

Homework – 2

HPC

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ps4521

1.

The code solutions are attached with the submission and can be found on this Github link. The changes for both parts are made and explained in the comments of val_test01_solved.cpp and val_test02_solved.cpp

2.

The computer architecture was found using the “uname -m” command, and it is found to be “x86_64” architecture. The processor used was a 2.1 Ghz 64 core AMD Opteron(TM) Processor 6272. This was run on the Crunchy1 CIMS server.

We have 6 possible orientations for the 3 loops in variables i,j and p, which I label as (i,j,p),(i,p,j),(j,i,p),(j,p,i),(p,i,j),(p,j,i). The following experiments were run under O3 Optimization flag.

For the loop ordering (i,j,p) and (i,p,j):

[ps4521@crunchy1 homework2]\$./a.out					[ps4521@crunchy1 homework2]\$./a.out				
Dimension	Time	Gflop/s	GB/s	Error	Dimension	Time	Gflop/s	GB/s	Error
16	2.973193	0.672678	0.399403	0.000000e+00	16	2.274297	0.879394	0.522140	0.000000e+00
64	2.606716	0.767310	0.401639	0.000000e+00	64	13.710911	0.145881	0.076359	0.000000e+00
112	2.289953	0.873650	0.448526	0.000000e+00	112	5.662570	0.353306	0.181385	0.000000e+00
160	2.741744	0.732030	0.372878	0.000000e+00	160	13.577847	0.147817	0.075294	0.000000e+00
208	2.753143	0.732165	0.371363	0.000000e+00	208	6.316444	0.319128	0.161866	0.000000e+00
256	2.994103	0.672410	0.340145	0.000000e+00	256	28.910209	0.069639	0.035227	0.000000e+00
304	2.832654	0.714101	0.360574	0.000000e+00	304	6.468008	0.312739	0.157913	0.000000e+00
352	2.958251	0.678189	0.341985	0.000000e+00	352	13.201152	0.151976	0.076635	0.000000e+00
400	3.404379	0.601578	0.303045	0.000000e+00	400	6.915192	0.296160	0.149190	0.000000e+00
448	4.308270	0.500890	0.252122	0.000000e+00	448	15.797993	0.136598	0.068756	0.000000e+00
496	4.778005	0.459696	0.231238	0.000000e+00	496	9.522929	0.230647	0.116021	0.000000e+00
544	5.039842	0.447206	0.224836	0.000000e+00	544	27.881971	0.080835	0.040641	0.000000e+00
592	4.620666	0.449015	0.225645	0.000000e+00	592	7.464346	0.277954	0.139681	0.000000e+00
640	4.663755	0.449670	0.225889	0.000000e+00	640	17.054706	0.122966	0.061771	0.000000e+00
688	5.775540	0.451089	0.226528	0.000000e+00	688	9.218892	0.282603	0.141918	0.000000e+00
736	4.120627	0.580526	0.291446	0.000000e+00	736	15.576359	0.153574	0.077100	0.000000e+00
784	5.149421	0.561489	0.281819	0.000000e+00	784	12.650476	0.228556	0.114715	0.000000e+00
832	4.212747	0.546845	0.274409	0.000000e+00	832	16.217371	0.142053	0.071282	0.000000e+00
880	4.139230	0.658550	0.330397	0.000000e+00	880	10.605698	0.257021	0.128949	0.000000e+00
928	5.086269	0.628499	0.315265	0.000000e+00	928	20.903885	0.152924	0.076709	0.000000e+00
976	5.147169	0.722505	0.362363	0.000000e+00	976	12.674851	0.293404	0.147153	0.000000e+00
1024	16.177480	0.132745	0.066567	0.000000e+00	1024	55.288037	0.038842	0.019478	0.000000e+00
1072	3.943284	0.624822	0.313285	0.000000e+00	1072	8.620860	0.205081	0.143300	0.000000e+00
1120	4.028942	0.607418	0.349643	0.000000e+00	1120	18.386984	0.152818	0.076614	0.000000e+00
1168	4.772452	0.667755	0.334735	0.000000e+00	1168	11.165829	0.285409	0.143071	0.000000e+00
1216	5.026142	0.715477	0.358621	0.000000e+00	1216	25.115091	0.143184	0.071769	0.000000e+00
1264	5.870759	0.687982	0.344807	0.000000e+00	1264	14.497455	0.278599	0.139630	0.000000e+00
1312	6.788550	0.665357	0.333439	0.000000e+00	1312	31.823503	0.141933	0.071129	0.000000e+00
1360	7.159673	0.702673	0.352112	0.000000e+00	1360	18.087035	0.278150	0.139382	0.000000e+00
1408	7.874046	0.708990	0.355250	0.000000e+00	1408	47.306224	0.118010	0.059131	0.000000e+00
1456	8.663282	0.712577	0.357023	0.000000e+00	1456	21.975054	0.200921	0.140750	0.000000e+00
1504	9.191700	0.740249	0.370863	0.000000e+00	1504	44.491159	0.152932	0.076619	0.000000e+00
1552	10.448967	0.715536	0.358460	0.000000e+00	1552	26.620961	0.280855	0.140699	0.000000e+00
1600	11.152608	0.734537	0.367957	0.000000e+00	1600	57.041843	0.143614	0.071942	0.000000e+00
1648	12.522906	0.714820	0.358060	0.000000e+00	1648	31.828644	0.281244	0.140878	0.000000e+00
1696	13.336398	0.731592	0.366443	0.000000e+00	1696	63.314137	0.154101	0.077187	0.000000e+00
1744	14.137721	0.750395	0.375843	0.000000e+00	1744	37.108829	0.285886	0.143189	0.000000e+00
1792	53.546906	0.214936	0.107648	0.000000e+00	1792	149.757378	0.076852	0.038490	0.000000e+00
1840	16.796718	0.741753	0.371481	0.000000e+00	1840	43.976560	0.283310	0.141886	0.000000e+00
1888	17.804408	0.755977	0.378589	0.000000e+00	1888	89.190382	0.150910	0.075575	0.000000e+00
1936	19.820841	0.732190	0.366662	0.000000e+00	1936	50.718339	0.286142	0.143293	0.000000e+00
1984	21.151891	0.738424	0.369770	0.000000e+00	1984	112.230194	0.139170	0.069690	0.000000e+00

