Stanford | Online High School 2023-2024

OHS Honor Code Statement

OM013 Honors Precalculus with Trigonometry

For the student: I attest to the following (please check each box):

☑ I followed the instructions provided carefully and honestly, and

☑ The work returned to the Online High School is my own, produced within the stated time limits without assistance from books, notes, or other aids beyond those specifically permitted.

Student's Name (print): Pranav Ananth

Student's Signature: Pranav Ananth **Date:** 9/10/2023

Note that formal, professional proctoring is not required for this exam. Please ask a parent, guardian, teacher, or other adult to check the boxes and sign below.

For the proctor: I attest to the following (please check each box):

☑ I have taken reasonable steps to ensure the integrity of this exam, and

☑ The work returned to the Online High School is solely that of the student above, produced within the stated time limits without assistance from books, notes, or other aids beyond those specifically permitted.

Proctor's Name (print): ANANTH VENKATA

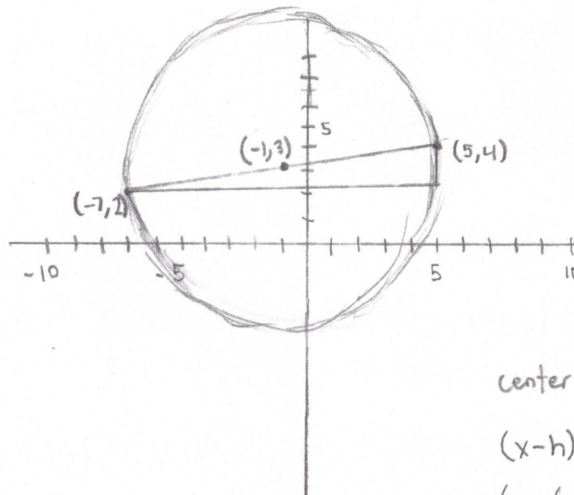
Proctor's Signature: [signature] **Date:** 9/10/2023

Start Time: 5:01 pm **End Time:** 5:32 pm

Honors Precalculus with Trigonometry (OM013)
Exam 1 — Sections 2.1-2.6, 3.7

Name: Pranav Ananth

1a. (10 points) Find the equation of the circle whose diameter has end points (-7,2) and (5,4).



distance formula $= \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$\sqrt{(5-(-7))^2+(4-2)^2}$

$=\sqrt{12^2+2^2}$

$=\sqrt{148}$

$=2\sqrt{37}$ - diameter

radius is $\frac{1}{2}$ of the diameter, $\sqrt{37}$

midpoint formula $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

center $= \left(\frac{-7+5}{2}, \frac{2+4}{2}\right) = (-1,3)$

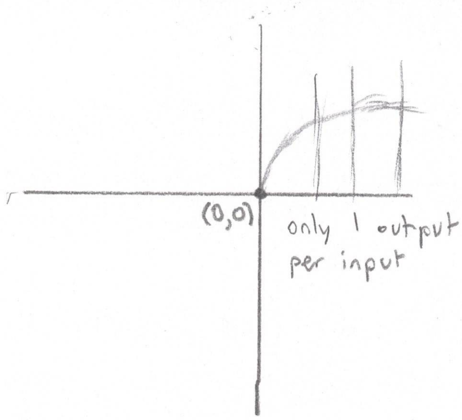
$(x-h)^2+(y-k)^2=r^2$

$(x-(-1))^2+(y-3)^2=\sqrt{37}^2$

$= \boxed{(x+1)^2+(y-3)^2=37}$

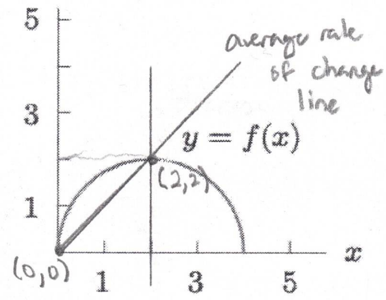
1b. (5 points) Does $y = \sqrt{x}$ represent a function? Why or why not? Also, provide its domain and range.

It is a function - every x has exactly one y



Domain: $[0, \infty)$
Range: $[0, \infty)$

2. Consider the function whose graph shown below.



a. (4 points) Estimate the interval(s) on which the graph is decreasing.

Past $x=2$, there is a downwards trend

x is decreasing $\boxed{(2,4]}$

b. (9 points) Estimate the average rate of change of the function between $x = 0$ and $x = 2$.

$x=0,\ f(x)=0$
$x=2,\ f(x)=2$

$\frac{\text{rise}}{\text{run}} = \frac{2-0}{2-0} = \boxed{1}$

c. (6 points) Write the equation of the secant line through $(0, f(0))$ and $(2, f(2))$.

Slope is 1
y-int is 0

$\boxed{y=x}$

3. (12 points; 6 points/part) Find the domain and range of each function. For part (b), assume the entire graph is shown.

a. $y = \dfrac{1}{\sqrt{4-9x^2}}$

if $x = \frac{2}{3}$

$y = \frac{1}{\sqrt{4-4}}$ = undefined

any more means a complex number

if $x = -\frac{2}{3}$

$y = \frac{1}{\sqrt{4-4}}$ = undefined

any less means complex number

Domain: $(-\frac{2}{3}, \frac{2}{3})$

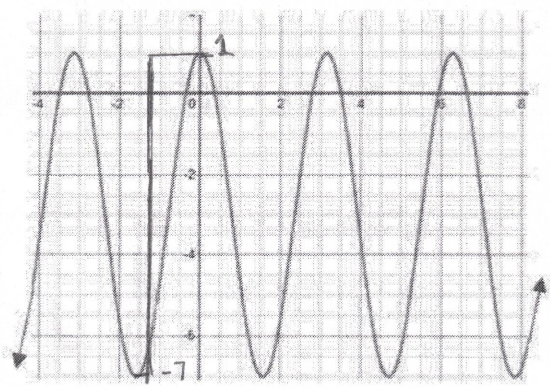
Range: $[\frac{1}{2}, \infty)$

x^2, so sloping graph

V

vertex at $(0, \frac{1}{\sqrt{4}})$ or $(0, \frac{1}{2})$

b.



