Dielectric Constant Measurement at Microwave Frequency - Solid Samples

Pranav Satheesh| PH17B008

February 25, 2021

Aim

This experiment aims to measure the dielectric constant of a solid sample using the transmission line method for a solid sample of different thickness using microwave frequencies.

Theory

Dielectric Materials

Dielectric materials are a class of insulators that can be polarised by an applied external field. Applied electric field aligns the dipole along its direction. For linear materials, the resultant polarization due to external electric field,

$$\begin{aligned}\vec{P} &= \epsilon_0(\epsilon_r - 1)\vec{E} \\ &= \epsilon_0 \chi \vec{E}\end{aligned} \tag{1}$$

Where ϵ_r is the **dielectric constant** or the **relative permittivity** of the material and χ is the susceptibility of the material. χ is a measure of how easily it is to **polarise** the dielectric material.

The dielectric constant, ϵ_r is a complex quantity

$$\epsilon_r^* = \epsilon_r^{'} - i\epsilon_r^{''} \tag{2}$$

The real part indicated the polarisability of the material and the imaginary part represents the amount of loss in the medium. Evaluating the dielectric constant gives us an idea about the polarisability and more about the molecular structure and thus is an important aspect of materials science.

Dielectric Permittivity ϵ_r

Dielectric dispersion is the dependence of the permittivity on the frequency of an applied electric field. Here we shine a source of electromagnetic wave of a certain frequency f. So ϵ_r is a function of the frequency, $\epsilon_r(f)$. Under reflection from a reflecting surface, the total electric field polarized along +x direction and wave propogating long +z direction as shown in the figure is

$$\vec{E} = |E|e^{-i\beta z}(1 + Re^{i\beta z})\hat{x} \tag{3}$$

For $R = -1$ the Amplitude becomes

$$|\vec{E}| = 2ESin(\beta z)$$

where β represents the wave vector in air, $\beta = 2\pi/\lambda$. Hence the intensity profile will be a sin squared function with minimas at $z_n = n(\lambda/2)$

$$EE^* \propto sin^2(\beta z) \tag{4}$$

With the sample this equation becomes,

$$EE^* \propto (1 + RR^* + Re^{i2\beta z} + R^* e^{-i2\beta z}) \tag{5}$$

This suggests that the **minimas gets shifted** when the sample is introduced. We can now measure the minimas, the points where we get zero intensity measurement and then compare it with no sample case and find the value of the shift x. Using this shift in the position of minima, we can measure the dielectric constant using impedance of a transmission line.

Inside the air-filled rectangular waveguide, the wavevector magnitude is labeled by β_{g0}. For the hollow metallic waveguide for the given dimensions, operating in TE_{10} mode. This introduces a cut-off wave vector:

$$\beta_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{6}$$

The wave-vector inside the air filled wave-guide is related to the wave guide in air as,

$$\beta_{g0}^2 = \beta^2 - \beta_c^2 \tag{7}$$

We denote the wavevector in the sample kept inside the waveguide, β_{gs}

Transmission line impedance method

The intrinsic impedance at a length of the transmission line as seen by the power is given by

When a transmission line is terminated by a short, we get a standing wave and at d_0 we can find the zero field.

Now we can do the same with the sample inserted in the waveguide.The detector detects a minimum at d_s. We can relate the impedance of the waveguide at t(thickness of the material) to the characteristic impedance Z_s and terminated by short

$$\begin{aligned} Z_s^{'} &= Z_s \\ iZ_s tan(\beta_{gs}t) &= -iZ_0 tan(\beta_{g0}(d_s - t)) \\ -\frac{tan(\beta_{g0}(d_s - t))}{tan(\beta_{gs}t)} &= \frac{\beta_{g0}}{\beta_{gs}} \\ \frac{tan(\beta_{g0}(t + x))}{\beta_{g0}t} &= \frac{tan(\beta_{gs}t)}{\beta_{gs}t} = \frac{tanX}{X} \end{aligned} \tag{8}$$

