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## Comparing and combining time series trajectories using Dynamic Time Warping

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### Abstract

This research proposes the application of dynamic time warping (DTW) algorithm to analyse multivariate data from virtual reality training simulators, to assess the skill level of trainees. We present results of DTW algorithm applied to trajectory data from a virtual reality haptic training simulator for epidural needle insertion. The proposed application of DTW algorithm serves two purposes, to enable (i) two trajectories to be compared as a similarity measure and also enables (ii) two or more trajectories to be combined together to produce a typical or representative average trajectory using a novel hierarchical DTW process.

Our experiments included 100 expert and 100 novice simulator recordings. The data consists of multivariate time series data-streams including multi-dimensional trajectories combined with force and pressure measurements.

Our results show that our proposed application of DTW provides a useful time-independent method for (i) comparing two trajectories by providing a similarity measure and (ii) combining two or more trajectories into one, showing higher performance compared to conventional methods such as linear mean. These results demonstrate that DTW can be useful within virtual reality training simulators to provide a component in an automated scoring and assessment feedback system.

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### 1. Introduction

Virtual Reality (VR) training simulators are increasingly used to enable trainees to practice tactile procedures with hand-eye coordination. In medical training Epidural is one procedure that requires accurate needle insertion and

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VR can provide an ideal platform for training and practice. VR simulators produce data accurately recording the movement, angle and forces on virtual tools, the speed and accelerations of the operators hand motion. This data is multivariate data-streams including trajectories combined with multi-dimensional force and pressure measurements.

Machine learning and data analytics processes are required to analyse the trajectory data and compute a skill level or score to feedback to the trainee. Data analytics could autonomously identify key areas which require practice and give indication of when a trainee is ready to progress to more difficult scenarios.

Two key components which are missing but are required in order to produce an automated scoring system are (i) comparison between two insertions such as a similarity measure and (ii) combination of numerous trajectories into one, to produce an average insertion representing an individual's typical technique.

This research proposes that Dynamic Time Warping (DTW)<sup>1</sup> algorithm could provide a method for achieving both comparison and combination between time series data. These two components can form an important part of an automated scoring and assessment system for virtual reality simulator training. This research presents results and performance assessment from implementations of both DTW and the conventional method using linear mean aiming to identify the optimum method for comparison and combination of trajectory data.

In order to test our implementations and compare performance of DTW with linear mean, we conducted a data collection trial which has recorded time series from 100 expert and 100 novices using a VR simulator for epidural needle insertion. This data revealed numerous problems and difficulties when producing a comparison between two insertions. Insertions commonly take different lengths of time and the landmarks in the insertions therefore occur at different locations in the time series. This makes a mean linear scaling difficult or implausible, as merging two time series with different insertion rates could cause landmarks to become merged, causing blurring and loss of data. To achieve higher accuracy when comparing or combining two trajectory time series, a non-linear approach is required and this research proposes DTW for this purpose.

## 2. Literature Review

For the analysis of time series or trajectory data, a variety of distance functions have been used to assess the similarity or dissimilarity between two time series<sup>2</sup>. The most popular distance function is Euclidean distance<sup>3</sup>. The longest common subsequence (LCSS) is another distance function, specialized for hierarchical data with high noise<sup>4</sup>. There are over 12 distance functions<sup>5</sup> including edit distance, longest common distance, dynamic time warping (DTW), Sequence Weighted Alignment model (Swale). Recently benefits of DTW were highlighted as a component in an automated clustering system for trajectory data, combined with particle swarm optimisation<sup>2</sup>. No previous studies have applied DTW to combine trajectories, or as a means of analysing data from VR simulators.

There is relatively little literature on comparing two trajectories but one use is for clustering<sup>6</sup>. Methods for clustering time series typically apply methods for clustering static data converted to work with time series. Comparison between multivariate time series trajectories is more complex. Time-series are dynamic data and a current research challenge is to recognize dynamic changes in time series<sup>7</sup>. This justifies our use of dynamic methods such as DTW in this research.

We previously conducted thorough literature surveys of virtual reality simulators<sup>8,9</sup>. Our surveys found that data analysis is lacking in VR simulators but it has potential to offer benefit enabling the autonomous generation of scores and assessment.

