Review: Exponents and Logarithms

Intro to Exponents:

1) Recall that $a^n = a \cdot a \cdot a \cdot a \ldots (n\ times)$

   →Example: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

2) For a^0, we define it as $a^0 = 1$.

   →Examples: $5^0 = 1$, $(\frac{1}{5})^0 = 1$, $e^0 = 1$

3) For a^{-b}, we define it as $(1/a^b)$

   →Example: $5^{-3} = (1/5^3)$

Operations of Exponents:

1) Multiplication : $a^m \cdot a^n = a^{m+n}$

   -To multiply two exponential terms that have the same base, add their exponents.

   →Example: $3^2 \cdot 3^3 = 3^{3+2} = 3^5$

   -Do not add the exponents of terms with unlike bases.

   →Example: $2^2 \cdot 3^3 \neq 6^{3+2} \neq 6^5$

2) Division: $\frac{a^m}{a^n} = a^{m-n}$

   -To divide two exponential terms that have the same base, subtract their exponents.

   →Example: $\frac{7^6}{7^3} = 7^{6-3} = 7^3$

-Do not subtract the exponents of terms with unlike bases

3) Exponents of Exponential Terms: $(a^m)^n = a^{mn}$

   -To raise an exponential term to another exponent, multiply the two exponents.

   →Example: $(2^3)^2 = 2^{2\cdot 3} = 2^6$

4) Products/quotients raised to exponents: $(ab)^m = (a^m b^m)$; $(\frac{a}{b})^m = \frac{a^m}{b^m}$

   - To raise a product or a quotient to an exponent, apply the exponent to each individual part

→Examples: $(2x)^4 = 2^4x^4 = 16x^4$;

Radicals are another form of exponents. Here's a helpful way to think about them:

$$\sqrt[n]{a} = a^{\frac{1}{n}};$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

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The Logarithm

If $\boldsymbol{b^c = a}$, then $\boldsymbol{\log_b a = c}$

A logarithm is just another way to write an exponent. If you want to find out what 5^2 is, you multiply two fives together to get 25. But if you want to find out which power you have to raise 5 to in order to get 25, you use a logarithm.

$$\log_5 25 = ?$$

The question you ask yourself when you look at this log is: To what power should I raise 5 in order to get 25? The answer is 2.

$$\log_5 25 = 2$$

Here's the general form of a logarithm:

$$\log_b a = c$$

The Common Log, the Natural Log and lg (log base 2)

- Logarithms can have any base (*b*), but the most common bases are 10, 2 and e.
- Logs with bases of 10 are called common logs, and often the 10 is left out when a common log is written.

  →Example: $\log_{10} 100$ is the same as $\log 100$

- Logs with bases of *e* are known as natural logs. The shortened version of $\log_e x$ is $\ln x$.
- *e* is a constant with an approximate value of 2.71828. Don't let it scare you... it's just a number.
- Logs with bases of 2 are often written as lg

  → Example: lg 32 = 5

Simplifying Logarithms

The following rules for simplifying logarithms will be illustrated using the natural log, *ln*, but these rules apply to all logarithms.

1) **Adding logarithms (with the same base)**

$$\ln a + \ln b = \ln(a \cdot b)$$

Two logs of the same base that are added together can be consolidated into one log by *multiplying* the inside numbers.

→Example: $\ln 5 + \ln 4 = \ln(5 \cdot 4) = \ln 20$

2) **Subtracting logarithms (with the same base)**

$$\ln a - \ln b = \ln(a/b)$$

Similarly, two logs of the same base being subtracted can be consolidated into one log by *dividing* the inside numbers.

→Example: $\ln 14 - \ln 2 = \ln(14/2) = \ln 7$

3) **Exponents of logarithms**

$$\ln a^b = b \ln a$$

If the inside number of the logarithm is raised to a power, you bring down the exponent as a coefficient.

→Example: $\ln 3^2 = 2 \ln 3$

4) **Change of Base**

$$\log_b a = \frac{\log a}{\log b}$$

5) **Things that cancel**
   - $\ln e = 1$
   - $\log_a a = 1$
   - $\ln 1 = 0$
   - $\log_a 1 = 0$
   - $e^{\ln x} = x$
   - $\ln e^x = x$

Limits as x goes to infinity

$$\lim_{x\to\infty}\frac{p(x)}{q(x)}=\infty, \qquad if\ p(x) grows\ faster\ than\ q(x)$$

$$\lim_{x\to\infty}\frac{p(x)}{q(x)}=0, \qquad if\ q(x) grows\ faster\ than\ p(x)$$

$$\lim_{x\to\infty}\frac{p(x)}{q(x)}=\text{c},$$
$$if\ p(x)\ and\ q(x) grow\ at\ the\ same\ rate\ \ and\ c\ is\ the\ ratio\ of\ the\ leading\ coefficients$$

Examples:

1. $\lim_{x\to\infty}\frac{x^5+3x^3}{10000x^2}=\ \infty$

2. $\lim_{x\to\infty}\frac{x^5+3x^3}{x^7}=\ 0$

3. $\lim_{x\to\infty}\frac{6x^5+3x^3}{3x^5}=\ 2$

4. $\lim_{x\to\infty}\frac{\log x^3}{\log x}=\frac{3\log x}{\log x}=3$