Where we have used the relation of impedence on the material parameters,

$$Z = \frac{\omega\mu}{\beta}$$

The final equation is a transcendental equation. We have to figured out X by solving it. Once we know X we can calculate the dielectric constant as,

$$\beta_{gs} = \frac{X}{t} \tag{9}$$

$$\beta_{0s}^2 = \beta_{gs}^2 + \beta_c^2 \tag{10}$$

$$\epsilon_r = \frac{\beta_{0s}^2}{\beta_0^2} \tag{11}$$

Data

Dielectric Sample	a = 2.25 cm	b = 1.125 cm	TE_{10} mode	
Frequency	9 GHz			
Set	Intensity			
Distance(cm)	No Sample	Thickness = 4 mm	Thickness = 5 mm	Thickness =6 mm
0	0.00			
0.1	0.06			
0.2	0.25			
0.3	0.55			
0.4	0.94	2.08		
0.5	1.40	2.58	3.81	
0.6	1.90	3.04	3.97	3.27
0.7	2.40	3.44	4.00	2.84
0.8	2.88	3.74	3.90	2.36
0.9	3.30	3.93	3.68	1.86
1	3.64	4.00	3.35	1.36
1.1	3.87	3.94	2.94	0.91
1.2	3.99	3.76	2.47	0.52
1.3	3.98	3.46	1.97	0.23
1.4	3.84	3.07	1.47	0.05
1.5	3.58	2.62	1.00	0.00
1.6	3.23	2.12	0.60	0.07
...	...	...	...	...

Table 1: The data taken for no sample and samples with varying thickness. This is only part of the data. The full data can be found here [2]

Analysis and Results

Measuring the shift in the minima

To figure out the shift in the minima, We have to plot the intensity vs distance figures and find the values. For multiple shifts we take the average of it.

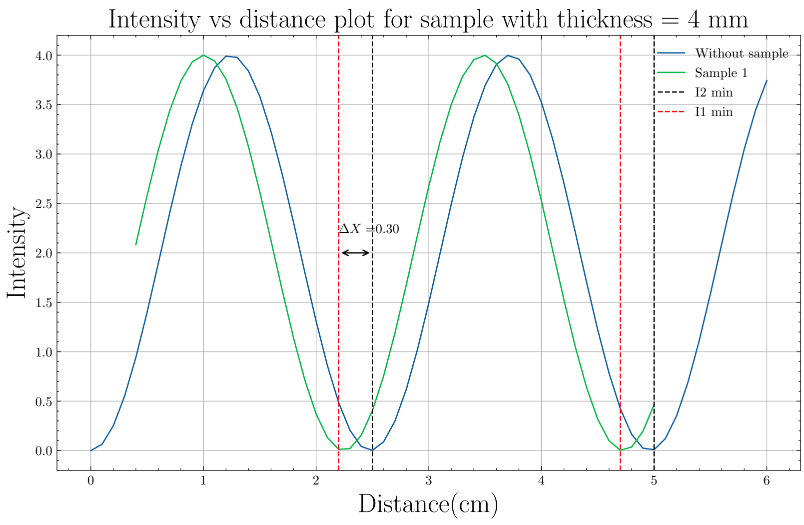


Figure 1: Intensity vs distance without the sample and with a sample of 4 mm thickness