### 2.1. Time Series Data Representation

In our data collection trial (section 4), data was recorded in form of a set of  $N$  trajectories where  $N$  was 200, comprised of 100 experts and 100 novices:  $(t): t_1, t_2, t_3, \dots, t_i, \dots, t_N$ . Each trajectory  $t$  consists of  $P$  coordinates, where  $P$  is the length of trajectory which is relative to time taken as shown in Eq. 1. Each coordinate consists of 3 values  $(x, y, z)$  to represent 3D space and additionally 2 values force  $(f)$  and pressure  $(p)$  were recorded. The state of an object  $t_i$  varies from time step 0 to  $P$  and at timestep  $k$  it is represented by  $(x_{ik}, y_{ik}, z_{ik}, p_{ik}, f_{ik})$ .

$$\mathbf{t}_i = (\mathbf{x}_{i1}, \mathbf{y}_{i1}, \mathbf{z}_{i1}, \mathbf{p}_{i1}, \mathbf{f}_{i1}), (\mathbf{x}_{i2}, \mathbf{y}_{i2}, \mathbf{z}_{i2}, \mathbf{p}_{i2}, \mathbf{f}_{i2}), \dots, (\mathbf{x}_{ik}, \mathbf{y}_{ik}, \mathbf{z}_{ik}, \mathbf{p}_{ik}, \mathbf{f}_{ik}), \dots, (\mathbf{x}_{iP}, \mathbf{y}_{iP}, \mathbf{z}_{iP}, \mathbf{p}_{iP}, \mathbf{f}_{iP}) \quad (1)$$

## 2.2. Description of the Dynamic Time Warping Algorithm

If  $t_1$  and  $t_2$  are two trajectory time series with a number of coordinate measurements  $P$  and  $S$  respectively, the DTW algorithm begins by constructing a matrix of  $P \times S$  cells. The cells are monochrome coloured according to the difference between the coordinate from the two insertions, with white indicating no difference and black indicating a large difference. The path of minimum difference through the grid is then computed by aiming to avoid the black ‘hills’ and follow the white ‘valleys.’ This minimum path is represented by a red line showing which time in the data from  $t_1$  matches most closely with which time in the data from  $t_2$ .

A distance measure  $\delta$  is applied to identify the distance between two elements<sup>1</sup>. The measure can be the magnitude of the distance (Eq. 2) or the square of the distance (Eq. 3).

$$\delta(i, k) = |t_i - t_k| \quad (2)$$

$$\delta(i, k) = (t_i - t_k)^2 \quad (3)$$

Dynamic time warping between two trajectories  $t_i$  and  $t_k$  can be formally defined as a minimization of the cumulative distance over potential paths between two time series elements<sup>1</sup>, shown in Eq. 4. This makes use of  $W$  which is the sequence of grid points whereby each element of  $W$  referred to as  $w_s$  indicates a point  $(i, k)$  identifying one element from  $t_i$  and one element from  $t_k$  which are aligned.

$$DTW(t_i, t_k) = \min[\sum_{k=0}^P \delta(w_s)] \quad (4)$$

## 3. Problem definition

### 3.1. Comparison between two trajectories.

To assess similarity between two insertion trajectories, a number of similarity measures have been proposed for time series analysis. The appropriate choice of distance function depends on the type of data and applications; however previously Euclidean distance has commonly been used. This research has applied a more advanced distance function from literature: dynamic time warping (DTW). DTW has recently been shown to be an appropriate similarity measure for trajectories<sup>2</sup>. DTW works by expanding or contracting a given trajectory to minimise the difference between two trajectories. DTW has the benefit that two trajectories of the same shape will be classed as similar even if each trajectory took a different length of time or if one trajectory contains accelerations or decelerations. This is beneficial for VR simulations since it is more important to complete successful procedures irrespective of speed variations.

There are often several problems that occur when comparing two trajectories including: different sampling rates, outliers in the data particularly near the beginning or end or due to faulty sensors, different lengths of two trajectories, or similar motion in a different time or space<sup>4</sup>.

### 3.2. Combining two or more trajectories into one.

It is often required to combine two trajectories into one. This could enable a typical or ‘average’ trajectory to be generated from several separate trajectories. Generating an average trajectory from several of an expert’s trajectories could represent an ideal trajectory and could be useful for training. Combining several of a trainee’s trajectories could summarise the trainee’s general technique.