For the loop ordering (j,i,p) and (j,p,i):

[ps4521@crunchyl homework2]\$./a.out					
Dimension	Time	Gflop/s	GB/s	Error	
16	2.956917	0.676381	0.401601	0.000000e+00	
64	2.539642	0.787575	0.412246	0.000000e+00	
112	2.283175	0.876244	0.449857	0.000000e+00	
160	2.733785	0.734162	0.373964	0.000000e+00	
208	2.771514	0.727312	0.368981	0.000000e+00	
256	2.834460	0.710282	0.359303	0.000000e+00	
304	2.873721	0.712876	0.359956	0.000000e+00	
352	2.944799	0.681287	0.343547	0.000000e+00	
400	3.472148	0.589837	0.297130	0.000000e+00	
448	4.582735	0.479257	0.241233	0.000000e+00	
496	4.982000	0.440873	0.221770	0.000000e+00	
544	5.326492	0.423139	0.212736	0.000000e+00	
592	3.364430	0.616671	0.309898	0.000000e+00	
640	5.160191	0.406410	0.204157	0.000000e+00	
688	4.761466	0.547160	0.274773	0.000000e+00	
736	4.560375	0.524547	0.263342	0.000000e+00	
784	5.675074	0.509481	0.255715	0.000000e+00	
832	3.715791	0.619981	0.311108	0.000000e+00	
880	4.606022	0.591810	0.296914	0.000000e+00	
928	4.611776	0.693163	0.347702	0.000000e+00	
976	5.717157	0.650473	0.326236	0.000000e+00	
1024	19.183823	0.111942	0.056135	0.000000e+00	
1072	3.912913	0.629672	0.315717	0.000000e+00	
1120	4.752576	0.591228	0.296406	0.000000e+00	
1168	5.057805	0.630081	0.315850	0.000000e+00	
1216	5.941120	0.605288	0.303391	0.000000e+00	
1264	7.120085	0.567265	0.284306	0.000000e+00	
1312	8.650558	0.522141	0.261667	0.000000e+00	
1360	8.634886	0.582626	0.291956	0.000000e+00	
1408	9.588473	0.582222	0.291731	0.000000e+00	
1456	10.509918	0.587374	0.294292	0.000000e+00	
1504	11.212687	0.606825	0.304018	0.000000e+00	
1552	11.934100	0.626490	0.313850	0.000000e+00	
1600	13.365370	0.612927	0.307038	0.000000e+00	
1648	14.442068	0.619829	0.310479	0.000000e+00	
1696	16.281288	0.599265	0.300162	0.000000e+00	
1744	17.125417	0.619481	0.318274	0.000000e+00	
1792	55.171146	0.208609	0.104479	0.000000e+00	
1840	20.336642	0.612638	0.306819	0.000000e+00	
1888	22.861355	0.588754	0.294845	0.000000e+00	
1936	23.184309	0.625968	0.313469	0.000000e+00	
1984	25.197985	0.619854	0.310395	0.000000e+00	

