LISA Response summary

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From section 3.1 of [1], we get the varying portion for the round trip to be,

$$\delta l(t_2) = \frac{Lsin^2\theta}{2}\ \tau(cos\theta, f)\,[h_+(t_2)cos2\psi + h_x(t_2)sin2\psi] \tag{1}$$

Where, we have considered the arm to be in the x-z plane $\mathbf{u} = \hat{x}\ sin\theta + \hat{z}\ cos\theta$

ψ is the polarisation angle. τ is the transfer function as defined in eq(7) of [1]

Now from we consider the plus and cross polarisation amplitudes of a binary black hole with an angle of inclination i with the line of sight $\hat{z}$ (3.27a and 3.27b) of [2]

We can then write,

$$h(t) = \frac{A(t)}{d}\ \left[F_+\ \frac{1 + cos^2 i}{2} cos\Phi(t) + F_\times\ cosi\ sin\Phi(t)\right] \tag{2}$$

Where the $F_+ and F_x$ are the antenna response functions

In the present case we can write

$$\delta l = \frac{A(t)Lsin^2\theta\tau(cos\theta, f)}{2d}\left[\frac{1 + cos^2 i}{2}\ cos2\psi\ cos\Phi(t) + cosi\ sin2\psi\ sin\Phi(t)\right] \tag{3}$$

$$= \frac{A(t)Lsin^2\theta\tau(cos\theta, f)}{2d}\left(\sqrt{\frac{(1 + cos^2 i)^2}{4} cos^2 2\psi + cos^2 i sin^2 2\psi}\right) cos(\Phi(t) - \Psi) \tag{4}$$

The general formula is given by eq(8) of [1]

FOr a general $\hat{\Omega}$ specified by θ, ω in spherical coordinate system, we find that the antenna pattern becomes:

$$F_+(\theta, \phi, \psi, i) = \frac{(1 + cos^2 i)}{2}\left[\frac{1}{2}(1 + cos^2\theta)cos2\phi cos\psi - cos\theta sin2\phi sin2\psi\right] \tag{5}$$

$$F_\times(\theta, \phi, \psi, i) = cosi\left[\frac{1}{2}(1 + cos^2\theta)cos2\phi cos\psi + cos\theta sin2\phi cos2\psi\right] \tag{6}$$

$$\tag{7}$$

If we put back $\theta = 0$ and $\phi = 0$ in [5] and [6] we get back the responses in eq [4].

The measured strain will be (as from eq 14 and 15)

$$s = \frac{\delta l_u(t) - \delta l_v(t)}{l} \tag{8}$$

$$= \mathbf{D}(\hat{\Omega}, f) : h(\hat{\Omega}, f, x, t) \tag{9}$$

where $D(\hat{\Omega}, f) = \frac{1}{2}\left((u \otimes u)\tau - (v \otimes v)\tau\right)$

Depending on the orientation of the two arms specified by the u and v vector we get the final response as a function of the arm orientation,the orientation and location of binary.

References

[1] https://arxiv.org/pdf/gr-qc/0103075.pdf

[2] https://dcc.ligo.org/public/0106/T1300666/003/Whelan_notes.pdf