Combination of trajectories can be achieved by taking recordings from the two separate insertions. Conventionally, a linear mean combination method is used. However mean can cause landmarks to become blurred and lost. This research produces results analyzing an alternative combination method using dynamic time warping.

When comparing the performance of various methods for combining two trajectories into one, we use the landmark preservation as a metric. Changes in direction in a trajectory can provide landmarks.

#### 4. Experimental Analysis

Data was collected using a computer based haptic VR simulator that we have developed for the Epidural Procedure<sup>10</sup>. The simulator uses a haptic device which measures needle motion with 3 degrees of freedom and measures the force applied onto the needle by the operators hand. The pressure was measured separately via a wireless microcontroller, giving 5 measurements over time during each insertion.

The simulator contains 5 virtual tissue layers, each with a different stiffness, requiring a different force to pierce. These tissue layers create landmarks within the data which are useful for comparison.

An expert's performance was recorded whilst the needle insertion was repeated 100 times using the VR simulator, each insertion took around 10 seconds to complete. Separately 100 insertions were recorded in the same way from a novice trainee.

##### 4.1. Comparison of trajectories using DTW

Fifty pairs of the expert trajectories were compared using DTW. Fig.1 shows the results from DTW combination of 3 expert trajectory pairs. This is shown as a red line and the total sum of cumulative difference is the sum of all differences along the shortest line, which gives an overall similarity measure between two insertions.

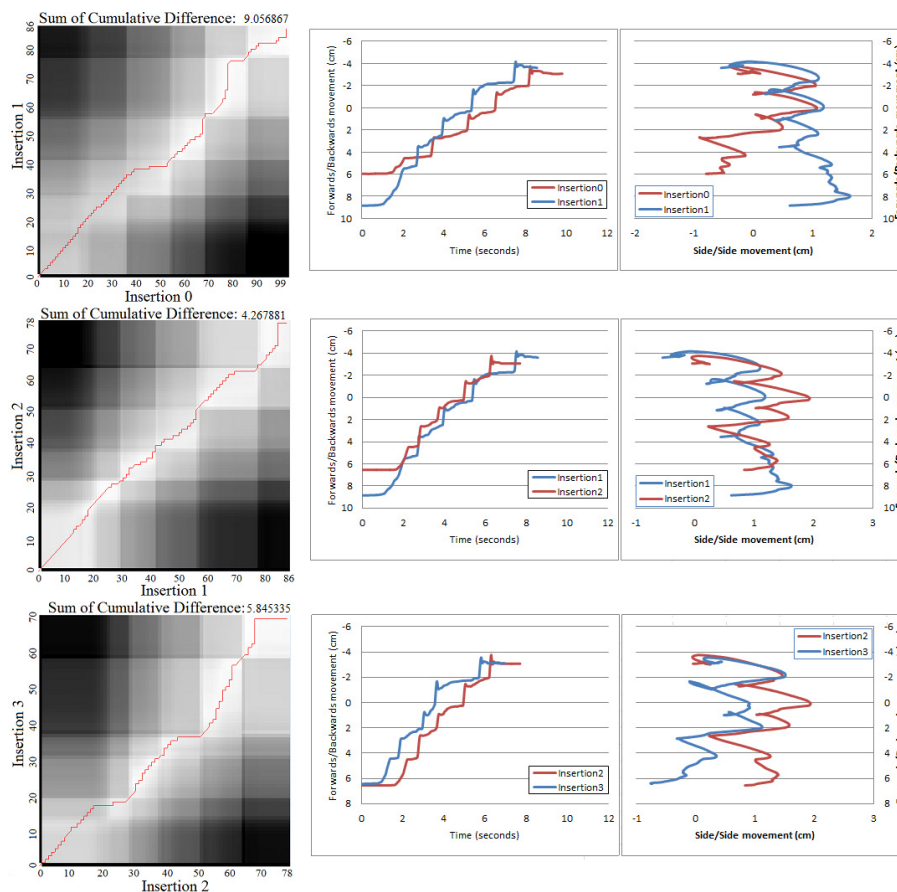


Fig. 1. DTW Comparison between three pairs of trajectories.