[ps4521@crunchyl homework2]\$ g++ --std=gnu++11 -O3 MMultl.cpp					
[ps4521@crunchyl homework2]\$./a.out					
Dimension	Time	Gflop/s	GB/s	Error	
16	1.220101	1.639212	0.973282	0.000000e+00	
64	1.273051	1.571153	0.822401	0.000000e+00	
112	1.080689	1.851243	0.950415	0.000000e+00	
160	1.056129	1.900374	0.968003	0.000000e+00	
208	1.026327	1.964049	0.996188	0.000000e+00	
256	1.073683	1.875102	0.948538	0.000000e+00	
304	0.999313	2.024191	1.022083	0.000000e+00	
352	0.957039	2.096313	1.057090	0.000000e+00	
400	1.002928	2.042022	1.028668	0.000000e+00	
448	1.081041	1.996196	1.004782	0.000000e+00	
496	1.171478	1.874923	0.943132	0.000000e+00	
544	1.374961	1.639210	0.824125	0.000000e+00	
592	1.423090	1.457917	0.732652	0.000000e+00	
640	1.532008	1.368891	0.687654	0.000000e+00	
688	1.879400	1.386233	0.696139	0.000000e+00	
736	1.727823	1.384476	0.695060	0.000000e+00	
784	2.093088	1.381376	0.693331	0.000000e+00	
832	1.675049	1.375316	0.690138	0.000000e+00	
880	1.978062	1.378060	0.691379	0.000000e+00	
928	2.318778	1.378621	0.691539	0.000000e+00	
976	2.724999	1.364718	0.684457	0.000000e+00	
1024	2.029546	1.058110	0.530605	0.000000e+00	
1072	2.224081	1.107806	0.555453	0.000000e+00	
1120	2.521305	1.114445	0.558715	0.000000e+00	
1168	2.871861	1.109673	0.556262	0.000000e+00	
1216	3.283619	1.095161	0.540931	0.000000e+00	
1264	3.687430	1.095336	0.540968	0.000000e+00	
1312	4.137560	1.091660	0.547078	0.000000e+00	
1360	4.613906	1.090380	0.546393	0.000000e+00	
1408	5.202766	1.073010	0.537648	0.000000e+00	
1456	5.677318	1.087354	0.544797	0.000000e+00	
1504	6.251953	1.088323	0.545247	0.000000e+00	
1552	6.857143	1.090340	0.546224	0.000000e+00	
1600	7.535107	1.087178	0.544608	0.000000e+00	
1648	8.225843	1.088231	0.545106	0.000000e+00	
1696	8.975438	1.087056	0.544489	0.000000e+00	
1744	9.771266	1.085722	0.543795	0.000000e+00	
1792	10.870199	1.058782	0.530277	0.000000e+00	
1840	11.477663	1.085500	0.543635	0.000000e+00	
1888	12.407125	1.084838	0.543281	0.000000e+00	
1936	13.370751	1.085401	0.543542	0.000000e+00	
1984	14.414167	1.083591	0.542615	0.000000e+00	

For the loop ordering (p,i,j) and (p,j,i):

