicates no magnitude-dependent bias in the fit.

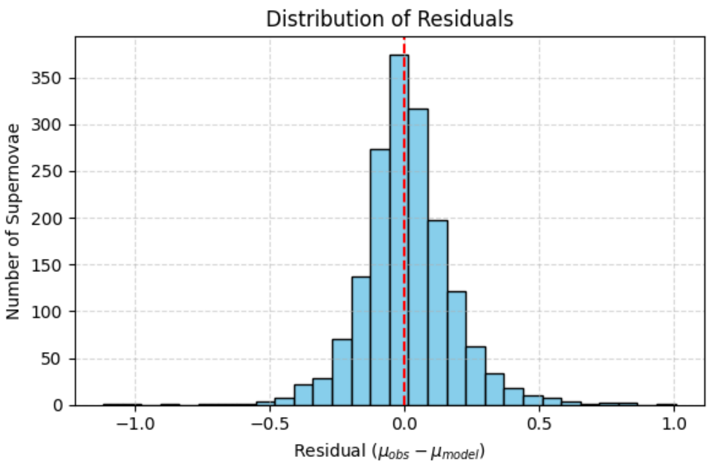

Figure 5: Histogram of residuals. The distribution is centered near zero and approximately Gaussian, supporting the χ^2 assumption.

3.4 Age of the Universe

With best-fit parameters, the age is computed via

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)} \approx 12.36 \text{ Gyr}.$$

3.5 Implementation Notes

Analysis used `pandas` for I/O, `Astropy` for cosmology helpers, `SciPy` for integration/optimization, and `Matplotlib` for visualization. Benign warnings were observed (a pandas `FutureWarning` for deprecated `delim_whitespace`, a Matplotlib label `SyntaxWarning`, and a font glyph `UserWarning`); none affect numerical results.

4 Results and Discussion

4.1 Global Fits

From the joint fit:

$$H_0 = 72.97 \pm 0.26 \text{ km s}^{-1}\text{ Mpc}^{-1}, \qquad \Omega_m = 0.351 \pm 0.019.$$

Fixing $\Omega_m = 0.3$ gives

$$H_0 = 73.53 \pm 0.17 \text{ km s}^{-1}\text{ Mpc}^{-1}.$$

The derived age is $t_0 \approx 12.36$ Gyr.

4.2 Implications

Our H_0 agrees with the local SH0ES distance-ladder determination and remains in significant tension with the Planck 2018 CMB-inferred value. The higher H_0 naturally implies a younger universe than CMB-based ΛCDM (~ 13.8 Gyr). The persistence of this discrepancy highlights the need for improved systematics control or extensions to the standard model.

5 Limitations

Dataset Constraints

While the Pantheon+SH0ES catalog provides one of the most comprehensive compilations of Type Ia supernovae to date, it is still subject to selection effects. Supernovae at higher redshifts suffer from reduced signal-to-noise, Malmquist bias, and calibration uncertainties in photometry. These factors can subtly bias distance moduli and hence inferred cosmological parameters.

Model Assumptions

This study adopted the standard flat ΛCDM model. While this framework is widely successful, it assumes only matter and dark energy contributions with a constant equation of state $w = -1$. Potential extensions such as evolving dark energy, additional relativistic species, or spatial curvature were not considered here. If the Hubble tension reflects new physics, our restricted model space may underrepresent the true cosmology.

Parameter Degeneracies

The analysis showed mild degeneracy between H_0 and Ω_m, visible in the χ^2 contour plot. Although the Pantheon+SH0ES data strongly constrain H_0, the degeneracy implies that joint fits with other cosmological probes (e.g., BAO, CMB) are necessary for breaking parameter correlations and improving robustness.

Systematic Uncertainties

The accuracy of H_0 is limited not only by supernova photometry but also by calibration of the local distance ladder. While Pantheon+SH0ES incorporates Cepheid-calibrated anchors, alternative distance indicators (e.g., TRGB, strong-lensing time delays) sometimes yield discrepant results. Such systematics remain an open issue and could contribute to the persistent tension with Planck results.

Computational Limitations

The implementation relied on numerical integration of the luminosity distance using `SciPy`. While sufficiently accurate for this analysis, more sophisticated treatments (e.g., Monte Carlo simulations of systematics, Bayesian hierarchical modeling) were not implemented due to time and computational constraints. These approaches could yield more precise uncertainty propagation.

