PLACO: Supplementary Material

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1 Experimental Details

1.1 Human Configuration

As part of our experiments, we utilize four human configurations of sizes 5, 7, 10, 15 humans.

5 human configuration:

$$[0.35, 0.65, 0.70, 0.56, 0.66]$$

7 human configuration:

$$[0.89, 0.47, 0.41, 0.45, 0.62, 0.37, 0.45]$$

10 human configuration:

$$[0.70, 0.29, 0.36, 0.92, 0.54, 0.55, 0.52, 0.44, 0.68, 0.61]$$

15 human configuration:

$$[0.48, 0.94, 0.53, 0.70, 0.43, 0.58, 0.30, 0.88,$$
$$0.62, 0.57, 0.38, 0.49, 0.39, 0.55, 0.29]$$

Each of these configurations was sampled from a gamma distribution, which has the following probability density function,

$$f(x; k, \theta) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \tag{1}$$

We aimed to keep the mean $\mu = 0.6$ and variance $\sigma^2 = 0.06$. Then, the parameters (k, θ) were chosen as follows:

$$\theta = \sigma^2/\mu$$
$$k = \mu/\theta$$

Once the parameters (k, θ) were set, we performed sampling using the popular python library NumPy used for linear algebra and matrix operations.

1.2 Linear Programming

To solve the Integer Linear Programming problem in our paper we use a python library PuLP. It doesn't directly solve optimization problems itself; it acts as an interface to optimization solvers like CBC (Coin-OR Branch and Cut), GLPK (GNU Linear Programming Kit), CPLEX (Constraints Programming Learning and EXperimentation), or Gurobi.

By default, PuLP uses CBC as its solver. As we used PuLP in its default mode, PLACO LP works on the CBC solver.

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2 Proofs

Lemma 1. *Given a single human i, the probability that the estimated label $h_i(x)$ is the same as $t_i(x)$ is given as:*

$$\mathbb{P}(h_i(x) = t_i(x)) \geq \mathbb{P}\left\{\mathbb{P}(t_i(x)|m(x)) > 1/2\right\} \tag{2}$$

Proof. $\mathbb{P}(h_i(x) = t_i(x))$

$$= \mathbb{P}\left\{t_i(x) = \arg\max_{k\in\mathcal{Y}} \sum_{y\in\mathcal{Y}} \mathbb{P}(k|y)\cdot\mathbb{P}(y|m(x))\right\}$$
$$= \mathbb{P}\left\{\sum_{y\in\mathcal{Y}} \mathbb{P}(t_i(x)|y)\cdot\mathbb{P}(y|m(x)) \geq \max_{k\neq t_i(x)} \sum_{y\in\mathcal{Y}} \mathbb{P}(k|y)\cdot\mathbb{P}(y|m(x))\right\}$$
$$\geq \mathbb{P}\left\{\sum_{y\in\mathcal{Y}} \mathbb{P}(t_i(x)|y)\cdot\mathbb{P}(y|m(x)) \geq \sum_{y\in\mathcal{Y}} \max_{k\neq t_i(x)} \mathbb{P}(k|y)\cdot\mathbb{P}(y|m(x))\right\}$$
$$\geq \mathbb{P}\left\{\sum_{y\in\mathcal{Y}} \mathbb{P}(t_i(x)|y)\cdot\mathbb{P}(y|m(x)) \geq \sum_{y\in\mathcal{Y}} (1 - \mathbb{P}(t_i(x)|y)\cdot\mathbb{P}(y|m(x)))\right\}$$
$$= \mathbb{P}\left\{2\cdot\sum_{y\in\mathcal{Y}} \mathbb{P}(t_i(x)|y)\cdot\mathbb{P}(y|m(x)) \geq \sum_{y\in\mathcal{Y}} \mathbb{P}(y|m(x))\right\}$$
$$= \mathbb{P}\left\{\mathbb{P}(t_i(x)|m(x) \geq 1/2\right\} \qquad \square$$