Measurement of majority carrier concentration - Hall Effect

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Aim

Using Hall effect to measure the hall coefficient of the sample at room temperature and thereby measure the majority carrier concentration. Also to see the variation of the Hall coefficient with temperature.

Theory

Hall Effect

When a current carrying conductor is placed in a magnetic field, the field exerts a transverse force on the charge carriers and push them to the perpendicular sides of the conductor as illustrated in the figure. Due to the accumulation of charges, a voltage is generated which we call **Hall Voltage**.

Consider a current density in the x direction $\vec{J_x}$, The electrons thus have a constant velocity of electrons in the -x direction. Consider an uniform. constant magnetic field $\vec{B_z}$ is applied along the z direction. The magnetic force on the electrons is:

$$\begin{aligned}\vec{F_h} &= -e(-\vec{v_x} \times \vec{B_z}) \\ &= ev_x B_z \, \hat{j}\end{aligned} \tag{1}$$

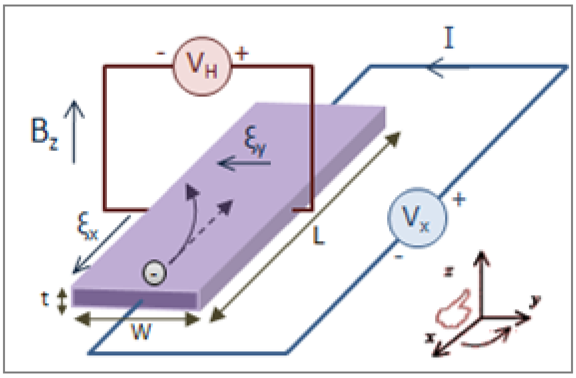

Figure 1: The Hall effect setup. Initially, the charge carriers follow the curved arrow, due to the magnetic force. At some distance from the current-introducing contacts, electrons pile up on the left side and deplete from the right side, which creates an electric field ξ_y in the direction of the hall voltage, V_H

Hence the magnetic force accumulates the electrons on +y face of the bar whereas the electric field oppose this motion and they reach an equilibrium under steady condition.

$$
\begin{aligned}
eE_y &= e\ v_x B_z \\
\frac{V_y}{d} &= v_x B_z \\
\frac{V_y}{d} &= \frac{I_x}{nedt} B_z \\
V_y &= \left(\frac{1}{ne}\right) \frac{I_x}{t} B_z \\
&= \frac{R_H I_x B_z}{t}
\end{aligned}
\tag{2}
$$

$$
\boxed{R_H = \frac{V_h\ t}{BI}} \tag{3}
$$

R_H is the **Hall coefficient** and gives us the **concentration of the charge carriers**. The polarity of the **Hall voltage** gives the **sign of the charge carriers**. For a given sample, Hall voltage is measured as a function of current is plotted and the slopes gives us the Hall coefficient value. From the Hall coefficient value we can estimate the concentration of the majority charge carrier at room temperature.

$$
R_H = \frac{1}{ne} \tag{4}
$$

Hall coefficient for two types of carriers

The Eq (4) is perfect for the ideal case of conductors with a single charge carriers. However, in semiconductors there is a current contribution from both **holes** and **electrons** which are present in different concentration and mobilities. The hall coefficient in this case is modified as:

$$
R_H = \frac{\mu_h^2 p - \mu_e^2 n}{e(\mu_h p + \mu_e n)^2} \tag{5}
$$

Where μ_h is the **mobility of holes** and μ_e is the **mobility of electrons**. p and n are the concentration of holes and electrons in the semiconductor respectively. In the case of a single electron carrier dominated by electron, $p = 0$ and the Eq (5) reduces to (4).

Since the mobilities are dependent on the temperature we expect the hall coefficient to change with the respect to temperature. Therefore it may become zero and change the sign. This is called **Hall coefficient inversion**.

Data

Temperature vs Hall Coefficient

Thickness of the sample(t) = 0.5 mm
Probe current (I) = 0.5 mA
Residual magnetic field = 0.14 KG
Magnetic field = 3.14 KG Corrected magnetic field = 3.14 - 0.14 KG = 3.00 KG

S.No.	Heater Current (mA)	Temp (C)	Hall Voltage (mV)	Offset voltage (mV)	Corrected Hall Voltage (mV)	Hall coefficient (X103)($cm^3.C^{-1}$)
1	0	20.0	49.9	2.4	47.5	19.792
2	200	22.8	49.8	2.3	47.5	19.792
3	300	28.0	48.8	1.9	46.9	19.542
4	400	35.5	46.8	1.5	45.3	18.875
5	500	44.8	41.1	0.6	40.5	16.875
6	550	51.8	33.8	0.0	33.8	14.083
7	600	58.8	22.8	-0.6	23.4	9.750
8	650	63.8	15.5	-0.9	16.4	6.833
9	700	69.3	8.7	-1.1	9.8	4.083
10	750	75.8	3.1	-0.6	3.7	1.542
11	800	82.8	-1.3	-0.9	-0.4	-0.166
12	850	88.8	-2.8	-0.8	-2.0	-0.833
13	900	95.6	-3.3	-0.4	-2.9	-1.208

Table 1: The temperature and Hall Coefficient calculated. The hall coefficient inversion is seen here.