[ps4521@crunchyl homework2]\$./a.out					
Dimension	Time	Gflop/s	GB/s	Error	
16	2.270707	0.880784	0.522966	0.000000e+00	
64	13.996622	0.142903	0.074801	0.000000e+00	
112	5.661695	0.353366	0.181415	0.000000e+00	
160	13.236721	0.151627	0.077235	0.000000e+00	
208	6.285407	0.320704	0.162665	0.000000e+00	
256	28.695259	0.070160	0.035491	0.000000e+00	
304	6.445700	0.313822	0.158459	0.000000e+00	
352	13.008960	0.154221	0.077768	0.000000e+00	
400	6.928764	0.295579	0.148898	0.000000e+00	
448	15.744649	0.137060	0.068989	0.000000e+00	
496	8.537916	0.257256	0.129406	0.000000e+00	
544	24.642906	0.091460	0.045982	0.000000e+00	
592	7.344673	0.282483	0.141957	0.000000e+00	
640	17.024280	0.123186	0.061882	0.000000e+00	
688	9.198465	0.283230	0.142233	0.000000e+00	
736	15.584360	0.153496	0.077061	0.000000e+00	
784	9.942607	0.290803	0.145958	0.000000e+00	
832	16.209792	0.142119	0.071316	0.000000e+00	
880	9.588688	0.284282	0.142625	0.000000e+00	
928	20.769843	0.153911	0.077204	0.000000e+00	
976	12.894220	0.288413	0.144650	0.000000e+00	
1024	58.720022	0.036572	0.018339	0.000000e+00	
1072	11.062373	0.222723	0.111673	0.000000e+00	
1120	18.004994	0.156060	0.078239	0.000000e+00	
1168	12.178201	0.261683	0.131178	0.000000e+00	
1216	25.293587	0.142174	0.071262	0.000000e+00	
1264	14.481393	0.278908	0.139785	0.000000e+00	
1312	29.221050	0.154574	0.077464	0.000000e+00	
1360	18.002749	0.279452	0.140034	0.000000e+00	
1408	47.330236	0.117950	0.059101	0.000000e+00	
1456	21.609949	0.285667	0.143128	0.000000e+00	
1504	44.406125	0.153225	0.076765	0.000000e+00	
1552	26.517492	0.281950	0.141248	0.000000e+00	
1600	57.256383	0.143076	0.071672	0.000000e+00	
1648	32.174767	0.278219	0.139363	0.000000e+00	
1696	63.585554	0.153444	0.076858	0.000000e+00	
1744	37.800167	0.280657	0.140570	0.000000e+00	
1792	151.293514	0.076072	0.038100	0.000000e+00	
1840	44.292641	0.281288	0.140874	0.000000e+00	
1888	87.150974	0.154441	0.077343	0.000000e+00	
1936	51.525417	0.281660	0.141048	0.000000e+00	
1984	189.943052	0.142065	0.071140	0.000000e+00	

[ps4521@crunchyl homework2]\$./a.out					
Dimension	Time	Gflop/s	GB/s	Error	
16	1.464560	1.365600	0.810825	0.000000e+00	
64	1.568489	1.275214	0.667495	0.000000e+00	
112	1.402219	1.426751	0.732484	0.000000e+00	
160	1.354584	1.481665	0.754723	0.000000e+00	
208	1.338889	1.514594	0.768220	0.000000e+00	
256	1.384806	1.453826	0.735431	0.000000e+00	
304	1.307408	1.547185	0.781227	0.000000e+00	
352	1.280672	1.566563	0.789957	0.000000e+00	
400	1.294512	1.582064	0.796965	0.000000e+00	
448	1.425636	1.513689	0.761913	0.000000e+00	
496	1.515076	1.449717	0.729243	0.000000e+00	
544	1.665523	1.353238	0.680350	0.000000e+00	
592	1.591632	1.303534	0.655070	0.000000e+00	
640	1.685078	1.244543	0.625188	0.000000e+00	
688	2.049028	1.271474	0.638509	0.000000e+00	
736	1.876984	1.274454	0.639824	0.000000e+00	
784	2.268419	1.274606	0.639742	0.000000e+00	
832	1.824207	1.262862	0.633708	0.000000e+00	
880	2.131845	1.278652	0.641505	0.000000e+00	
928	2.524578	1.266238	0.635166	0.000000e+00	
976	2.978092	1.248738	0.626288	0.000000e+00	
1024	2.305358	0.931519	0.467124	0.000000e+00	
1072	2.447226	1.006793	0.504805	0.000000e+00	
1120	2.779171	1.011041	0.506875	0.000000e+00	
1168	3.156607	1.009574	0.506083	0.000000e+00	
1216	3.670071	0.978243	0.490328	0.000000e+00	
1264	4.017684	1.005299	0.503843	0.000000e+00	
1312	4.532437	0.996551	0.499415	0.000000e+00	
1360	5.003920	1.005394	0.503806	0.000000e+00	
1408	5.795269	0.963306	0.482679	0.000000e+00	
1456	6.165803	1.001208	0.501636	0.000000e+00	
1504	6.832018	0.995920	0.498953	0.000000e+00	
1552	7.122770	1.008613	0.505281	0.000000e+00	
1600	8.225446	0.995934	0.498901	0.000000e+00	
1648	8.869943	1.009208	0.505523	0.000000e+00	
1696	9.702244	1.005623	0.503701	0.000000e+00	
1744	10.523032	1.008158	0.504946	0.000000e+00	
1792	12.322845	0.933970	0.467767	0.000000e+00	
1840	12.406323	1.004247	0.502942	0.000000e+00	
1888	13.487385	0.997949	0.499767	0.000000e+00	
1936	14.383196	1.008999	0.505281	0.000000e+00	
1984	15.640493	0.998636	0.500073	0.000000e+00	
[ps4521@crunchyl homework2]\$					