Cumulative difference was generally between 1 and 9 for an expert with high similarity between repetitions, but raised to over 80 when comparing between two trainees who were using different techniques. The DTW technique was easily able to differentiate between two trainees due to differences in their techniques.

The graphs on the central column show that the forward/backward motion has a one way direction. Each insertion exhibits a series of ‘steps’ which is caused by the tougher tissue layers which the needle passes through which cause sudden needle movement and this also causes the checkerboard appearance on the DTW matrices. The forwards/backwards movement is the largest motion, ranging to about 10cm in total compared to the side/side motion on a scale of around 3cm. The graphs in the right column of Fig. 1 show that side movement occurs when the tougher layers are encountered. Each successive insertion took less time than the previous insertion, suggesting that the trainee was becoming more familiar with the technique.

Insertion 1 appears to be an anomaly, beginning with forwards movement of 9cm, whereas all other insertions begin around 6-7cm. This caused a large cumulative difference of over 9 in the DTW graph. This is because there is far less white area in the first half of the grid, and the majority of the path is grey, showing that irrespective of time difference, there was no time when the insertions were at the same location. Although in the middle graph insertions 0 and 1 seem to somewhat match, significant difference is visible on the right hand graph where the side/side motions are significantly separated from each other in a way which cannot be corrected by time warping.

Several graphs from the right column (insertions 0&1 and insertions 2&3) show some insertions started from quite a different lateral position but then aligned much more closely towards the end of the insertion.

Comparison between insertions 2 and 3 shows that the depth motion seems very similar between the two, except that insertion 2 began 1 second later – this time difference was removed by the dynamic time warping. However the side/side difference between insertion 2 and 3 increased cumulative difference to over 5.

There are interesting features in the comparison between insertions 2 and 3, there is a dramatic ‘step’ shape near the beginning of the DTW matrix on the left whereby Insertion 2 moves from 20 to 30 during which insertion 3 does not progress at all. This can be explained by the depth of around 4cm, the side/side positions are quite different exhibiting exaggerated zigzag motions. During one such zigzag, insertion 2 reached its rightmost point, and then turns bringing it closer to the position of insertion 3. At this point, insertion 3 is stopped because if allowed to continue, the zigzag would again lead further away from insertion 2. Meanwhile, insertion 2 continues, because its zigzag motion causes it to move back closer to insertion 3 which then proceeds.

#### 4.2. Removing sections of the trajectory data

A test was conducted to remove (a) the first 4 seconds and (b) last 4 seconds of data from a 10 second insertion. Fig. 2 shows the result. When the first 4 seconds was removed (Fig. 2a) there was initially difference, but then the line converged onto the path with no difference. When the last 4 seconds was removed (Fig. 2b) there was no difference in the data, until a large difference was detected in the final 4 seconds. The difference between the two images demonstrates the difference depending on which corner is the start point for drawing the shortest route line.

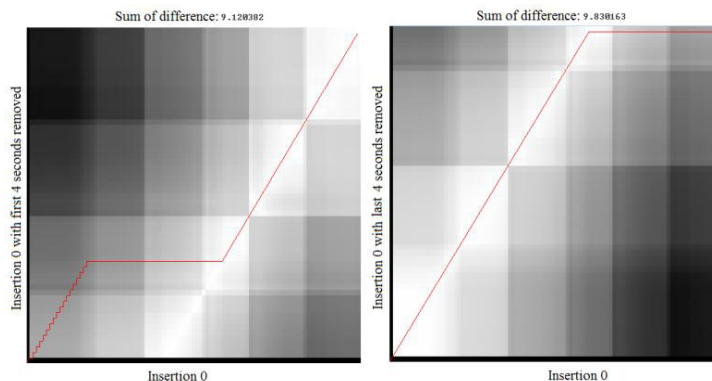


Fig. 2. When removing the first or last section of a trajectory, the shortest path is found.

#### 4.3. Compressed trajectory (one third of data)

A test was conducted to compare insertion 0 with a compressed version of insertion 0 containing 1/3 of the data. This was achieved by removing every 2nd and 3rd row, for example keeping all rows evenly divisible by 3 using Modulo, such that  $\text{row mod } 3 = 0$ .

When DTW algorithm compared one trajectory to a compressed version of itself, it did identify the files to be slightly different. The data is shown in Table. 1. There was no exact path with 0 difference between insertion 0 and the compressed insertion 0.