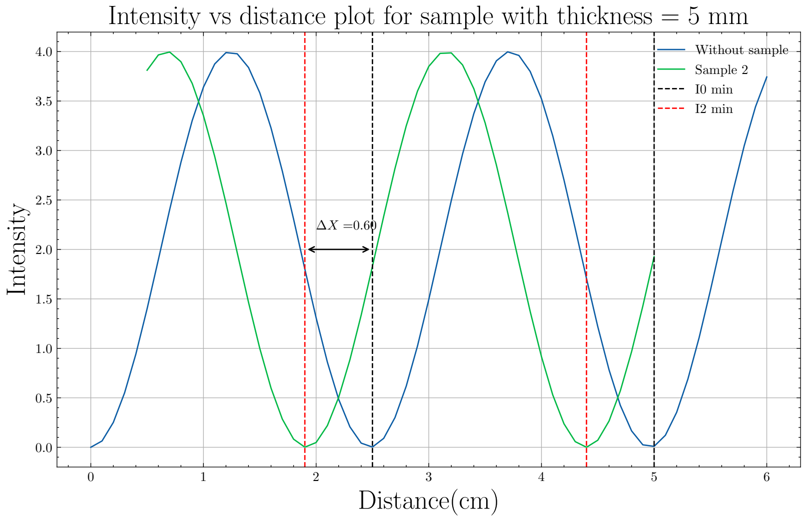


Figure 2: Intensity vs distance without the sample and with a sample of 5 mm thickness

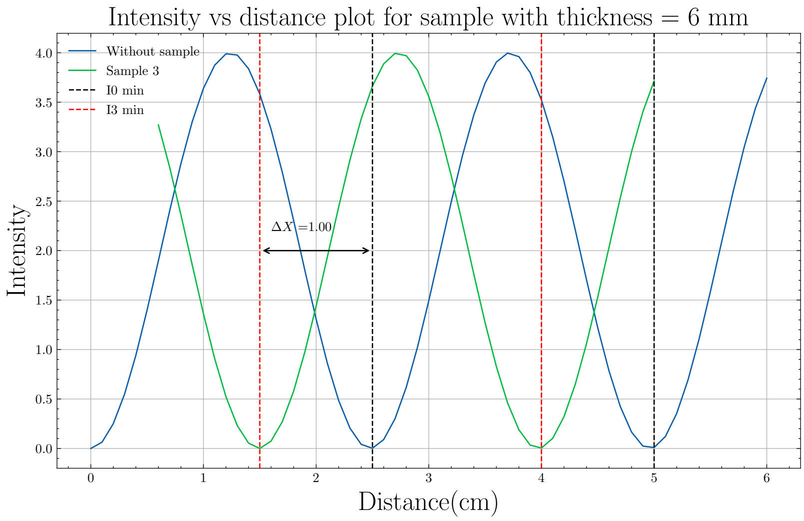


Figure 3: Intensity vs distance without the sample and with a sample of 6 mm thickness

From the plots we get the shifts for the different sample thickness as

$$\begin{aligned} x_1 &= 0.29cm\,;\, t_1 = 4mm \\ x_2 &= 0.69cm\,;\, t_2 = 5mm \\ x_3 &= 1.0cm\,;\, t_3 = 6mm \end{aligned} \tag{12}$$

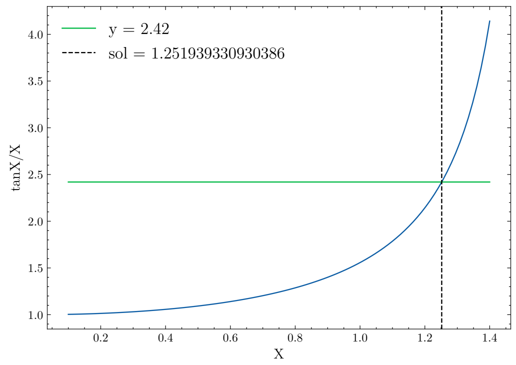
We now have to use the transcendental equation,

$$\frac{tan(\beta_{g0}(t+x))}{\beta_{g0}t} = \frac{tanX}{X} \tag{13}$$

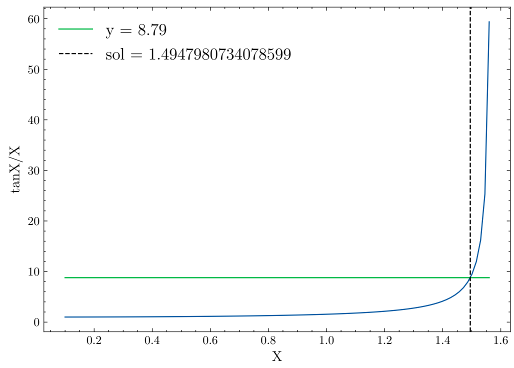
The plots for (13) are in 4 . I've used a numerical method (bisection) to find the solution of this non linear equation.