I have added the code for the blocked and blocked + parallelized matrix multiplication in the github repository. Here are the results for blocked matrix multiplication for Block sizes 16,64 and 256 respectively.

```
D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe
```

Dimension	Time	Gflop/s	GB/s	Error
16	1.020976	1.958912	1.163104	0.000000e+00
64	0.738212	2.709465	1.418236	0.000000e+00
112	0.666880	2.999965	1.540161	0.000000e+00
160	0.764546	2.625141	1.337181	0.000000e+00
208	0.799946	2.519867	1.278105	0.000000e+00
256	1.437199	1.400826	0.708621	0.000000e+00
304	1.223059	1.653887	0.835104	0.000000e+00
352	1.116402	1.797071	0.906194	0.000000e+00
400	0.920893	2.223929	1.120304	0.000000e+00
448	1.625687	1.327420	0.668154	0.000000e+00
496	1.175917	1.867845	0.939571	0.000000e+00
544	1.373525	1.640923	0.824986	0.000000e+00
592	1.289323	1.609176	0.808665	0.000000e+00
640	1.287115	1.629343	0.818490	0.000000e+00
688	1.337681	1.947613	0.978053	0.000000e+00
736	1.470763	1.626455	0.816542	0.000000e+00
784	1.350202	2.141415	1.074805	0.000000e+00
832	1.401323	1.643962	0.824945	0.000000e+00
880	1.383946	1.969649	0.988182	0.000000e+00
928	1.826533	1.750154	0.877906	0.000000e+00
976	1.758845	2.114374	1.060437	0.000000e+00
1024	2.432615	0.882788	0.442687	0.000000e+00
1072	1.212991	2.031220	1.018452	0.000000e+00
1120	1.576851	1.781941	0.893357	0.000000e+00
1168	1.807128	1.763476	0.884003	0.000000e+00
1216	2.084056	1.725525	0.864891	0.000000e+00
1264	1.969861	2.050386	1.027626	0.000000e+00
1312	2.834532	1.593493	0.798568	0.000000e+00
1360	3.701005	1.359337	0.681168	0.000000e+00
1408	3.808505	1.465829	0.734476	0.000000e+00
1456	2.997651	2.059364	1.031804	0.000000e+00
1504	4.372577	1.556095	0.779599	0.000000e+00
1552	2.981688	2.507512	1.256179	0.000000e+00
1600	3.750569	2.184202	1.094149	0.000000e+00
1648	3.554494	2.518395	1.261490	0.000000e+00
1696	5.678969	1.718059	0.860549	0.000000e+00
1744	4.093059	2.591919	1.298189	0.000000e+00
1792	6.911503	1.665220	0.834004	0.000000e+00
1840	4.852835	2.567367	1.285777	0.000000e+00
1888	6.424976	2.094906	1.049117	0.000000e+00
1936	6.005181	2.416684	1.210215	0.000000e+00
1984	10.660919	1.465077	0.733646	0.000000e+00

For Block size = 16

```
D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe
```