Magnetic Field vs Hall Voltage

Current in Amp	Magnetic Field H (Gauss)	Hall Voltage(mV)
0.00	120.80	0.13
0.10	230.00	2.10
0.21	343.00	3.90
0.30	429.00	5.70
0.40	535.00	7.60
0.50	637.00	9.60
0.60	752.00	11.60
0.71	863.00	13.20
0.80	965.00	15.10
...	...	...

Table 2: The Magnetic field and Hall voltage measured. The full table in [2]. The plot is linear and the slope gives us Hall coefficient.

Analysis and Results

Hall coefficient at room temperature

We first measure the hall coefficient and the charge concentration at room temperature. To do so, we plot the Hall Coefficient vs the magnetic field applied. From Eq (3), the plot is supposed to be straight line with slope related to the hall coefficient.

$$m(slope) = \frac{R_H I}{t} \tag{6}$$

The slope we get from the fit is:

$$m = 0.1539976\, V\ T^{-1} \tag{7}$$

Hence the **Hall Coefficient** for the sample is,

$$\begin{aligned} R_H &= \frac{m\ t}{I} \\ &= 0.019249709\ m^3\ C^{-1} \\ &= 19.2497 \times 10^3\ cm^3 C^{-1} \end{aligned} \tag{8}$$

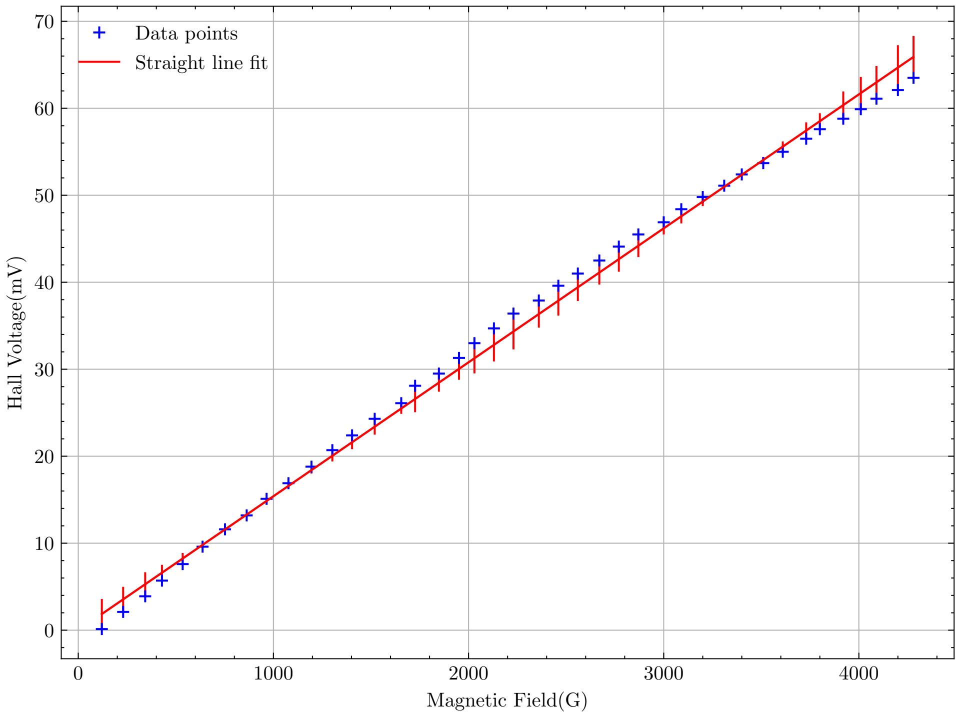

Figure 2: Hall Voltage vs Magnetic field measured at room temperature. The straight line fit gives us the hall coefficient. The error bars are indicated in blue.

The **concentration of the majority charge carrier** is then easily found from the hall coefficient.

$$\begin{aligned} n &= \frac{1}{e\ R_H} \\ &= \frac{1}{(1.6 *\ 10^{-19} *\ 0.0192)} m^{-3} \\ &= 3.247 * 10^{20}\ m^{-3} \\ &= 3.247 * 10^{14}\ cm^{-3} \end{aligned} \tag{9}$$

Hall coefficient vs Temperature

We expect the Hall coefficient to vary with temperature as the mobilities of the charge carriers change with temperature. From (5) we see that the Hall coefficient has a chance of turning negative at some point when $\mu_e^2 n > \mu_h^2 p$. This point of **Hall coefficient inversion** is clearly seen in the Fig 3. So the inversion takes place around the temperature of $\approx 82°C$. The plot is constructed from the data in table 1. The final column shows the hall coefficient measured in $(10^3\ cm^3\ C^{-1})$ using (4).

