Worksheet 2

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1 Selected Problems

6. Give a denial (negation) of the following statements:

   (a) 34159 is a prime number — 45159 is not a prime number

   (b) Roses are red and violets are blue
   Roses are not red, or violets are not blue

   (c) If there are no hamburgers, I'll have a hot dog
   I will not eat hot-dogs even when there are no burgers

   (d) Fred will go but he will not player
   Either Fred will go or he will not play

   (e) The number x is either negative or greater than 10 — $0 \leq x \leq 10$

   (f) We will win the first game or the second
   We will win neither the first game nor the second

6. Write down the contrapositives for the statements

   (a) If two rectangles are congruent, they have the same area
   If two rectangles do not have the same area they are not congruent

   (b) If a triangle with sides a, b, c is right-angled than $a^2 + b^2 = c^2$
   If $a^2 + b^2 \neq c^2$ then the triangle is not right-angled

   (c) If $2^n - 1$ is prime then so is n
   If n is not prime then $2^n - 1$ is not prime

   (d) If the Yuan rises the Dollar will fall
   If the dollar did not fall the Yuan did not rise

2 Optional Problems

1. Express ϕ unless ψ in terms of logical operators [$\phi \Leftrightarrow \psi$]

3. Express $\phi \oplus \psi$ in terms of logical operators $[(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)]$

5. Complete the truth table for mod-2 arithmetic

M	N	$M \times N$	$M + N$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

6. In the above truth table, interpret 1 as T and 0 as F

   (a) What logical operator corresponds to $\times$ [AND]

   (b) What logical operator corresponds to $+$ [XOR]

   (c) Does $\neg$ corresponds to $-$

   Yes, corresponds to subtraction under modulo 2

7. In the above truth table, interpret 1 as F and 0 as T

   (a) What logical operator corresponds to $\times$ [OR]

   (b) What logical operator corresponds to $+$ [XNOR/equivalence]

   (c) Does $\neg$ corresponds to $-$

   Yes, corresponds to subtraction under modulo 2

8. Four cards are placed on the table in front of you. You are told (truthfully) that each has a letter printed on one side and a digit on the other, but of course you can only see one face of each. What you see is:

   B E 4 7

   You are now told that the cards you are looking at were chosen to follow the rule If there is a vowel on one side, then there is an odd number on the other side. What is the least number of cards you have to turn over to verify this rule, and which cards do you in fact have to turn over?

   (a) Two cards, E and 4:

   (b) Turn over E to make sure that there is an odd number on the other side.

   (c) Turn over 4 to make sure that there is not a vowel on the other side

   (d) You do not need to turn over B or 4 as having a consonant and an odd number are allowed.

11. You are in charge of a party where there are young people. Some are drinking alcohol, others soft drinks. Some are old enough to drink alcohol legally, others are under age. You are responsible for ensuring that the drinking laws are not broken, so you have asked each person to put his or her photo ID on the table. At one table are four young people. One person has a beer, another has a Coke, but their IDs happen to be face down so you cannot see their ages. You can, however, see the IDs of the other two people. One is under the drinking age, the other is above it.

Unfortunately, you are not sure if they are drinking Seven-up or vodka and tonic. Which IDs and/or drinks do you need to check to make sure that no one is breaking the law?