Dimension	Time	Gflop/s	GB/s	Error
64	1.188099	1.683495	0.881204	0.000000e+00
128	1.094885	1.827300	0.935063	0.000000e+00
192	1.773176	1.133627	0.575670	0.000000e+00
256	1.905283	1.056676	0.534529	0.000000e+00
320	1.377290	1.475082	0.744455	0.000000e+00
384	1.259021	1.619061	0.815855	0.000000e+00
448	1.420614	1.519040	0.764606	0.000000e+00
512	2.412762	0.890052	0.447634	0.000000e+00
576	1.198081	1.914090	0.962030	0.000000e+00
640	1.269114	1.652454	0.830100	0.000000e+00
704	1.050715	1.992436	1.000463	0.000000e+00
768	1.886868	1.440434	0.723030	0.000000e+00
832	1.488403	1.547781	0.776681	0.000000e+00
896	1.762900	1.632136	0.818801	0.000000e+00
960	1.883456	1.878963	0.942417	0.000000e+00
1024	2.499709	0.859093	0.430805	0.000000e+00
1088	1.515963	1.699136	0.851910	0.000000e+00
1152	1.852560	1.650498	0.827398	0.000000e+00
1216	1.906179	1.886545	0.945599	0.000000e+00
1280	3.149388	1.331784	0.667453	0.000000e+00
1344	2.666888	1.820635	0.912350	0.000000e+00
1408	4.011855	1.391530	0.697248	0.000000e+00
1472	3.862830	1.651383	0.827374	0.000000e+00
1536	7.705747	0.940565	0.471201	0.000000e+00
1600	5.046841	1.623194	0.813119	0.000000e+00
1664	5.440212	1.693847	0.848450	0.000000e+00
1728	6.059805	1.702953	0.852955	0.000000e+00
1792	8.711448	1.321155	0.661683	0.000000e+00
1856	6.810271	1.877585	0.940310	0.000000e+00
1920	8.303182	1.704862	0.853763	0.000000e+00
1984	8.987214	1.737921	0.870274	0.000000e+00

For Block size = 64

```
D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe
```

Dimension	Time	Gflop/s	GB/s	Error
256	2.649161	0.759964	0.384435	0.000000e+00
512	3.149741	0.681797	0.342896	0.000000e+00
768	3.048400	0.891585	0.447534	0.000000e+00
1024	3.396615	0.632242	0.317047	0.000000e+00
1280	4.463586	0.939671	0.470937	0.000000e+00
1536	12.520038	0.578893	0.290012	0.000000e+00
1792	15.247450	0.754826	0.378045	0.000000e+00

•

For Block size = 256

Now, the results below show the running time, flops and bandwidth for a parallelized and blocked implementation of Matrix Multiplication

D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe

Dimension	Time	Gflop/s	GB/s	Error
16	0.850567	2.351376	1.396129	0.000000e+00
64	0.605508	3.303274	1.729057	0.000000e+00
112	0.631444	3.168320	1.626593	0.000000e+00
160	0.678165	2.959517	1.507504	0.000000e+00
208	0.729324	2.763869	1.401866	0.000000e+00
256	1.191627	1.689511	0.854655	0.000000e+00
304	0.689745	2.932682	1.480812	0.000000e+00
352	0.869922	2.306247	1.162951	0.000000e+00
400	0.867374	2.361151	1.189430	0.000000e+00
448	1.030474	2.094152	1.054088	0.000000e+00
496	0.796999	2.755876	1.386272	0.000000e+00
544	0.922365	2.443554	1.228515	0.000000e+00
592	0.735739	2.819951	1.417121	0.000000e+00
640	1.021300	2.053414	1.031520	0.000000e+00
688	1.022366	2.548290	1.279701	0.000000e+00
736	1.046018	2.286892	1.148107	0.000000e+00
784	1.157969	2.496907	1.253231	0.000000e+00
832	1.113924	2.068113	1.037785	0.000000e+00
880	1.218630	2.236846	1.122236	0.000000e+00
928	1.804440	1.771583	0.888655	0.000000e+00
976	1.910722	1.946309	0.976146	0.000000e+00
1024	2.176754	0.986553	0.494722	0.000000e+00
1072	0.948125	2.598656	1.302964	0.000000e+00
1120	1.400735	2.005987	1.005680	0.000000e+00
1168	1.360960	2.341603	1.173809	0.000000e+00
1216	1.672590	2.150013	1.077659	0.000000e+00
1264	1.537472	2.627024	1.316629	0.000000e+00
1312	2.207382	2.046228	1.025454	0.000000e+00
1360	2.111464	2.382665	1.193960	0.000000e+00
1408	3.379070	1.652117	0.827819	0.000000e+00
1456	2.607782	2.367243	1.186060	0.000000e+00
1504	3.442407	1.976566	0.990254	0.000000e+00
1552	3.563105	2.098343	1.051200	0.000000e+00
1600	3.871088	2.116201	1.060084	0.000000e+00
1648	3.748036	2.388350	1.196349	0.000000e+00
1696	4.461379	2.186948	1.095408	0.000000e+00
1744	4.201569	2.524980	1.264662	0.000000e+00
1792	10.951904	1.050883	0.526321	0.000000e+00
1840	4.924299	2.530108	1.267116	0.000000e+00
1888	7.358802	1.829064	0.915985	0.000000e+00
1936	8.834260	1.642767	0.822656	0.000000e+00
1984	12.123497	1.288330	0.645139	0.000000e+00