Table 1. The mappings made by the DTW algorithm to match (a) trajectory and (b) expanded version of the trajectory.

	A	B	C	D	E			A	B	C	D	E
1	-0.15883	0.112187	1.194478	0	104		1	-0.15883	0.112187	1.194478	0	104
2	-0.15883	0.112187	1.194478	0	104		2	-0.15267	0.100633	1.196778	0	104
3	-0.15425	0.101047	1.197078	0	104		3	-0.15221	0.114721	1.191733	0	104
4	-0.15267	0.100633	1.196778	0	104		4	-0.15021	0.112777	1.191849	0	104
5	-0.15267	0.100633	1.196778	0	104		5	-0.14704	0.111952	1.191245	0	104
6	-0.15109	0.100219	1.196477	0	104		6	-0.10331	0.12338	1.170141	0	104
7	-0.15221	0.114721	1.191733	0	104		7	-0.10906	0.193022	1.071529	0	136
8	-0.15063	0.114309	1.191432	0	104		8	-0.09668	0.111285	0.982656	0	178

DTW algorithm processes every line from insertion0 which identifies several differences between the files. When the DTW algorithm was run, the green lines in Table 1 show the actual closest mappings found. In first 2 iterations the algorithm found exact matches (shown as green lines) between the files, until the algorithm reaches line 2 (left) and line 1 (right). However on the next step, it is not possible to find an exact match as stepping forward to line 3 on the left or stepping to line 2 in the right file, or diagonally on both line 3 and 2, would all cause some difference between the two measurements. The rows cannot be exactly matched after some of the rows had been removed. The DTW matrix and comparison output in this case is shown in Fig. 3a, resulting in sum difference of 0.8.

Although the difference was 0.8 this is still relatively small difference, which suggests that compression could be used to shrink the size of files with minimal effect on data accuracy, which is useful because DTW has quadratic complexity which makes it time consuming to apply for large datasets<sup>3</sup>.

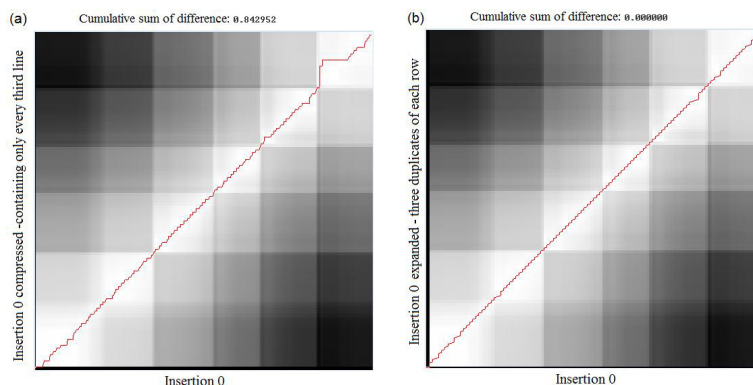


Fig. 3. DTW comparison of a trajectory and (a) compressed version of the same trajectory (b) expanded version of the same trajectory..

#### 4.4. Expanded trajectory (three times the data)

An expanded copy of Insertion0 data was generated with every line duplicated 3 times, to create exactly 3 times the number of measurements. The DTW algorithm was applied to produce a comparison between these two files.



The result was that the DTW algorithm found an exact match between insertion 0 and the expanded version of insertion0 containing 3 duplicates of every measurement. The Table 2 shows green lines indicating the actual mappings found by the algorithm. On some occasions such as lines 1&2 and lines 4&5 there are two consecutive lines in the original file which by chance are identical. In this case, six copies were produced in the expanded file. The result of this on the DTW algorithm was that in this case it processes the first line only once and the second line 5 times, which again identifies no difference between the two files.

Table. 2. Comparison between a trajectory and an expanded version of the same trajectory.

