Measurement of electrical resistivity of thin samples using linear four probe method

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Aim

Using the linear four probe method we seek out to measure the resistivity of a thin sample of known thickness and study the variation with temperature. Using the resistivity temperature relation we can find the Band Gap energy of the semiconductor sample.

Theory

Linear Four probe method

One can use a simple *two-probe method* to determine the resistivity of the material with moderate to high resistance. The method fails to give accurate values for good conductors. Hence we switch to a better method, the **Four-Probe Method**. In this method, a fixed current flows through the outer leads and the voltage difference is measured across a pair of inner leads as shown in 1 . This overcomes contact and lead resistance problems.

Floating Point potential

The floating point potential at any point on a plane due to the current I is,

$$V = \frac{\rho I}{2\pi x} \tag{1}$$

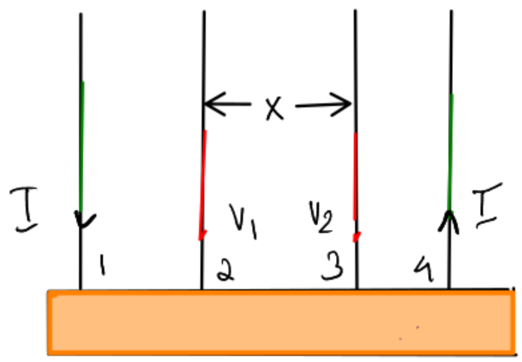

Figure 1: A rough schematic of The four probe setup

The potential difference between at 1 and 2 in Fig.1 is given by,

$$V = \frac{\rho I}{2\pi}\left(\frac{2}{x} - \frac{1}{x}\right) \tag{2}$$

Hence the resistivity is,

$$\rho = \left(\frac{V}{I}\right) 2\pi x \tag{3}$$

By sending a constant value current and noting down the voltmeter readings, we can measure the resistivity of the sample given by Eq (3).

Correction factor

If a sample is backed by a non-conducting surface, we need a **correction factor** for the resistivity. the correction factors are given by the table. For a general w/x value, one has to obtain a fit for the curve and figure out the value as will be shown in the Analysis section.

$$\rho_c = \frac{\rho}{f_2(w/x)} \tag{4}$$

Resistivity of Semiconductors

Electronic band structure gives us an idea of how conductive a material is. In the case of Metals, the *Fermi Level*, E_F lies inside one of the band and there is no band gap. In the case of Semiconductors, there is a **band gap** as shown in Fig 2.

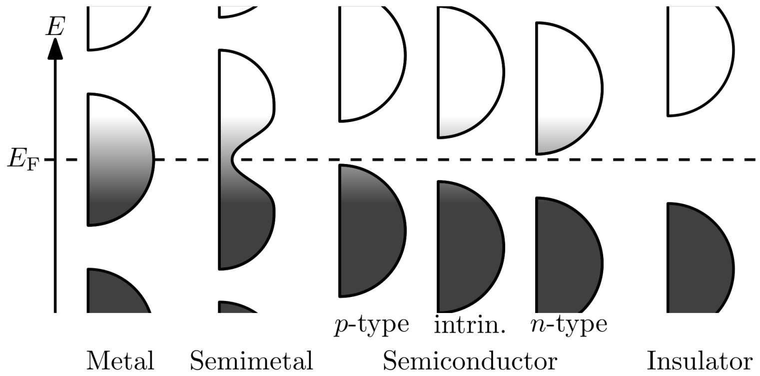

Figure 2: Band Gap for metals, semiconductors and insulators. Image taken from Wiki [4]

Temperature Dependence

The temperature dependence for semiconductor is very different from metals. Energy must be supplied, in this case thermal energy to excite the electrons to the **conduction band**. The temperature dependence of resistivity of semiconductor is given by,

$$\rho = A\ exp(-E_g/k_B T) \tag{5}$$

Where E_g is the band gap energy, k_B is the Boltzmann's constant and T is the temperature. We can measure the energy band gap by studying the temperature dependence of resistivity. In the experiment we calculate resistivity values for temperature upto 140 degrees Celsius. From a $ln\rho$ vs T^{-1} plot, we can get the E_g value from the slope.

$$ln\rho = lnA - \frac{E_g}{k_B T} \tag{6}$$

Data

The spacing between the probes(x) = 2 mm.
Thickness of the sample(w) = 0.5 mm

Four Probe Resitivity Data	
Constant Current: 5.00mA	
Temperature in $^\circ$C	Voltage in mV
28	416
30	413
35	396
40	375
45	347
50	317
55	285
60	253
65	224
70	197
75	172
80	150
85	131
90	114
95	100
100	88
105	77
110	68
115	59
120	53
125	47
130	42
135	37
140	33

Table 1: The Temperature and Voltage values taken using the Four Probe Data

Analysis and Results

Correction Factor calculation

We need to account for the correction factor, $f_2(w/x)$ after calculating the resistivity. The correction factors for various w/x values are available [1]. But to find for a specific value we first do a curve fit of the f_2 vs w/x data and get the value out of it.