For block size = 16, parallelized

D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe

Dimension	Time	Gflop/s	GB/s	Error
64	1.248990	1.601421	0.838244	0.000000e+00
128	1.272724	1.571969	0.804406	0.000000e+00
192	2.002005	1.004054	0.509871	0.000000e+00
256	1.816304	1.108441	0.560715	0.000000e+00
320	1.410078	1.440783	0.727145	0.000000e+00
384	1.448593	1.407181	0.709087	0.000000e+00
448	1.541514	1.399902	0.704638	0.000000e+00
512	3.438016	0.624629	0.314144	0.000000e+00
576	1.574077	1.456877	0.732232	0.000000e+00
640	1.282186	1.635606	0.821637	0.000000e+00
704	1.256946	1.665530	0.836314	0.000000e+00
768	2.964051	0.916958	0.460270	0.000000e+00
832	1.867652	1.233485	0.618966	0.000000e+00
896	1.854326	1.551665	0.778430	0.000000e+00
960	2.796777	1.265365	0.634660	0.000000e+00
1024	2.350677	0.913560	0.458118	0.000000e+00
1088	1.495510	1.722374	0.863562	0.000000e+00
1152	2.147778	1.423633	0.713670	0.000000e+00
1216	2.076979	1.731405	0.867838	0.000000e+00
1280	3.604601	1.163597	0.583162	0.000000e+00
1344	3.370111	1.440733	0.721975	0.000000e+00
1408	4.021931	1.388044	0.695501	0.000000e+00
1472	4.749345	1.343135	0.672936	0.000000e+00
1536	9.833706	0.737032	0.369236	0.000000e+00
1600	6.097013	1.343609	0.673064	0.000000e+00
1664	8.519148	1.081668	0.541809	0.000000e+00
1728	6.827320	1.511510	0.757067	0.000000e+00
1792	10.397422	1.106925	0.554389	0.000000e+00
1856	7.644595	1.672667	0.837685	0.000000e+00
1920	10.864596	1.302927	0.652481	0.000000e+00
1984	8.207546	1.903013	0.952945	0.000000e+00

For block size = 64, parallelized

D:\Pranav\Study\NYU\Sem4\HPX\homework2>a.exe

Dimension	Time	Gflop/s	GB/s	Error
256	1.724935	1.167155	0.590416	0.000000e+00
512	2.372039	0.905332	0.455318	0.000000e+00
768	2.542513	1.068985	0.536581	0.000000e+00
1024	2.505845	0.856990	0.429750	0.000000e+00
1280	4.087136	1.026221	0.514313	0.000000e+00
1536	12.315584	0.588503	0.294826	0.000000e+00
1792	13.797828	0.834129	0.417763	0.000000e+00

For block size = 256, parallelized

3.

I have chosen to modify the `sin4_vec()` function. I have added upto the 11th power in the Taylor expansion of $\sin(x)$ to obtain an accuracy of 12 digits. This is the output obtained.

```
[ps4521@crunchy1 homework2]$ g++ --std=gnu++11 -O3 -march=native fast-sin.cpp && ./a.out
Reference time: 58.7393
Taylor time:    4.4407      Error: 6.928125e-12
Intrin time:    2.3350      Error: 2.454130e-03
Vector time:    3.4166      Error: 6.928014e-12
```

We find the time for the vectorized implementation to be much lesser than the reference time and the inbuilt Taylor function time, and achieving 12 digits of accuracy, as asked by the question. Hence, the vectorized implementation is time-efficient.

4.

I have choosed to solve part b) here.