The solution for X found by solving the equation above is used to find the relative permittivity using Eq (8),(9),(10),(11).

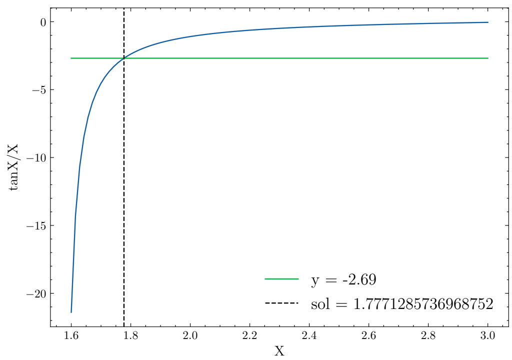
The calculations are summarised in the table 2. Thus the average value of the **dielectric constant** is $\epsilon_r = 3.13$.



(a) For sample 1



(b) For sample 2



(c) For sample 3

Figure 4: Transcendental equation plots for the equation (13). The solution X is labeled as "sol" indicated by the dotted black line.

Sample Thickness (t)	X	$\beta_{gs} = \frac{X}{t}(m^{-1})$	$\beta_{os} = \sqrt{\beta_{gs}^2 + \beta_c^2}(m^{-1})$	$\epsilon_r = \frac{\beta_{os}^2}{\beta}$
4 mm	1.25	312.98	342.72	3.30
5 mm	1.49	298.96	329.96	3.06
6 mm	1.77	296.19	327.45	3.02
Average				3.13

Table 2: Values calculated from the equations which leads to ϵ_r

Error analysis

We can estimate the error in measuring dielectric constant by differentiating (11).

$$\Delta\epsilon_r = \frac{2\beta_{os}}{\beta_o^2}\Delta\beta_{os} \tag{14}$$

Where,

$$\Delta\beta_{os} = \frac{\beta_{gs}}{\beta_{os}}\Delta\beta_{gs} + \frac{\beta_c^2}{\beta_{os}}\frac{\Delta a}{a} \tag{15}$$

Here we are not considering b as $b = 0$ in TE_{10}. Δa is the least count in measurement of the dimension "a" of the waveguide. Δa is taken to be 0.01 mm.

The error in relative perimittivity is computed to be:

$$\Delta\epsilon_r = 0.0048 \tag{16}$$

Which is a percentage error of 0.16%.

Conclusions and Discussions

The value of the **dielectric constant** after the error correction is found to be

$$\boxed{\epsilon_r = 3.1277 \pm 0.0048} \tag{17}$$

We have seen a great way to measure the electric permittivity of a material at microwave frequency., This method of using impedance of transmission lines is useful at high frequencies. At low frequencies, it is easy to measure the dielectric constant using a capacitance setup. We have used the reflectance property of material and the fact that the relative permittvity is dependent on the frequency of the source incident on the material.

Another notable point in the experiment is the usage of a waveguide. Waveguides will only propagate signals above a certain frequency denoted in this report by the wavevector β_c. We saw in the "Theory" section how the cut-off depended on the dimensions of the waveguide. Thus selecting the right waveguide with the appropriate dimensions and hence the cut-off is important for the experiment.

We have accounted for least count errors. There are observational errors possible as well in this experiment mainly when you read off the distance. Measuring the dielectric constant upto a great deal of accuracy is vital for their use in the industry and research.

The files used in the calculation and plotting can be found in [3]. I have used the python library *Matlplotlib* for plotting.

References

[1] Slides on "Dielectric Constant Measurement at Microwave Frequency" .

[2] The data https://github.com/pranavastro/physicslab3/blob/main/Exp1:Dielectric/PH17B008.xlsx

[3] Python files https://github.com/pranavastro/physicslab3/tree/main/Exp1:Dielectric

[4] Relative Permittivity, https://en.wikipedia.org/wiki/Relative_permittivity

[5] Waveguide cutoff, https://en.wikipedia.org/wiki/Waveguide

[6] Dielectric, https://en.wikipedia.org/wiki/Dielectric