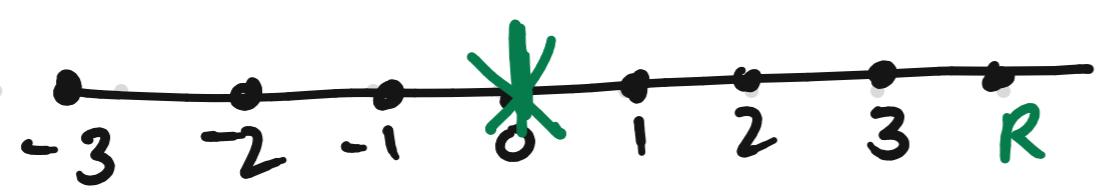


Lieb-Robinson Bound

$$H = \sum_{i \in L} [J_i Z_i Z_{i+1} + h_i X_i + g Z_i + K_i Y_{i+1}]$$



over long timescales H has an emergent notion of locality

[If we perturb 10^5 at $t=0$, will g detect it at (R) at later time t]

↳ $\langle [Z_R, X_0(t)] \rangle \neq 0$? How sensitive is correlation to earlier .

↳ approx hold if $R < \sqrt{t}$

→ Why should these bounds exist?

(1) choose $J, K, g \dots$ s.t. we have non-interacting system where

$$\epsilon(k) = 2\hbar(1 - \cos k) \rightarrow \text{how fast can wavepacket spread?}$$

\downarrow

$$\max(v_g) \rightarrow 2\hbar = \text{const.}$$

emergent speed limit on spread of info

$$(2) H = \sum_{i \in L} h (|i\rangle\langle i+1| + \text{h.c.}) \quad \& |\Psi(0)\rangle = |0\rangle$$

$$\text{At what } t=T \text{ is } P(x \geq R) \geq \frac{1}{2} \Rightarrow \sum_{|x| \geq R} |\langle x | \Psi(T) \rangle|^2$$

$$\langle x | e^{-iHt} | 0 \rangle = \langle x | \sum_{n=0}^{\infty} \frac{(-iHt)^n}{n!} | 0 \rangle$$

To reach R we need at least first R terms in perturbation series

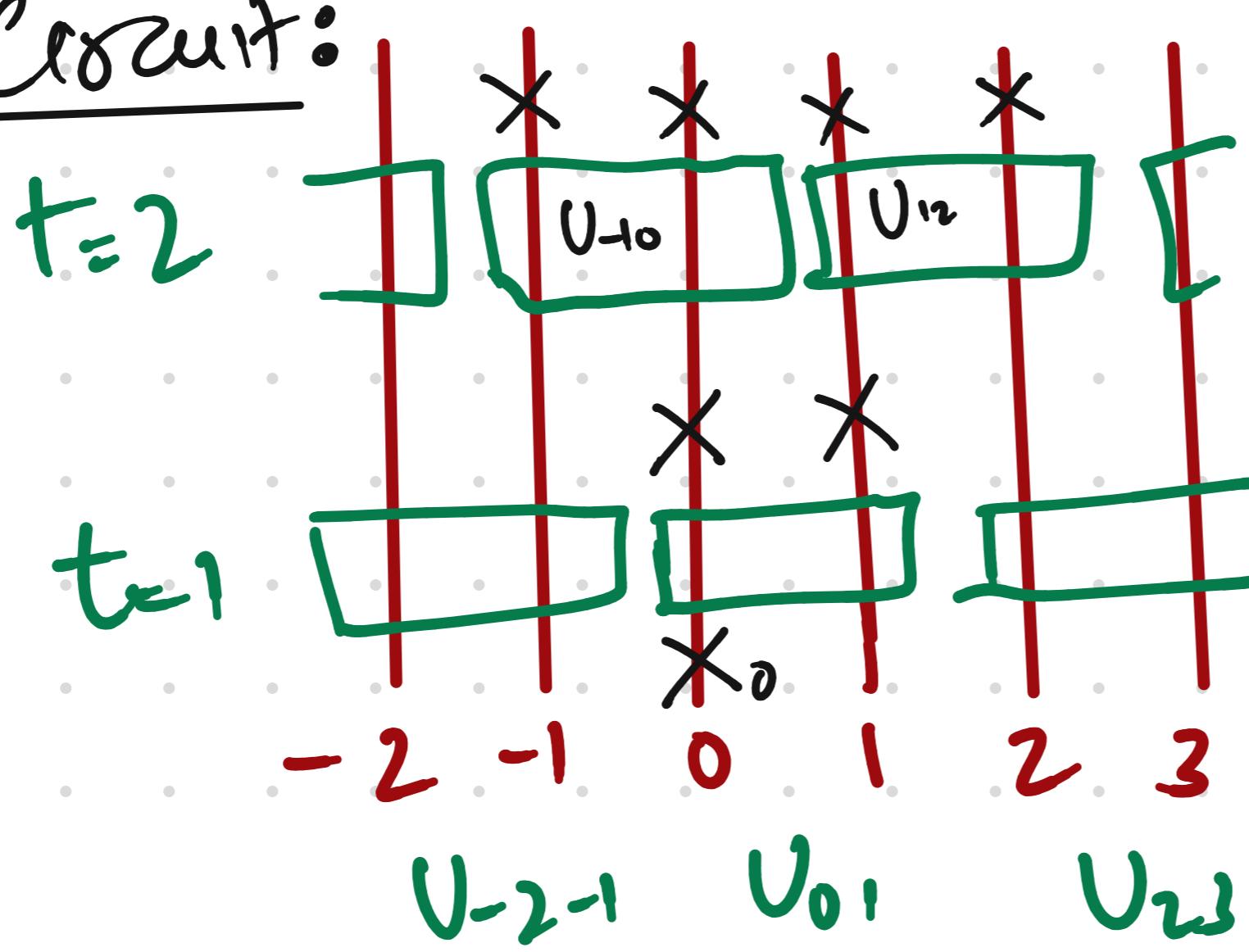
$$\text{short t: } \langle R | \Psi(T) \rangle \leq \frac{(-iht)^R}{R!} + \mathcal{O}(t^{R+1})$$

