

DP Practice Problems

1. (True/False) The running time of a dynamic programming algorithm is always $\Theta(P)$ where P is the number of subproblems.

Solution: False. The running time of a dynamic program is the number of subproblems times the time per subproblem. This would only be true if the time per subproblem is $O(1)$.

2. You just started a consulting business where you collect a fee for completing various types of projects (the fee is different for each project). You can select in advance the projects you will work on during some finite time period. You work on only one project at a time and once you start a project it must be completed to receive your fee. There is a set of n projects p_1, p_2, \dots, p_n each with a duration d_1, d_2, \dots, d_n (in days) and you receive the fee f_1, f_2, \dots, f_n (in dollars) associated with it. That is project p_i takes d_i days and you collect f_i dollars after it is completed.

Each of the n projects must be completed in the next D days or you lose its contract. Unfortunately, you do not have enough time to complete all the projects. Your goal is to select a subset S of the projects to complete that will maximize the total fees you earn in D days.

(a) What type of algorithm would you use to solve this problem? Divide and Conquer, Greedy or **Dynamic Programming**. Why?

(b) Describe the algorithm verbally. If you select a DP algorithm give the formula used to fill the table or array.

(c) What is the running time of your algorithm?

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3. Product Sum

Given a list of n integers, v_1, \dots, v_n , the product-sum is the largest sum that can be formed by multiplying adjacent elements in the list. Each element can be matched with at most one of its neighbors.

For example, given the list 1, 2, 3, 1 the product sum is $8 = 1 + (2 \times 3) + 1$, and given the list 2, 2, 1, 3, 2, 1, 2, 2, 1, 2 the product sum is $19 = (2 \times 2) + 1 + (3 \times 2) + 1 + (2 \times 2) + 1 + 2$.

4. Canoe Rental Problem: There are n trading posts numbered 1 to n as you travel downstream. At any trading post i you can rent a canoe to be returned at any of the downstream trading posts j , where $j \geq i$. You are given an array $R[i, j]$ defining the costs of a canoe which is picked up at post i and dropped off at post j , for $1 \leq i \leq j \leq n$. Assume that $R[i, i] = 0$ and that you can't take a canoe upriver. Your problem is to determine a sequence of rentals which start at post 1 and end at post n , and that has the minimum total cost.

(a) Describe verbally and give pseudo code for a DP algorithm to compute the cost of the cheapest sequence of canoe rentals from trading post 1 to n . Give the recursive formula you used to fill in the table or array.

b) Describe the code for Print Sequence algorithm that prints the sequence of trading posts visited.

c) What is the running time of your algorithm to find the minimum cost and to find the sequence?