Graph Algorithm – Practice Problems

1. A Hamiltonian path in a graph G=(V,E) is a simple path that includes every vertex in V. Design an algorithm to determine if a directed acyclic graph (DAG) G has a Hamiltonian path. Your algorithm should run in O(V+E). Provide a written description of your algorithm, pseudocode and an explanation of the running time.

2. Below is a list of courses and prerequisites for a factious CS degree.

Course	Prerequisite
CS 150	None
CS 151	CS 150
CS 221	CS 151
CS 222	CS 221
CS 325	CS 221
CS 351	CS 151
CS 370	CS 151
CS 375	CS 151, CS 222
CS 401	CS 375, CS 351, CS 325, CS 222
CS 425	CS 325, CS 222
MATH 200	None
MATH 201	MATH 200

(a) Draw a directed acyclic graph (DAG) that represents the precedence among the courses.

(b) Give a topological sort of the graph.

(c) Design an algorithm to find the longest path in an unweighted DAG where the length of a path is determined by the number of edges. What is the running time of the algorithm?

(d) Find the longest path in the DAG from part a) or part b).

(e) If you are allowed to take multiple courses at one time as long as there is no prerequisite conflict, find an order in which all the classes can be taken in the fewest number of terms.

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3. One can model a maze by having a vertex for a starting point, a finishing point, dead ends and all the points in the maze where more than one path can be taken, and then connecting the vertices according to the paths in the maze.

(a) Construct such a graph for the maze below:

(b) Design an algorithm to help you quickly determine an escape route for a maze such as the one above given that you have a graph representation. What is the running time of the algorithm?

4. Consider an undirected graph G=(V,E) with nonnegative edge weights $w(u,v) \geq 0$. Suppose that you have computed shortest paths to all vertices from vertex $s \in V$. If you increase the weight of each edge in E by 1, the new weights $w'(u,v) = w(u,v) + 1$. Do the shortest paths change? Give an example where the path changes or prove that the shortest paths cannot change.