I did a curve fit using Scipy's **Curvefit** function to get a good fit for the function as shown in Fig 3. The best fit function is:

$$f_2(w/x) = \frac{1.39}{(w/x)} \tag{7}$$

As the plot 3 shows, the fit is perfect for smaller values of w/x. In our case $w/x = 0.25$. Hence,

$$f_2(0.25) = 5.542$$

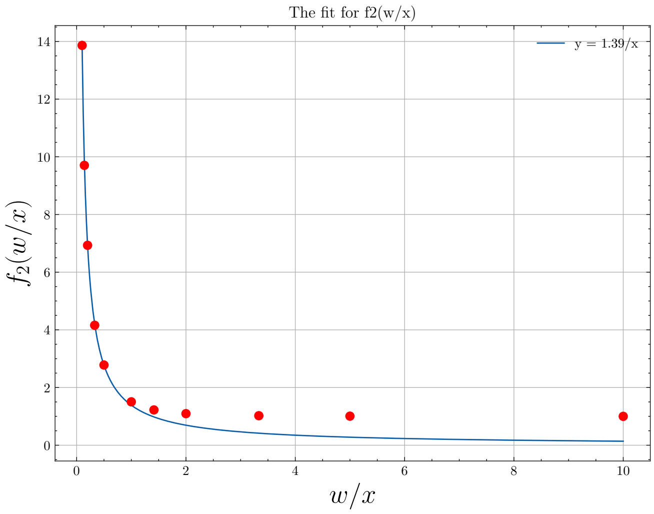

Figure 3: The fit for the resistivity correction function

Resistivity vs Temperature

The steady current is 5 mA. From Eq (5) we can determine the resistivity values in $\Omega \cdot m$. The values are quoted in table 2. We also take a log of the resistivity for the plot later. Note that for calculations we have to convert the temperature to its SI unit, **kelvin**.

The figure 4 shows the plot of resistivity vs temperature from the calculated resistivity values. Notice how only one portion of it behaves exponentially whereas initial values show a steady rate. This will be further explained in the discussions section.