$$|\quad "|\quad | \sim \underbrace{\left(\frac{e^{ht}}{R}\right)^R}_{\text{should be 1}} \quad R! \geq \frac{R^R}{e^R} \quad (\text{stirling})$$

$$\text{should be 1} \rightarrow e^{hT} \geq R$$

$$"V_{LR}" = e^h$$

(3) Many Body Circuit:



$$|\Psi(0)\rangle = |0 \dots x_0 x_1 x_2 \dots \rangle$$

$$|\Psi(t)\rangle = U_{-2-1} U_{01} U_{23} \dots |\Psi(0)\rangle$$

↳ looks non locally different

How do we think of locality in this system?

→ Apply X at $t=0$ and then at what sites this operation changes the function.

$$X_0(1) = X_0 X, \quad X_0(2) = \sum X_1 X_0 X_1 X_2 \quad \left. \begin{array}{l} \text{after } t \text{ time steps it acts on } \leq 2t \text{ sites} \\ \text{distance } \leq t \text{ from } x=0; V_{LR} = 1 \end{array} \right\}$$

Perturbation A_0 at $t=0$: can only influence site ' R ' when coordinate $v \neq 0$

$$\langle \Psi(t) | B_\delta | \Psi(t) \rangle_{\text{pert}} : \quad \langle \Psi(t) \rangle_{\text{pert}} = U(t) \cdot e^{i A_0 t} |\Psi_0\rangle$$

$U^\dagger B_\delta U = B_\delta(t)$

Taylor exp. $\langle \Psi_0 | B_\delta(t) | \Psi_0 \rangle - i\varepsilon \langle \Psi_0 | [A_0, B_\delta(t)] | \Psi_0 \rangle + \mathcal{O}(\varepsilon^2)$

Need $\delta \leq t$ to not vanish
(in the sum X)

time it takes for a typical string to reach $\sigma \rightarrow$ butterfly velocity

$$T_\delta(-[A_0, B_\delta(t)]^2) = \sum_s |c_s|^2$$

↓
s is non trivial, at origin & anti-commutes with A_0

Proof L-R bound

Goal: for any $|\Psi_{1,2}\rangle$, $|\langle \Psi_1 | [A_0, B_\delta(t)] | \Psi_2 \rangle|$ should be small

$\substack{\text{sub} \\ \Psi_{1,2}} = \| [A_0, B_\delta(t)] \| = \lambda_{\max}$

↓
factors of decay

Try #1: $B_\delta(t) = B_\delta + i[H, B_\delta]t + \dots + \frac{(-it)^n}{n!} [H, \dots [H, B_\delta]]$

$e^{iHt} \cdot B_\delta e^{-iHt} =$

\downarrow
nested commutators

$O(n!)$ terms

can't sum it

Try #2: $\| [A_0, B_\delta(t)] \|$

$$= \| [A_0, U_\varepsilon^\dagger B_\delta(t-\varepsilon) U_\varepsilon] \|$$

$$= \| [U_\varepsilon A_0 U_\varepsilon^\dagger, B_\delta(t-\varepsilon)] \| = \| [A_0 d - \varepsilon, B_\delta(t-\varepsilon)] \|$$