- i) I have the results for the optimization flags, O0,O1,O2 and O3 respectively, for the functions: Mult-add, sqrt, sin and cos in order.

```
D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O0 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
3.188970 seconds
10.525293 cycles/eval
0.627018 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O0 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
60.724148 seconds
200.391588 cycles/eval
0.032935 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O0 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
14.820940 seconds
48.910907 cycles/eval
0.134937 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O0 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
17.152951 seconds
56.606470 cycles/eval
0.116593 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>
```

O0 Flag - Mult-add, sqrt, sin and cos in order

```
D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O1 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
1.933195 seconds
6.381230 cycles/eval
1.034161 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O1 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
3.329795 seconds
10.990474 cycles/eval
0.600451 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O1 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
13.222621 seconds
43.636845 cycles/eval
0.151245 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O1 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
16.187275 seconds
53.420207 cycles/eval
0.123546 Gflop/s
```

O1 Flag - Mult-add, sqrt, sin and cos in order

```
D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
1.008395 seconds
3.329332 cycles/eval
1.982110 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
3.307585 seconds
10.917212 cycles/eval
0.604501 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
13.181376 seconds
43.499941 cycles/eval
0.151723 Gflop/s

D:\Pranav\Study\WVU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
15.520948 seconds
51.221310 cycles/eval
0.128851 Gflop/s
```

O2 Flag - Mult-add, sqrt, sin and cos in order

```
D:\Pranav\Study\WU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
1.008395 seconds
3.329332 cycles/eval
1.982110 Gflop/s

D:\Pranav\Study\WU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
3.307585 seconds
10.917212 cycles/eval
0.604501 Gflop/s

D:\Pranav\Study\WU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
13.181376 seconds
43.499941 cycles/eval
0.151723 Gflop/s

D:\Pranav\Study\WU\Sem4\HPX\homework2>g++ -std=c++11 -O3 -march=native lecture4/compute.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
15.520948 seconds
51.221310 cycles/eval
0.128851 Gflop/s
```

O3 Flag - Mult-add, sqrt, sin and cos in order

ii)

I have obtained:

```
D:\Pranav\Study\WU\Sem4\HPX\homework2>g++ -std=g++11 -O3 -march=native lecture4/compute-vec.cpp -ftree-vectorize -fopt-info-vec-optimized && a.exe -n 1000000000
lecture4/compute-vec.cpp:16:21: optimized: loop vectorized using 32 byte vectors
lecture4/compute-vec.cpp:16:21: optimized: loop versioned for vectorization because of possible aliasing
lecture4/compute-vec.cpp:52:21: optimized: loop vectorized using 16 byte vectors
lecture4/compute-vec.cpp:46:5: optimized: basic block part vectorized using 32 byte vectors
time = 1.196810
flop-rate = 6.680520 Gflop/s

time = 1.206389
flop-rate = 6.630037 Gflop/s

time = 1.204871
flop-rate = 6.638354 Gflop/s
```

iii)

I have ran compute-vec-pipe for m values 1,4,8 and 16 respectively, and the results are below in order of increasing M.

```
compute-vec-pipe.cpp:67: note: LOOP VECTORIZED.
compute-vec-pipe.cpp:61: note: vectorized 1 loops in function.
BASIC BLOCK VECTORIZED

compute-vec-pipe.cpp:61: note: basic block vectorized using SLP
time = 2.093515
flop-rate = 3.821135 Gflop/s

time = 4.186304
flop-rate = 1.910980 Gflop/s

time = 4.186855
flop-rate = 1.910727 Gflop/s
```


For M = 1

```
compute-vec-pipe.cpp:61: note: basic block vectorized using SLP
time = 11.724657
flop-rate = 2.729168 Gflop/s

time = 4.188103
flop-rate = 7.640641 Gflop/s

time = 4.188089
flop-rate = 7.640674 Gflop/s
```

For M = 4

```
compute-vec-pipe.cpp:67: note: LOOP VECTORIZED.
compute-vec-pipe.cpp:61: note: vectorized 2 loops in function.
time = 48.178204
flop-rate = 1.328399 Gflop/s

time = 6.701662
flop-rate = 9.549843 Gflop/s

time = 6.700816
flop-rate = 9.551049 Gflop/s
```

For M = 8

```
compute-vec-pipe.cpp:67: note: LOOP VECTORIZED.
compute-vec-pipe.cpp:61: note: vectorized 2 loops in function.
time = 60.729756
flop-rate = 2.107695 Gflop/s

time = 57.795974
flop-rate = 2.214686 Gflop/s

time = 57.787629
flop-rate = 2.215006 Gflop/s
```

For M = 16