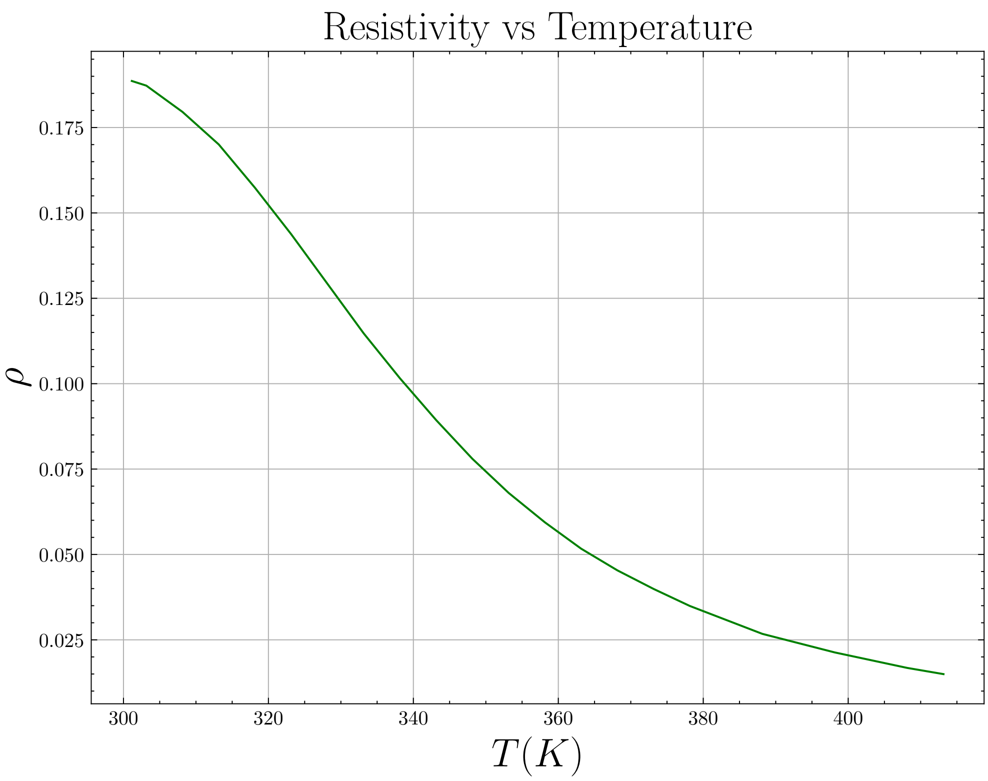

Figure 4: Resistivity vs Temperature for the material

Four Probe Resitivity Data		
Constant Current: 5.00mA		
Temperature in $^\circ$C	Resistivity ρ (Ωm)	ln ρ
28	1.045	0.019
30	1.038	0.016
35	0.995	-0.002
40	0.942	0.025
45	0.872	-0.059
50	0.796	-0.098
55	0.716	-0.145
60	0.636	0.196
65	0.563	-0.249
70	0.495	-0.305
75	0.432	-0.364
80	0.377	-0.423
85	0.329	-0.482
90	0.286	-0.542
95	0.251	-0.599
100	0.221	-0.655
105	0.193	-0.713
110	0.170	-0.767
115	0.148	-0.828
120	0.133	-0.875
125	0.118	-0.927
130	0.105	-0.976
135	0.092	-1.031
140	0.083	-1.081

Table 2: Calculated values of resistivity for various temperature readings

Calculation of Band gap

To calculate the Band Gap, we can use the Eq (6). We plot $ln\rho$ vs $1/T$ and we get a **straight line curve** with some deviations for low temperatures (High T^{-1}). The plot is shown in 5. From a straight line fit we get the slope and intercept (m & c respectively) to be:

$$m = 3495.777\ K\ ,\ c = 12.619\ \Omega m$$

From Eq (6), we can read off the slope to be

$$m = \frac{E_g}{2k_B} \tag{8}$$

Using the Boltzmann constant $k_B = 8.617 * 10^{-5}/eVK^{-1}$, the Band gap in eV will be,

$$\begin{aligned} E_g &= 2mk_B \\ &= 0.60246\,eV \end{aligned} \tag{9}$$

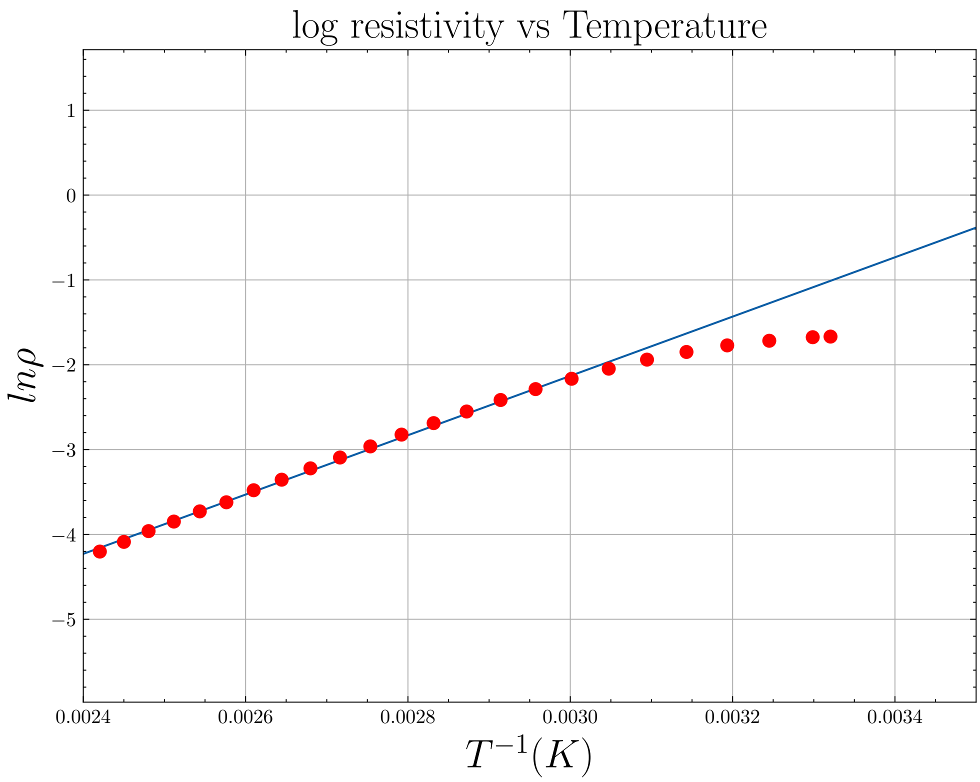

Figure 5: The log resistivity vs T inverse plot. The red dots correspond to the data values and the straight line is the best fit. Notice how some of thge points deviate from straight line behaviour. These points are ignored as they correspond to a steady state at low temperatures.

Error analysis

For error in the calculated Band Gap Energy, I'm assuming a small percentage error in the probe spacing, $\Delta x \approx 0.01$.

$$\begin{aligned} \Delta E_g &= 2k_B T \frac{\Delta \rho}{\rho} \\ &= 2k_B T \frac{\Delta x}{x} \\ &= 0.00061\, eV \end{aligned} \tag{10}$$

$$\frac{\Delta E_g}{E_g} = 0.101\% \tag{11}$$

There could also be errors from the thickness of the sample which can be also added above although even a 1% error would not change the result we got drastically. Also the formula for ρ can be theoretically considered for only very large surface in comparison to the probe distance. There is also a chance of error due to variation in doping.

