

LOG EXPRESSION: (Q3: NUMERICAL PRECISION)

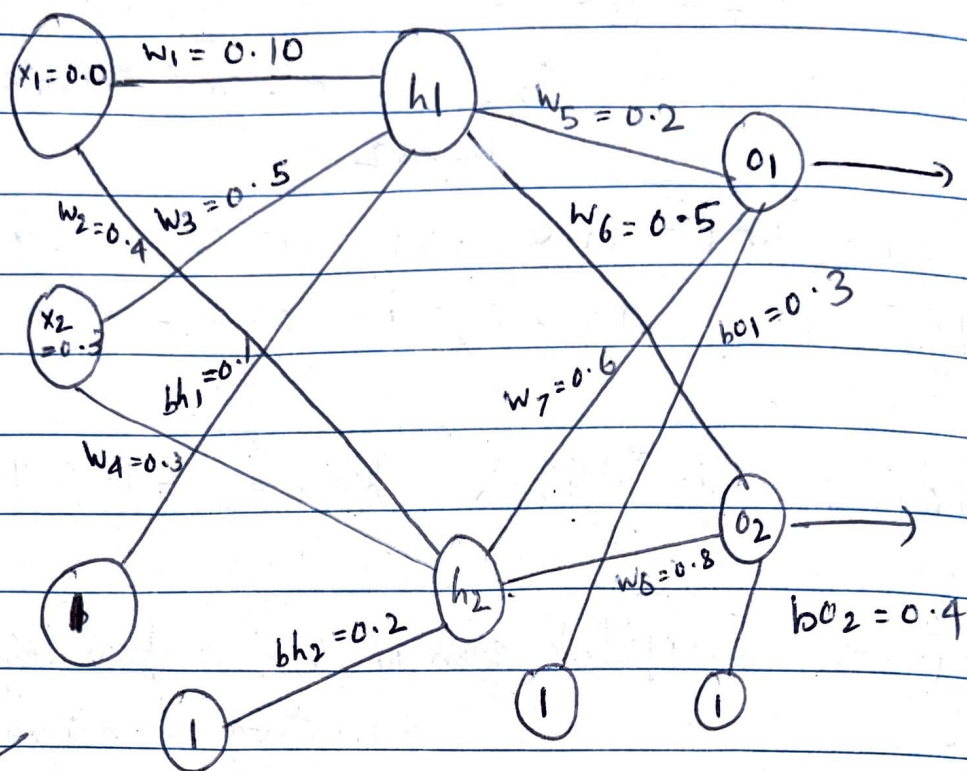
$$- \log \left( \frac{e^x}{e^x + e^y} \right)$$

$$= - \left[ \log(e^x) - \log(e^x + e^y) \right]$$

$$= - \left[ \cancel{\log(e^x)} - \left[ \cancel{\log(e^x)} + \log(1 + e^y/e^x) \right] \right]$$

$$= \log(1 + e^{y-x})$$

# Ques 4: MANUAL BACK PROPAGATION



FORWARD PASS

$$\begin{aligned}
 h_1 &= w_1 x_1 + w_3 x_2 + b_{h1} \\
 &= (0.1)(0.0) + (0.5)(0.3) + 1 \times 0.1 \\
 &= 0.05
 \end{aligned}$$

$$\begin{aligned}
 h_2 &= w_2 x_1 + w_4 x_2 + b_{h2} \\
 &= (0.4)(0.0) + (0.3)(0.3) + 1 \times 0.2 \\
 &= -0.11
 \end{aligned}$$

$$\text{hidden o/p 1} = \text{sigmoid}(h_1) = \frac{1}{1 + e^{-0.05}} = 0.512$$

$$\text{hidden o/p 2} = \text{sigmoid}(h_2) = \frac{1}{1 + e^{-(-0.11)}} = 0.4725$$