	A	B	C	D	E			A	B	C	D	E
1	-0.15883	0.112187	1.194478	0	104		1	-0.15883	0.112187	1.194478	0	104
2	-0.15883	0.112187	1.194478	0	104		2	-0.15883	0.112187	1.194478	0	104
3	-0.15425	0.101047	1.197078	0	104		3	-0.15883	0.112187	1.194478	0	104
4	-0.15267	0.100633	1.196778	0	104		4	-0.15883	0.112187	1.194478	0	104
5	-0.15267	0.100633	1.196778	0	104		5	-0.15883	0.112187	1.194478	0	104
6	-0.15109	0.100219	1.196477	0	104		6	-0.15883	0.112187	1.194478	0	104
7	-0.15221	0.114721	1.191733	0	104		7	-0.15425	0.101047	1.197078	0	104
8	-0.15063	0.114309	1.191432	0	104		8	-0.15425	0.101047	1.197078	0	104

The DTW output matrix is shown in Fig. 3b. The red line is a very direct diagonal. The line appears to contain many small steps – this effect is caused by the slight delays as each one line in the original Insertion0 is matched to several lines in the expanded file. The occasional slightly larger steps are caused by presence of multiple identical lines in the original file which were in the original data by chance.

#### 4.5. Spatial Comparison between twenty trajectories

A collection of 20 insertions was plotted on three axes to identify similarity between the data. Fig. 4. shows a comparison of twenty insertions on three axes. There are some similarities visible especially in the central graph.



Fig. 4. Plot of 20 insertions on three axes: (a) X and Y, (b) X and Z, (c) Y and Z.

### 5. Combining multiple expert trajectories into a single ‘typical expert trajectory’

When an expert performs a simulated procedure several times, the trajectories are still slightly different every time. It is often required to combine several of these trajectories together to create a new trajectory which could represent all the trajectories combined as a single typical or average trajectory. This typical expert trajectory could be useful to compare with trainee trajectories. This research presents results which compare the accuracy of two combination methods: Mean (Section 5.1) and DTW (Section 5.2)

#### 5.1. Combination of trajectories using mean

In order to generate a mean ‘average’ insertion, three time series were linearly stretched to the same duration of time. Insertions contain 5 landmarks which are caused by 5 tissue layers, but the landmarks will not be in the same

place in time, or in time relative to each other. The mean insertion was calculated by summing the 3 pressures at every time step and dividing by 3.

Results of linear mean combination shows in Fig. 5 that the landmarks are lost when this approach is used. For example Patient 037 had a very increased pressure trace, with sudden increase at the beginning and a drop at the end which is no longer visible by viewing the Mean insertion. Using linear mean to combine two trajectories produced inaccurate results causing the landmarks to become blurred.

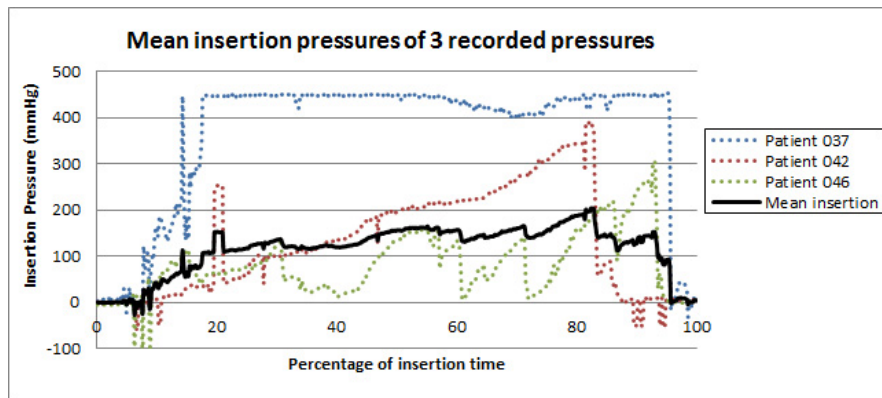


Fig. 5. Combining 3 trajectories to create a combined trajectory using Mean<sup>6</sup>.

## 5.2. Combination of trajectories using Dynamic Time Warping

Dynamic Time Warping was used to combine two insertions. The combined trajectory was generated by using the DTW matrix to identify the shortest path between insertions, which identified which data to write into the new file at each time-step. Starting on the first data point on both trajectories, the DTW process iteratively steps forward whichever trajectory causes the least difference to occur. To create the combined trajectory, at each time step the mean of the two current points in the two time series was computed which produced a new point. That new point forms part of the new trajectory and is written into a separate trajectory file.