Domain: $(-\infty, \infty)$

Range: $[-7, 1]$

extends infinitely left and right

goes from −7 to 1 on y-axis

4. (10 points; 5 points/part)
   a. R. McNeill alexander, a British paleontologist, uses observations of living animals to estimate the speed of dinosaurs. Alexander believes that two animals of different sizes, but geometrically similar shapes, will move in a similar fashion. For animals of similar shapes, their velocity is directly proportional to the square root of their hip height. Alexander has compared a white rhinoceros, with a hip height of 1.5 m, and a member of the genus Triceratops, with a hip height of 2.8 m. If a white rhinoceros can move at 45 km/hr, then what is the estimated velocity of the dinosaur?

proportion of speed

$\frac{\sqrt{2.8}}{\sqrt{1.5}} = \sqrt{\frac{28}{15}}$

$\boxed{\sqrt{\frac{28}{15}} \cdot 45 \text{ km/h} \approx 61.48 \text{ km/h}}$

   b. The power, P, of a jet of water, is jointly proportional to the cross-sectional area, A, of the jet, and to the cube of the velocity, v. If the velocity is doubled, and the cross-sectional area is halved, by what factor will the power increase?

$P = kAv^3$

$k \cdot \frac{1}{2}A \cdot (2v)^3$

$= k \cdot \frac{1}{2}A \cdot 8v^3$

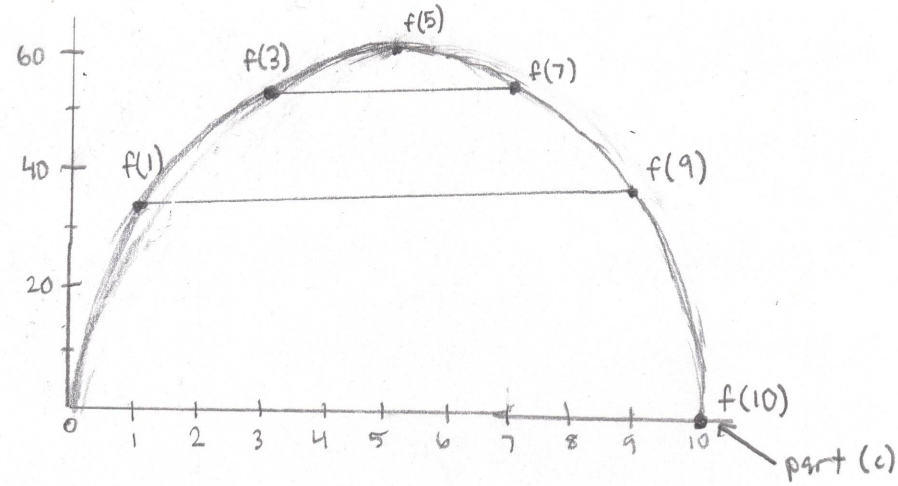
$= 4kAv^3$

factor

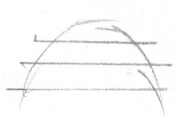
$\boxed{\text{The power was multiplied by a factor of 4.}}$

5. A cannon ball is shot straight up overhead by a cannon. The height s (in meters) of the cannon ball above the ground is a function of time t (in seconds), so $s = f(t)$.

a. (10 points) Sketch a possible graph of s as a function of t.



b. (5 points) The cannon ball reaches its maximum height of 60 meters, when t is 5 seconds, i.e., $f(5) = 60$. Assuming that the graph of s is a parabola, what can be said about the following four values of f: $f(1), f(3), f(9)$, and $f(7)$?



f(1) and f(9) have the same output
f(3) and f(7) have the same output

exact same y-value on the opposite side

c. (6 points) At what time do you expect the cannon ball to reach the ground again? Mark this time on your graph in part a.

Answer is labeled, 10 seconds

6. Consider the piecewise defined function

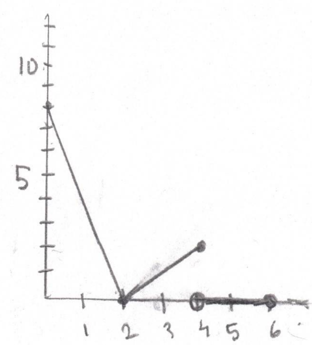
$$f(x) = \begin{cases} |(x-3)^2 - 1| & 0 \le x \le 4 \\ 0 & 4 < x \le 6 \end{cases}$$

a. (8 points) Evaluate $f(2)$ and $f(3)$.

$f(2) = |(2-3)^2 - 1| = |1-1| = 0$

$f(3) = |(3-3)^2 - 1| = |-1| = 1$

b. (10 points) Sketch the graph of $f(x)$.



c. (5 points) Is $f(x)$ continuous over its domain? If not, classify any discontinuity as a jump, hole, or vertical asymptote.

It is continuous over the domain.

$0 \le x \le 4$ no gap

$4 < x \le 6$