$$\begin{aligned}
 \hat{O}_1 &= w_5 \times (\text{hidden o/p 1}) + w_7 \times (\text{hidden o/p 2}) + b \times b_{01} \\
 &= 0.2 \times 0.5124 + 0.6 \times 0.4725 + 1 \times 0.3 \\
 &= 0.10248 + 0.2835 + 0.3 \\
 &= 0.38598 + 0.3 = 0.68598 \text{ (Same value as activation fn is linear)}
 \end{aligned}$$

$$\begin{aligned}
 \hat{O}_2 &= w_6 \times (\text{hidden o/p 1}) + w_8 \times (\text{hidden o/p 2}) + b \times b_{02} \\
 &= 0.5 \times 0.5124 + 0.8 \times 0.4725 + 1 \times 0.4 \\
 &= 0.2562 + 0.378 + 0.4 \\
 &= 1.0342 \text{ (Same value as activation fn is linear)}
 \end{aligned}$$

$$E_1 = \frac{1}{2} * (0.68598 - 0.01)^2 = 0.22847$$

$$E_2 = \frac{1}{2} * (1.0342 - 0.99)^2 = 0.00097$$

$$E = E_1 + E_2 = 0.22944$$



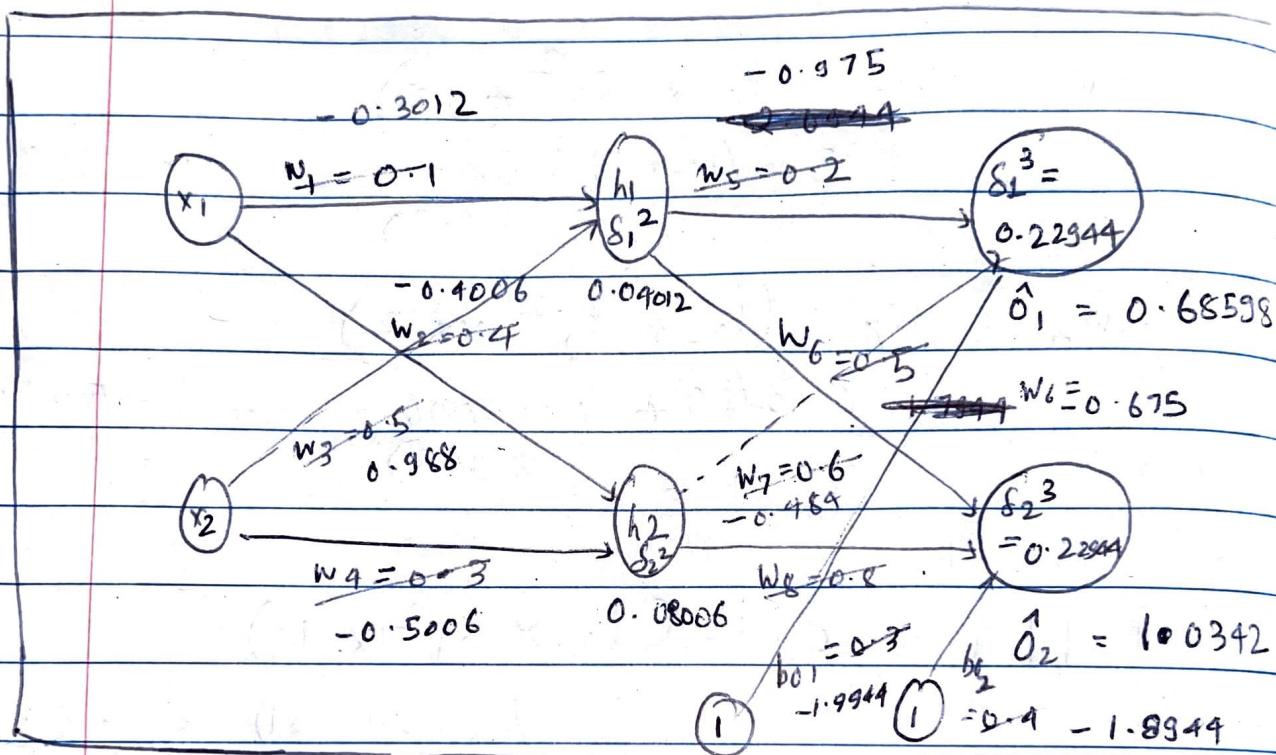
Q4:  
b)

