Greedy Practice Problems

Problem 1: The Waiter's Dilemma

Alice and Bob are the only two waiters in the Acme Restaurant. Today, the restaurant received N orders. The amount of the tip for each order may differ when handled by different waiters, if Alice takes the ith order, she would be tipped Ai dollars and if Bob takes this order, the tip would be Bi dollars.

In order to maximize the total tips for the night they decided to distribute the orders among themselves. Each order will be handled by only one person and due to time constraints Alice cannot take more than X orders and Bob cannot take more than Y orders. It is guaranteed that X + Y is greater than or equal to N, which means that all the orders can be handled by either Alice or Bob. Find out the maximum possible amount of total tip money after processing all the orders.

Example 1: Input N = 5, X = 3, Y = 3

Ai	1	2	3	4	5
Bj	5	4	3	2	1

Output: 21

Example 2: Input N= 8, X = 4, Y = 4

Ai	1	4	3	2	7	5	9	6
Bi	1	2	3	6	5	4	9	8

Output: 43

a) Give a verbal description and detailed pseudocode for an algorithm to find the maximum total tips.

b) What is the running time of your algorithm?

Problem 2: Suppose we have an alphabet with only five letters A, B, C, D, E which occur with the following frequencies.

Letter	A	B	C	D	E
frequency	0.35	0.12	0.18	0.05	0.30

Use Huffman coding to find the optimal prefix-free variable-length binary encoding of the alphabet.

(a) Draw a binary tree that represents the optimal encoding. (b) Fill in the table below with the binary encoding of each letter.

Letter	encoding
A	
B	
C	
D	
E	

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Problem 3: Work Schedules

You are the manager in a firm where the length of working time and the start time of work is different for different employees. For example, person X works every day from 8 to 11 AM, person Y from 9 AM to 1 PM, person Z from 2 to 10 PM etc. Your task is to create a workforce consisting of maximum number of people possible with non-overlapping work hours (if a person leaves at 7 AM and another starts at 7 AM, that is not an overlap).

You use the following greedy strategy:

a) Choose the person (say X) with least number of working hour over- laps with other workers.
b) Eliminate all people having working hour overlaps with X.
c) Choose the next person with the least number of overlaps with the remaining people, and so on.

Does this strategy yield a workforce with the maximum number of people? If yes, prove it. Otherwise, just give a counterexample.