Honors Precalculus with Trigonometry (OM013) Problem Set 5

Name: Pranav Ananth

Please show all work because you will be graded on the clarity of your explanation as well as the correctness of your work if this were graded. An answer with no work/explanation will receive zero credit.

1. Find the quotient and remainder using ***long division***.

a. $$\frac{x^4 - 3x^3 + x - 2}{x^2 - 5x + 1}$$

$$x^2 + 2x + 9 + \frac{44x-11}{x^2-5x+1}$$

$$x^2-5x+1 \,\big)\, x^4 - 3x^3 + 0x^2 + x - 2$$
$$-(x^4 - 5x^3 + x^2)$$
$$2x^3 - x^2 + x - 2$$
$$-(2x^3 - 10x^2 + 2x)$$
$$9x^2 - x - 2$$
$$-(9x^2 - 45x + 9)$$
$$44x - 11$$

Q: $x^2 + 2x + 9$

R: $\frac{44x-11}{x^2-5x+1}$

b. $$\frac{x^3 + 2x^2 - x + 1}{x + 3}$$

$$x^2 - x + 2 - \frac{5}{x+3}$$

$$x+3 \,\big)\, x^3 + 2x^2 - x + 1$$
$$-(x^3 + 3x^2)$$
$$-x^2 - x + 1$$
$$-(-x^2 - 3x)$$
$$2x + 1$$
$$-(2x + 6)$$
$$-5$$

Q: $x^2 - x + 2$

R: $-\frac{5}{x+3}$

2. a. Find the quotient and remainder using ***synthetic division***.

$$\frac{3x^3 - 12x^2 - 9x + 1}{x - 5}$$

$x = 5$ because $5 - 5 = 0$

Q: $3x^2 + 3x + 6 + \frac{31}{x-5}$

$$\begin{array}{r|rrrr} 5 & 3 & -12 & -9 & 1 \\ & & 15 & 15 & 30 \\ \hline & 3 & 3 & 6 & 31 \end{array}$$

b. Use ***synthetic division*** and the ***Remainder Theorem*** to evaluate *P(c)*.

$$P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14, \quad c = -7$$

Remainder Theorem: $\frac{P(x)}{x - c}$; remainder is $P(c)$

$x + 7$, $-7 + 7 = 0$

$$\begin{array}{r|rrrr} -7 & 30 & -40 & 36 & 14 \\ & & -210 & 1750 & -12502 \\ \hline & 30 & -250 & 1786 & -12448 \end{array}$$

← remainder

$\boxed{P(c) = -12448}$

3\. i. Find all rational zeros of the polynomial, and write the polynomial in factored form. Use Rational Zeros Theorem.

$$P(x) = 2x^4 - 7x^3 + 3x^2 + 8x - 4$$

$\frac{p}{q}$ p is a factor of -4 (-1
q is a factor of 2

$\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} 1 & 2 & -7 & 3 & 8 & -4 \\ & & 2 & -5 & -2 & 6 \\ \hline & 2 & -5 & -2 & 6 & 2 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 2 & -7 & 3 & 8 & -4 \\ & & 4 & -6 & -6 & 4 \\ \hline & 2 & -3 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 2 & -7 & 3 & 8 & -4 \\ & & -2 & 9 & -12 & 4 \\ \hline & 2 & -9 & 12 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -3 & 2 \\ & & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

$(x+1)(x-2)(2x^2-5x+2)$

Quadratic Formula

$$\frac{5 \pm \sqrt{25-16}}{4}$$

$$\frac{5 \pm 3}{4}$$

doubled

2 or $\frac{1}{2}$

Roots: $\{-1, \frac{1}{2}, 2\}$

Factored: $(x-2)^2(x+1)(x-\frac{1}{2})$

ii. a. State the Fundamental Theorem of Algebra and b. what it means, what its implications are, and how it is used.

Every function defined by a polynomial of degree 1 or more has at least 1 complex zero.

This means that every such equation has a zero, and it is used to help find the factors of a polynomial.