## MANUAL BACKWARD PASS

### BACKWARD PASS :

- $LR = 10$  ,  $E = 0.22944$

$\delta_i^x \rightarrow x \rightarrow \text{layer}, i \rightarrow \text{Subscript}$



$$\delta_1^3 = \text{error} * \text{derivative (linear fn)}$$

$$= 0.22944 \times 1 = 0.22944$$

$$\delta_2^3 = \text{error} * \text{derivative (linear fn)}$$

$$= 0.22944 \times 1 = 0.22944$$

hidden o/p1

$$w_5 = w_5 - lr * \delta_1^3 \times \text{hidden o/p1} = 0.2 - 10 * (0.22944) \times 0.5124$$

$$w_6 = w_6 - lr * \delta_2^3 \times \text{hidden o/p2} = 0.5 - 10 * (0.22944) \times 0.5124$$

$$w_5 = \boxed{-2.0944}$$

$$w_6 = \boxed{-7.1544}$$

0.5124

$$\begin{aligned} W_5 &= -0.975 \\ W_6 &= -0.675 \end{aligned}$$

$$\begin{aligned} W_7 &= -0.484 \\ W_8 &= -0.284 \end{aligned}$$

$\times \text{hidden o/p 2}$

$$\begin{aligned} W_7 &= W_7 - lr * \delta_1^3 = 0.6 - 10 * (0.22944) = \boxed{-1.0725} \\ W_8 &= W_8 - lr * \delta_2^3 = 0.8 - 10 * (0.22944) = \boxed{-1.0725} \end{aligned}$$

$$\begin{aligned} b_{01} &= b_{01} - lr * \delta_1^3 \times 1 = 0.3 - 10 * (0.22944) \times 1 \\ b_{02} &= b_{02} - lr * \delta_2^3 \times 1 = 0.4 - 10 * (0.22944) \times 1 \end{aligned}$$

$$b_{01} = -1.9944, \quad b_{02} = -1.8944$$

$\delta_1^2 \rightarrow$  (weighted sum of  $\delta$  from prev layer)  $\times$   
derivative of activation fn

$$= (0.2 \times \delta_1^3 + 0.5 \delta_2^3) \times (\text{hidden o/p1})$$

$$\begin{aligned} &= (0.2 \times 0.22944 + 0.5 \times 0.22944) (0.5124) (1 - 0.5124) \\ &= \underline{0.04012} \end{aligned}$$

$$\delta_2^2 = (0.6 \times \delta_1^3 + 0.8 \times \delta_2^3) (\text{hidden o/p2})$$

$$\begin{aligned} &= (0.6 \times 0.22944 + 0.8 \times 0.22944) \\ &\quad (0.4725) (1 - 0.4725) \end{aligned}$$

$$= \underline{0.08006}$$



$$W_1 = W_1 - l_r \times (\delta_1^2 \times 0)$$

$$= 0.1 - 10 \times (0.04012 \times 0) =$$

$$\boxed{0.1}$$

$$W_2 = W_2 - l_r \times \delta_2^2$$

$$= 0.4 - (10 \times 0.08006) =$$

$$\boxed{0.4}$$

$$W_3 = W_3 - (l_r \times \delta_1^2 \times 2)$$

$$= 0.5 - (10 \times 0.04012 \times 2) = 0.0988$$

$$W_4 = W_4 - (l_r \times \delta_2^2 \times 0.3)$$

$$= 0.3 - (10 \times 0.08006 \times 0.3) = -0.5006$$

$$bh_1 = 0.1 - (l_r \times \delta_1^2 \times 1)$$

$$= 0.1 - (10 \times 0.04012 \times 1) = -0.3012$$

$$bh_2 = 0.2 - (10 \times \delta_2^2 \times 1)$$

$$= 0.2 - (10 \times 0.08006 \times 1)$$

$$= -0.6006$$