Discussions and Conclusion

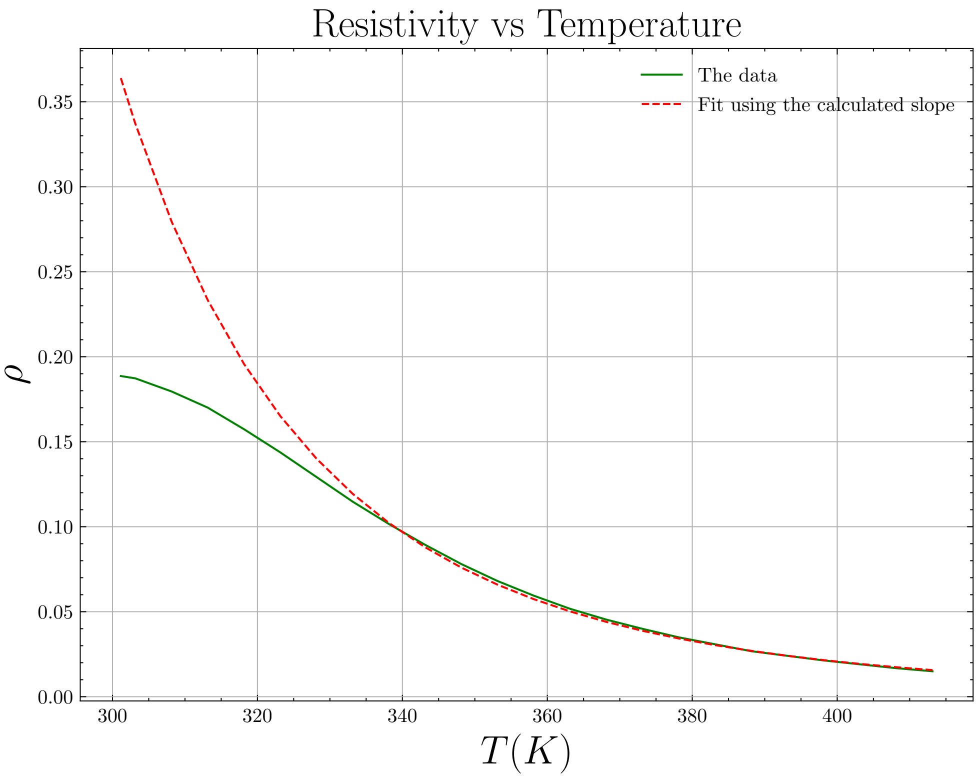

Figure 6: Resistivity vs Temperature comparison with data and the fit function. We notice the exponential behaviour shown by the data after the temperature reaches a certain value.

The band gap energy we got from the experiment with the error is,

$$\boxed{E_g = 0.60246 \pm 0.00061\, eV} \tag{12}$$

The band gap energy calculated is acceptable for a **semiconductor**. For Germanium the value is about 0.67 eV which is very near to the value we get from the calculation. The Fig 4 doesn't exactly show an **exponential behaviour** throughout as is expected from the Eq (5). For low Temperatures, there is a steady state where the resistivity remains constant while the material heats. This is also seen in the plot 5 for high inverse temperature values we see deviation from straight line behaviour. During this heating time, the number of electrons being thermally excited to the conduction band is not large enough to affect the resistivity of the sample. Eventually after a certain temperature number of excited electrons becomes large enough to show the exponential behaviour as expected for a semiconductor.

Using the values we got from the fit we can construct the resistivity vs temperature relation of the material and plot it against the original data. The result is shown in Fig 6. The dotted curve corresponds to the the one obtained from the fit and calculation. The green curve is the original data. We notice how the material starts to follow the exponential behaviour only from a certain value of temperature and before this it is spent on heating and exciting electrons as explained above.

The full data and Python files used for the calculations can be found in [2], [3] . I've used the python package Matplotlib for plotting and Scipy for curve-fitting.

References

[1] Slides on "Measurement of electrical resistivity of thin samples usinglinear four probe method" .

[2] The data, https://github.com/pranavastro/physicslab3/blob/main/Exp2:%20Resistivity/Electrical%20Resistivity%20Data.xlsx

[3] Python files ,https://github.com/pranavastro/physicslab3/tree/main/Exp2:%20Resistivity

[4] Semiconductor, https://en.wikipedia.org/wiki/Semiconductor

[5] Resistivity of Semiconductors, https://eng.libretexts.org/Bookshelves/Materials_Science/Supplemental_Modules_(Materials_Science)/Electronic_Properties/Resistivity#Resistivity_of_Semiconductors