Math 075 Homework 3

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Exercise 2.1.14

Problem

In each case determine all s and t such that the given matrix is symmetric

Part (c)

$$\begin{bmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{bmatrix}$$

Solution (c)

$$\begin{cases} t = 2s \\ t = st \\ s^2 = s \end{cases}$$

$$\begin{cases} s = 1 \\ t = 2 \end{cases}$$

The given matrix is symmetric when $s = 1$ and $t = 2$.

Exercise 2.2.15

Problem

In each case find the matrix A.

Part (a)

$$\left(A + 3 \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{bmatrix} \right)$$

Solution (a)

$$A + 3 \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{bmatrix}$$

$$A + \begin{bmatrix} 3 & 3 \\ -3 & 6 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -2 \\ 3 & -1 \\ 3 & -4 \end{bmatrix}$$

Exercise 2.2.1

Problem

In each case find a system of equations that is equal to the vector equation.

Part (a)

$$x_1 \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}$$

Solution (a)

$$\begin{cases} 2x_1 + x_2 + 2x_3 = 5 \\ -3x_1 + x_2 = 6 \\ 4x_2 - x_3 = 3 \end{cases}$$

$$A = \begin{bmatrix} 2 & 1 & 2 & 5 \\ -3 & 1 & 0 & 6 \\ 0 & 4 & -1 & 3 \end{bmatrix}$$

Exercise 2.2.5

Problem

In each case, express every solution of the system as a sum of a specific soution plus a solution of the associated homogeneous system.

Part (c)

$$\begin{cases} x_1 + x_2 - x_3 - 5x_5 = 2 \\ x_2 + x_3 - 4x_5 = -1 \\ x_2 + x_3 + x_4 - x_5 = -1 \\ -2x_1 - x_2 + 2x_4 = 3 \end{cases}$$

Solution (c)

$$\begin{bmatrix} 1 & 1 & -1 & 0 & -5 & 2 \\ 0 & 1 & 1 & 0 & -4 & -1 \\ 0 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -4 & 1 & 1 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{9}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 + 0x_5 = 0 \\ x_2 - \frac{9}{2}x_5 = \frac{1}{2} \\ x_3 + \frac{1}{2}x_5 = -\frac{3}{2} \\ x_4 + 3x_5 = 0 \\ x_5 = free \end{cases}$$

$$\begin{bmatrix} 1 & 1 & -1 & - & -5 & 0 \\ 0 & 1 & 1 & 0 & -4 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{9}{2} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 + 0x_5 = 0 \\ x_2 - \frac{9}{2}x_5 = 0 \\ x_3 + \frac{1}{2}x_5 = 0 \\ x_4 + 3x_5 = 0 \\ x_5 = free \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ -\frac{3}{2} \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ \frac{9}{2} \\ -\frac{1}{2} \\ -3 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ \frac{9}{2} \\ -\frac{1}{2} \\ -3 \\ 1 \end{bmatrix}$$

Exercise 2.2.12

Problem

The projection $P : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ for all $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$. Show that P is induced by a matrix and find the matrix.

Solution

$$P_A\mathbf{x} = A\mathbf{x}$$

$$A\mathbf{b} = I\mathbf{x}$$

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = I_2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$