6 Conclusion

In this study, we analyzed the Pantheon+SH0ES Type Ia supernova dataset within the ΛCDM framework to obtain constraints on the Hubble constant and matter density parameter. The best-fit values,

$$H_0 = 72.97 \pm 0.26 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \qquad \Omega_m = 0.351 \pm 0.019,$$

are consistent with recent local distance-ladder measurements but remain in significant tension with the CMB-based Planck result. The derived age of the Universe, $t_0 \approx 12.36$ Gyr, further highlights the cosmological implications of this discrepancy.

Subsample analyses (low-z vs. high-z) demonstrated robust consistency across redshift regimes, supporting the reliability of the Pantheon+SH0ES calibration. Additional tests with fixed Ω_m confirmed that the dataset tightly constrains the Hubble constant, independent of small variations in matter density.

The persistence of the Hubble tension underscores the importance of continued work in this area. Future progress will require both improved control of systematics in supernova cosmology and complementary constraints from alternative probes such as BAO, TRGB distances, and strong-lensing time delays. At the same time, the discrepancy may point toward physics beyond the standard ΛCDM model, including evolving dark energy or modifications to the early-Universe framework.

Overall, this project demonstrates the power of Type Ia supernovae as precise cosmological probes and highlights their central role in one of the most pressing challenges of modern cosmology: reconciling the different routes to measuring the expansion rate of the Universe.

References

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Appendix

A. Additional Figure

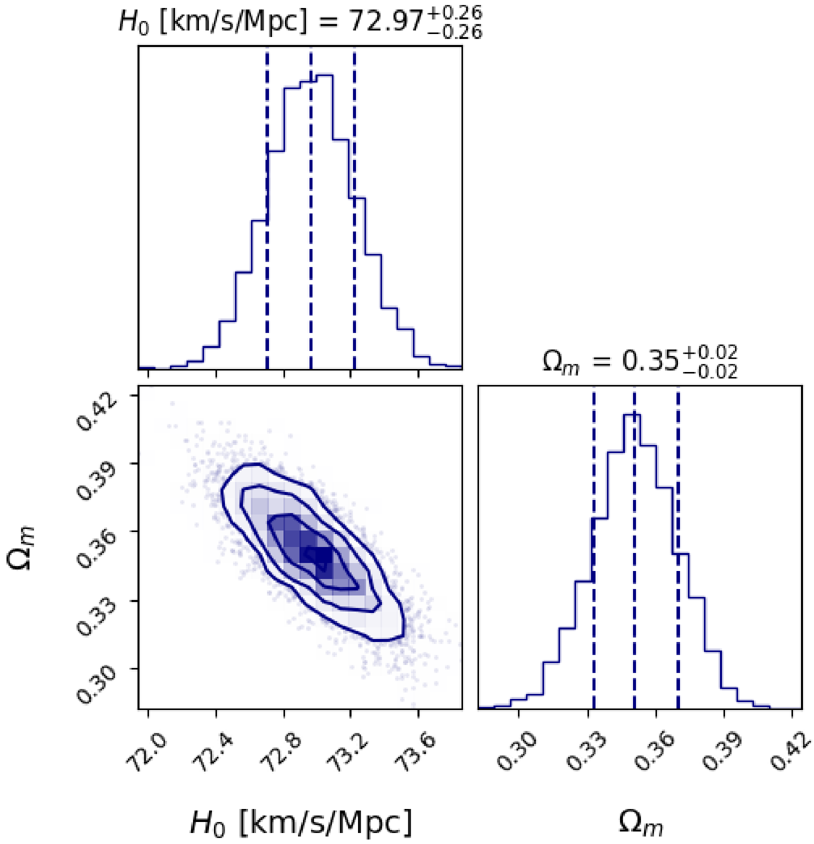

Figure 6: Joint posterior distribution of H_0 and Ω_m. The 1σ and 2σ confidence contours are indicated.

B. Supplementary Results

Parameter	**Value**
Cosmological Model	Flat ΛCDM
Best-fit H_0	72.97 ± 0.26 km s^{-1} Mpc^{-1}
Best-fit Ω_m	0.351 ± 0.019
Age of the Universe (t_0)	12.36 Gyr
Fixed Ω_m Case	$\Omega_m = 0.30 \Rightarrow H_0 = 73.53 \pm 0.17$ km s^{-1} Mpc^{-1}
Low-z Subsample ($z < 0.1$)	$H_0 = 73.01$ km s^{-1} Mpc^{-1}
High-z Subsample ($z \geq 0.1$)	$H_0 = 73.85$ km s^{-1} Mpc^{-1}

Table 1: Summary of key cosmological results from the Pantheon+SH0ES supernova analysis. Uncertainties are 1σ.