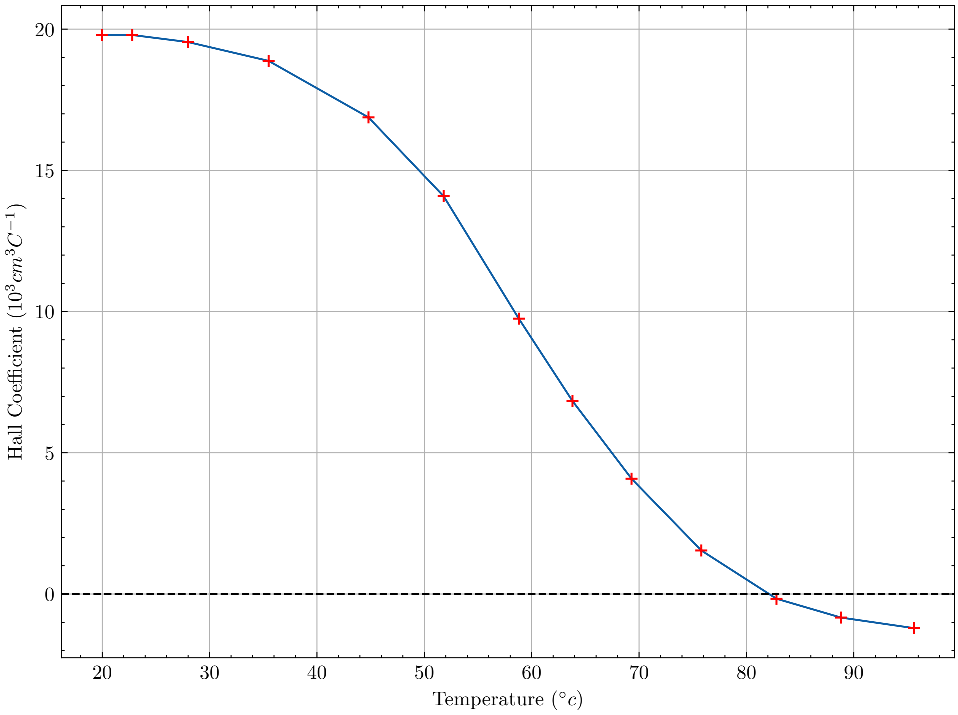

Figure 3: The Hall coefficient variation with temperature starting from room temperature.

Error analysis

The error in the Hall coefficient measurement and the subsequent vcarrier concentration is propogated from the straight line fit (Fig 2). The fractional error in hall voltage is averaged to get,

$$\frac{\Delta \bar{V}_H}{V_H} = 0.0344 \tag{10}$$

Therefore the error in Hall coefficient is,

$$\begin{aligned} \frac{\Delta R_H}{R_H} &= \frac{\Delta V_H}{V_H} \\ \Delta R_H &= R_H\ \frac{\Delta V_H}{V_H} \\ &= 0.00066199\, m^3\ C^{-1} \end{aligned} \tag{11}$$

The hall coefficient with the error at room temperature is

$$\boxed{R_H = (19.2497 \pm 0.6620) \times 10^3 \; cm^3 \; C^{-1}} \tag{12}$$

Similarly the majority career concentration error can be found.

$$\begin{aligned} \frac{\Delta n}{n} &= \frac{\Delta R_H}{R_H} \\ \Delta n &= n \, \frac{\Delta R_H}{R_H} \\ &= (3.247 \pm 0.1116) \times 10^{20} \, m^{-3} \end{aligned} \tag{13}$$

$$\boxed{n = (3.247 \pm 0.112) \times 10^{14} \, cm^{-3}} \tag{14}$$

Discussions and Conclusion

The hall coefficient and the majority carrier is thus determined from the hall effect experiment. The plot 3 also suggests the variation of hall coefficient with temperature and suggests **hall coefficient inversion** at some temperature about 82 degree Celsius. After this point the hall coefficient changes its sign.

In general, $\mu_e > \mu_h$ so the inversion happens only if **p>n**. This suggests that the sample is a **p-type semiconductor** and hall coefficient inversion will only happen with these semiconductors. So the **majority charge carrier** in our sample is essentially *holes*.

At the point of the inversion, we can find the mobilities of the charge carriers knowing the conductivity σ. Also we can derive the temperature dependence of the concentration of the charge carrier from which we can also derive the band gap of the semiconductor.

References

[1] Slides on "MEASUREMENT OF MAJORITY CARRIER CONCENTRATION – HALL EFFECT" .

[2] The data, https://github.com/pranavastro/physicslab3/blob/main/Exp3:%20Hall%20effect/HallcurrentBvV.xlsx

[3] Python files ,https://github.com/pranavastro/physicslab3/tree/main/Exp3:%20Hall%20effect

[4] Hall Effect, https://en.wikipedia.org/wiki/Hall_effect