Results in Fig. 6 show the combined trajectories generated by using both DTW and Mean. Generating a combined trajectory by DTW provides a better method than Mean to enable several insertions to be combined in a time neutral manner, restoring all landmarks. This shows that DTW preserves more landmarks than using a mean.

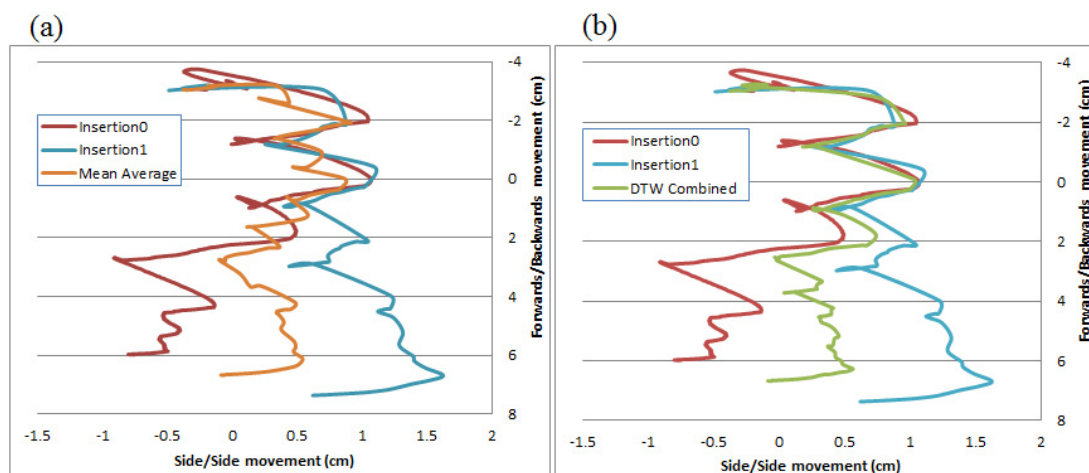


Fig. 6. Combining 2 trajectories using (a) Mean and (b) DTW.



Looking at the two insertions 0 and 1 side by side (Fig. 6), it is clear that they share some events caused by the tissue layers: at depths of -1, 1 and 3 cm the needle moves to the left during both insertions. The mean average insertion in Fig. 6a was generated by linearly combining the two insertions. The linear mean did not produce a good combination because the landmarks become mixed up and the mean insertion now in Fig. 6a now has at least 5 separate landmarks instead of the initial 3. In this case although the landmarks occurred at the same depth of both insertions, they occurred at different times, so have not been aligned during the combination. In Fig 6b, the DTW combination has retained the three landmarks clearly, producing a more effective combination method than mean.

### 5.3. Combination of 8 trajectories into 1 using DTW and Mean

Rather than combining together 2 insertions into 1 it is common to collect a larger number of trajectories which would need to be combined together into one. Our experiment collected 8 expert trajectories, and combined them together to produce a single combined trajectory. Both DTW and mean methods were used to assess their performance. Mean was generated by stretching all 8 insertions linearly to the same length, summing all 8 data points and dividing by 8. The DTW combination applied a hierarchical method since only 2 can be combined at a time. Initially, insertions 0&1, 2&3, 4&5, 6&7 were combined. Then in the second round, 0-3 and 4-7 were combined. Finally 0-8 were combined. Therefore, a total of 7 separate pair combinations was required to combine the 8 insertions hierarchically with DTW. The novel hierarchical DTW process is depicted in Fig. 7.

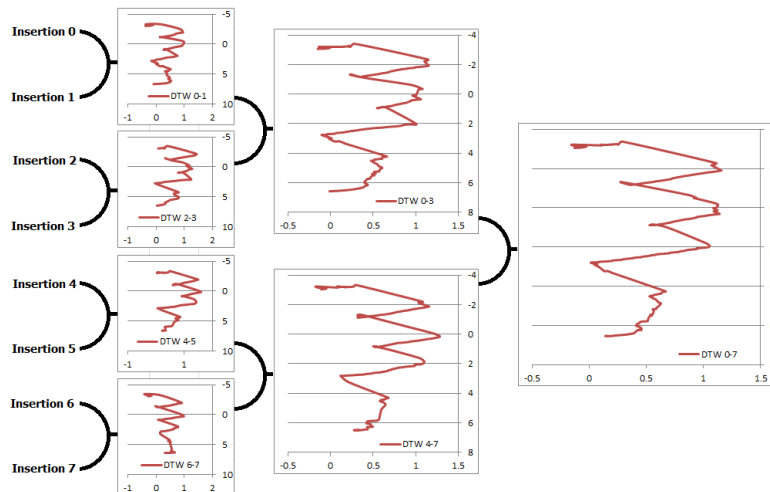


Fig. 7. Combining 8 trajectories into one using DTW with a hierarchical layered process.