$\approx A_0 - c\varepsilon \underbrace{[H, A_0]}_{A_{-1,0} + A_{0,1}} \rightarrow$ recursive relation can be obtained

$$C_{RS}(t) = \|\langle A_R, B_S(t) \rangle\| \rightarrow \frac{d}{dt} C_{RS} \leq \sum h_{RR'} C_{R'S}$$

End: $C_{\{0\}^R\{R'\}} \leq \left(\frac{2eht}{R}\right)^R$

Applications

- can't prepare interesting long range entanglement outside LR light cone
- Goal: product state \rightarrow Bellor

$$e^{-iHt} |...00...> = \frac{|00> + |11>}{\sqrt{2}} \otimes |\Psi>$$

or

$$e^{-iHt} (\alpha|10> + \beta|11>) \otimes |...> = \left[\alpha \left(\frac{|00> + |11>}{\sqrt{2}} \right) + \beta \left(\frac{|01> + |10>}{\sqrt{2}} \right) \right] \otimes |\Psi>$$

At time t : logical $X(t)(\alpha \leftrightarrow \beta)$ with X_R .

Swap order of operatrs. with $Z(0) = Z_0$.

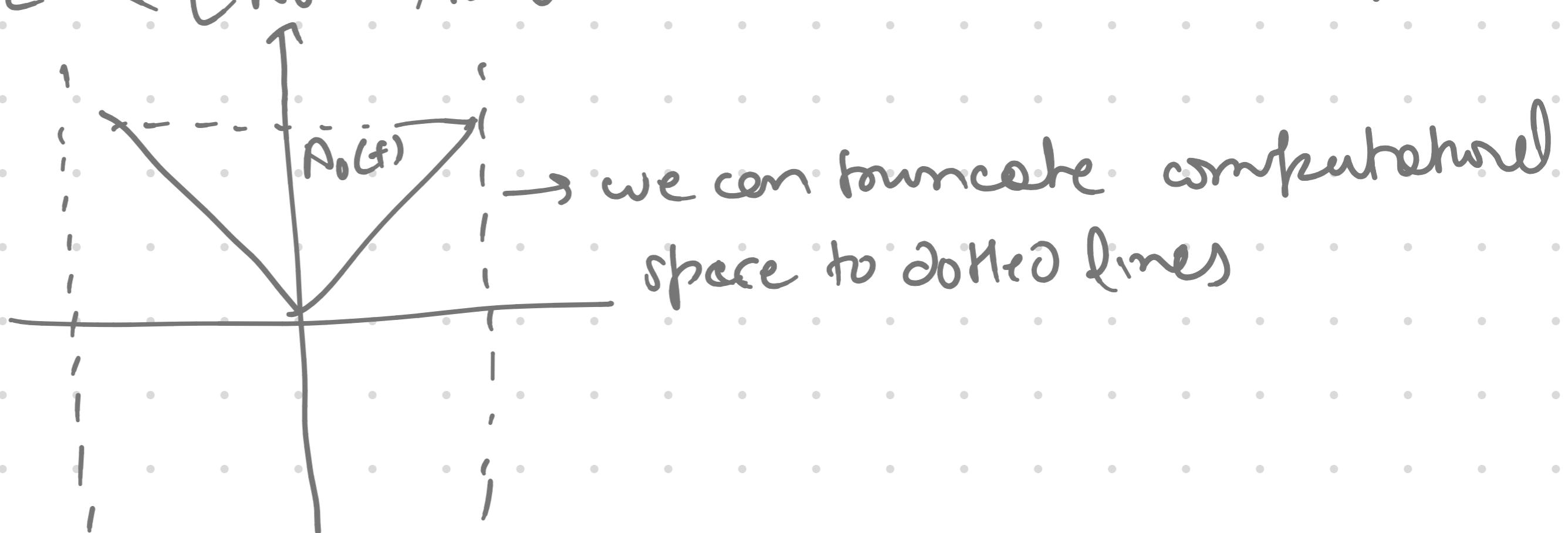
if we can do it then: $\|\langle X_R(t), Z_0 \rangle\| = 2$

$$\therefore XZ = -ZX$$

this means $R \leq V_{LR} t$

* Simulating Quantum Dynamics:

Let's say we calculate $\langle [A_0(t), B_r] \rangle$ on classical simulator up to an error ϵ . Then



** Quantum simulation: HHL Algo : uses LB to convert continuous time evolutions into discrete circuits

* H₀ gapped \rightarrow finite correlation length in ground state