The result of combining 8 trajectories into 1 is shown in Fig. 8. This was achieved using both Mean and separately using DTW, to compare the results. In Fig. 8a mean average insertion was produced combining eight separate insertions into one. There is significant loss of the landmarks. The majority of the insertion resembles a straight line with many small zigzags. It is unclear which part represents the three tissue layers, because all landmarks which occurred at different time stages in each insertion are all merged together. Between depths of -2cm to -3cm there is a large side movement. This is because the end of all insertions was time synchronised by the end of the file but the majority of the insertion wasn't.

In Fig. 8b, the DTW has combined insertions 0-7 in a much better way because the 3 tissue layer landmarks are preserved clearly. In Fig. 8c, a comparison between Mean and DTW for combining insertions 0-7 shows that DTW preserves the landmarks far better than using Mean. As the number of combined insertions increases, the benefits of DTW over Mean are increased further. The file length of a DTW combination trajectory is at least as long as the

longest of the two combined trajectories and at most could be as long as the sum of the two trajectory lengths such that:  $t_1 + t_2 \geq t_3 \geq \max(t_1, t_2)$ .

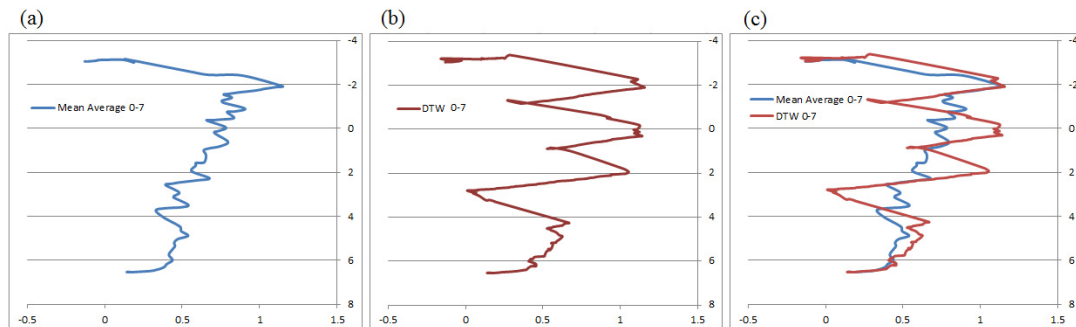


Fig. 8. Combination of 8 trajectories using (a) Mean (b) Dynamic Time Warp (c) both shown together.

## 6. Conclusion

Dynamic Time Warping (DTW) is a useful method which can provide two main components within a virtual reality training assessment system: (i) Comparison between two trajectories, which can be useful to compare the trainee's insertion to the expert insertion, or to compare a trainee's insertion to their own previous combined insertions, to assess a trainee's consistency. (ii) Combining two or more expert trajectories into a single 'combined' trajectory – this retains the landmarks in the data, which a mean average does not.

One drawback of using DTW to combine two trajectories is that the length of the new trajectory is generally larger than either of the initial ones. However section 4.3 has shown that the length of trajectories can be reduced without losing much information. There are many future work directions, including clustering and classification of trajectories. Clustering could identify groups of trajectories which share similar features. This could indicate which trajectories were performed by the same person, or whether they contain a similar shape or a particular type of error.

Trajectories may often have variations in timing, which is not an important aspect. The important key for scoring the accuracy of positioning is to assess how closely the ideal trajectory was followed irrespective of differences in accelerations, decelerations or timing delays. The data from this study demonstrated that DTW provides a mechanism for achieving robust comparison of trajectory data irrespective of variations in insertion timing.

Typically the DTW algorithm is used for comparing two trajectories. This research proposes the new step whereby the result of DTW comparisons can in turn be used to generate a new combined insertion, combining the two insertions in a way which minimizes difference between the two files, and can be applied to more than two files.

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